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PLACE OF MEETING.

PALMER HALL, COLORADO COLLEGE.

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ANNOUNCEMENT.

During the past year the *Colorado College Scientific Society* secured a considerable number of exchanges from other associations of similar character. It is hoped that this year the present list of exchanges may be greatly enlarged.

The past year has been a prosperous one for the Society. Its meetings have proved to be of great mutual benefit to its members. The following is a complete list of papers and reports presented to the society:

October 14, '90—

The Abandonment of Children in Ancient Greece and Rome. George L. Hendrickson.

Recent Researches in Magnetism. Florian Cajori.

November 14, '90—

Witchcraft Among the Hindus. Dr. H. W. Magoun.

December 11, '90—

Protection of Congressional Minorities. W. M. Hall.

Pulsations in the Aortic Arches of the Earthworm. Miss M. R. Mann.

Solidarity of the Race. J. M. Dickey.

January 13, '91—

Dialectical Studies in West Virginia. Dr. Sylvester Primer.

Men for the Hour. H. J. Barber.

February 10, '91—

Germ Theory of Disease. Miss M. R. Mann.

On Two Passages in the Crito. Dr. H. W. Magoun.

March 24, '91—

On van't Hoff's Law of Osmotic Pressure. (Published in the *Chemicl News* April 10, '91). D. J. Carnegie.

The Aryan Question. Dr. Sylvester Primer.

April 21, '91—

An Interpretation of the Fourth Gospel in the Light of Gnostic Philosophy. President William F. Slocum.

The Elliptic Functions Defined Independently of the Calculus. F. H. Loud.

The Study of Diophantine Analysis in the United States. F. Cajori.

May 12, '91—

Cross Ratio. B. E. Carter, Jr.

Calibration of Burettes. D. J. Carnegie.

June 9, '91—

On a Passage in the Frogs. Dr. H. W. Magoun.

Note on the Hadley-Allen Grammar. Dr. H. W. Magoun.

Historical Note on the Differentiation of a Logarithm. F. Cajori.

A Mathematical Error in the Century Dictionary. F. Cajori.

WITCHCRAFT AMONG THE HINDUS.

BY H. W. MAGOUN.

The mysterious always arouses curiosity, and the mysterious is no small element in magical practices. Witchcraft has never lacked its devotees. Its stronghold has always been among the ignorant and superstitious; but men of learning, and even eminence, for their time, have firmly believed in its reality, and have felt it a duty to search out and punish those who had dealings with the evil one. Even at the present time, in some parts of the world, people of intelligence still cling to a belief in the existence and power of witchcraft; its literature still flourishes among them, and its teachings are accepted as containing things which are true and worthy of respect.

While at the Johns Hopkins University, three MSS. were put into my hands which contained a practice called the Asurī-Kalpaḥ. The MSS. all belonged to the Atharva-Veda and were known as *pariçiṣṭas,* or supplementary writings. Their contents were as yet unknown. Two of the MSS. contained each a very brief outline of a rite which began with a spell; the third contained three versions of the same thing, one of them being much fuller than any of the others and partaking of the nature of a commentary. This MS. was really the key to the entire practice; but it presented enormous difficulties at the start, for the writing was something astonishing. Diligent comparisons of similar passages gradually established the sense of the greater part of the document, and an outline of it will be given below. The results of my work were published in the American Journal of Philology for July, 1889, under the title, The Āsurī-Kalpa; a Witchcraft Prac-

tice of the Atharva-Veda. In the preparation of the above paper various items of interest had to be passed over. They will form the chief part of the present article.

Born in a country where the excessive heat makes clear those words of the psalmist, "The sun shall not smite thee by day nor the moon by night," subject to many disorders of the digestive organs incidental to a hot country, surrounded by dangers from reptiles of which we know next to nothing, suffering from long continued drought, followed by terrific thunder and hail storms, and without revelation, the Hindus were particularly adapted to fall into the bonds of superstition. In its iron grasp they seem to have harbored no thought of inquiring into natural phenomena; and, while developing abstruse and subtle philosophical doctrines, keen grammatical insight, and a system of phonetics which is the basis of the science to-day, they remained utterly unscientific in all matters pertaining to the material world, and were perfectly helpless to explain correctly even the most simple process of nature. They looked upon the wonders of creation as children might, and in each new phenomenon they saw a living sentient being manifesting himself for the good or harm of the beholder. The cloud was a huge serpent keeping back the rain, the lightning was the weapon of Indra smiting the serpent and compelling him to allow the rain to fall, and an eclipse was an attempt by the demon Rahu to swallow the sun or moon. Agni or fire was a god born of two sticks. Rubbing the sticks together, they prayed earnestly for his appearance, and the resulting fire was regarded as an answer to prayer. If some doubter muttered spells instead, the spells were the cause of Agni's coming in his eyes and the step to the practice of witchcraft was a short one.

The use of spells or imprecations was accepted by the orthodox Hindus as proper in certain cases, and the Mānava-Dharmaçāstra states definitely that the hymns of the Atharva-Veda are to be used by a Brahman as a weapon against his enemies. The hymns of this Veda were also used

in the practice of medicine and were muttered over the patient while certain rites were performed with such materials as mud, gravel, curds, sour-milk, melted butter, and cow-dung. As Sir Alfred Lyall puts it, "We talk of a dose acting 'like a charm,' while the Hindu employs a charm to act like a dose." Brahmans were forbidden to practice medicine, unless compelled by necessity, however, so that even they recognized the uncanny element in it.

The faith in spells finally became so great that it was believed that even the gods must heed them when properly used. In the *Laws of Manu*, however, which were cited above, occurs the statement that witchcraft or *abhicāra* practices are secondary crimes; so that, while their outward forms, to our eyes at least, are so near alike that it is almost impossible to distinguish between them, the Hindus recognized a clear and sharp difference in the two. When the rites are used for harm, independent of, or in spite of, the gods, the practitioner is an enemy of the gods and a worker of black magic. Anything wrong in the community lies at his door and he must be punished, for he is destroying the power of the Brahmans' prayers by his arts.

If two sticks, with the help of a few spells would produce fire, the inference was that two knives crossed on a threshold or a red rag put over a door must harm the occupant of the house when imprecations were added. If the strange signs caused the person to tremble or turn pale, the concealed sorcerer was convinced that his intended victim was feeling his power and he believed in his charms accordingly.

The religious Hindus, while accepting these things as true, felt bound to propitiate their gods by an austerity painful to think of; but this asceticism was supposed to compel the gods to reward them *nolens volens*, so that the difference looks small to us in point of fact. The theosophic speculations of this class reached their culminating point in the teaching of the *Vedantas* which hold that there is one ultimate and only god,

unknown and unknowable, by whom they are finally to be absorbed. The sceptical Hindus naturally drifted into witchcraft, and the pretentions of the professional wizards are by no means modest. It is not an uncommon thing for a Hindu, when his own prayers and those of the Brahmans, hired for the purpose, are unavailing, to go to some practitioner of magic who rarely fails to promise all he wishes and does not scruple sometimes to even administer poison, if need be. There are stories of those who invoked genii; but did not know how to control them and were torn to pieces in consequence. That, however, matters little. The popular belief in witchcraft is so general that there is scarcely a town in central India of any size, says Lyall,* that has not a hereditary servant whose duty it is to ward off impending hail-storms by watching the motion of water in certain pots, muttering incantations and dancing about with a sword. This, however, is white magic, not black.

The sorcerer has an enemy in the witch-finder who volunteers, for so much money, to tell who has bewitched any sick or unfortunate person. The method called S a t a n e is as follows: The witch-finder sits on the ground with a branch of the Bale-tree opposite. Rice is handed him which he eats in the name of each village. When the name of the right village is mentioned, he throws up the rice. The families of the village are then treated in the same way; and, lastly, the individuals of the family which has been chosen as guilty. A sufficient sum will induce him to doubt the result and try it again. C h a r r e e n is a similar process. A stone is hung on a string with the villages, families, and individuals marked on it; the names are mentioned and the guilty one is selected by the vibrations of the suspended rock. The sorcerer who was thus detected had to put his tongue to red-hot iron nine times unless sooner burnt. In G o b e r e e n, water, oil, and cow-dung are mixed and brought

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* Sir Alfred Lyall, A s i a t i c S t u d i e s, from which many of these facts were gathered.

to the boiling point; a ring is then dropped in which must be found and brought out by the hand, while the person invokes the deity for protection from burning.

The use of fire has been believed in by nearly all races as a test for witchcraft, the supposition being that the deity protects the innocent by a miracle. The Hindus are no exception to the rule. One fire-test was to require the magician to carry two pounds and three-quarters of red-hot iron in the hands across seven circles, each sixteen finger-breadths broader than the preceding, and then throw it into a ninth, where it must burn some grass. If he is burned he is guilty of causing the sickness and must cure the person or die. The Mānava-Dharmaçāstra (VII, 108), says that when a witness falls sick or has a loss from fire within a week, it is because he has perjured himself in his testimony. The same law-book (VIII, 114-16, 190) says: 'Let the judge cause him who is under trial to take fire in his hand, or plunge into water, or touch separately the heads of his children and his wife, and he whom the fire burns not, whom the water does not reject from its depths, whom misfortune does not speedily overtake, shall have his oath received undoubted.'

The common water-test is to fix a bamboo rod in a pond, send the person to it, and require him to descend to the bottom while an arrow is shot and brought back by a runner. If he emerges before the arrow is returned, he is guilty. Another test is to put the accused into one sack, a stone in another, and throw the two tied together into running water. If the stone rises while he sinks, he is guilty. The simple original method which corresponds to the European, is to see whether the accused will float or sink, the latter indicating guilt. The punishment, especially as a result of the first method, is very severe. If the guilty person is a woman, she may be roped up to a tree, have a bandage of red-pepper tied over her eyes, and then be swung to and fro in the air, or she may even be beaten to death with rods from the castor-oil tree, which is supposed to be excel-

lent for purging witchcraft, simply because any slight indisposition that could be removed by the use of the oil, was attributed to the power of witches. Smallpox or cholera are due, they think, to the spells of witches also, since the gods do not stoop to these things, though they have them in their powers. It is the producing of evil results which makes the witches; for so long as magic is used for good ends and the gods are still served, the practitioners are not witches.

The literature of witchcraft in India has increased enormously, especially in the vernacular, and it is to-day the favorite reading of the people. The Āsurī-Kalpa may perhaps be taken as a fair specimen of this literature, and a brief outline of its contents may be of interest. In preparing for the āsurī-spell, a small, square piece of ground is first cleared, then a triangular hole, measuring about a span each way, is dug in the center, and the person puts on a red garment and adorns himself with garlands of the red oleander, and an ornament of sandal-wood, supposed to represent the feet of Viṣṇu resting on a lotus. He then lies down on a woolen blanket with his face to the east, toward which one angle of his three-cornered fire-pot points, and offers an oblation of sugar and melted butter, after which he mutters: "Oṁ, obeisance to Rudra, oṁ, O pungent one, thou of the pungent leaf, blessed Āsurī, reddish one, thou of the reddish garment, O daughter of Atharvan, non-terrific one, non-terrific wonder-worker, so-and-so smite, smite, burn, burn, cook, cook, crush, crush, so long burn, so long cook, until thou hast brought [him] into my power, Amen.*

* *oṁ namo rudrāya, oṁ kaṭuke kaṭukapattre subhaga āsuri rakte raktavāsase, atharvaṇasya duhite 'ghore 'ghorakarmakārike, amukaṁ hana hana daha daha paca paca mantha mantha tavad daha tāvat paca yavan me vaçam ānayaḥ svāhā.* The whisper-spell which is an extension of the first part of this formula, and consists mostly of mysterious particles, reads as follows:—*oṁ klīṁ hrīṁ çrīṁ kṣaūṁ*

For a woman the spell was somewhat longer, and a second brief one is added: 'For her not a lunar-day, not a lunar-mansion, not the kindling of a holy fire is decreed.'

Next in order comes a *nyāsa* ceremony, which is a sort of consecration service, giving obeisance to each pair of thumbs, fore-fingers, etc., and to the two palms and backs of the hands. It seems to be intended in this case as a propitiation of Rudra. * Durgā is then invoked, and mention is made of

kṣauṁ çrīṁ hrīṁ klīṁ oṁ, kaṭupattre subhaga asuri raktavāsase 'tharvaṇasya duhite 'ghore 'ghore svāhā, oṁ klīṁ hrīṁ, etc., as at the beginning. The Durga Puja by Pratápachandra Ghosha, contains a long list of these particles used in pouring water into a conch-shell as an act of worship. Keeping his system of transliteration, it is as follows:—Ksham, Lam, Ham, Sam, Sham, Sam, Vam, Lam, Ram, Yam, Mam, Bham, Bam, Pham, Pam, Nam, Dham, Dam, Tham, Tam, Nam, Dham, Dam, Tham, Tam, Nam, Jham, Jam, Chham, Cham, Nam, Gham, Gam, Kham, Kam, Ah, Am, Aum, Om, Aim, Em, Lm, Lm, Rm, Rm, Um, Um, Im, Im, Am, Am, Mam.

* The Durga Puja, which is a book giving an account of the religious worship of Durgā, contains such a ceremony, called the External Mātrika Nyāsa. Using the method of transliteration, etc., which is employed, it reads as follows:—The rshi of this Matrika mantra is Brahma, its metre is Gayatri, the deity Matrika sarasvati [Indus River], the consonants are the roots, the vowels are the Saktis [powers], these are used in the Matrika Nyasa. With a flower Om obeisance to Brahma Rshi in the head, Om obeisance to Gayatri Metre in the mouth, Om to Matrika Sarasvati Devi in the heart, Om to the consonant roots in the muladhara [root-holder], Om to the vowel Saktis in the feet. Then meditate on Matrika: Help me, O! goddess of speech, whose lips are parted by fifty characters [the letters of the Sanskrit alphabet], whose arms extend to the knees, whose bosom is well formed, whose forehead is emblazoned by the moon with all the phases, whose breasts are as compact as they are lofty, and whose hands hold the mudra [seal], the beadstring, a nectar-pot, and gift of knowledge, and who is white in appearance, and three-eyed.

Am to the forehead, Am to the mouth, Im to the right eye, Im to the left eye, Um to the right ear, Um to the left ear, Rm to the right nose [nostrils], Rm to the left nose, Im to the right cheek, Im to the left

her ninety million bodily forms. The person now indulges in a religious meditation (*dhyāna*), in which he worships the *āsurī*-plant as a goddess called the fearless wish-granter. There are three parts of this meditation to be used in the morning, at noon, and at night respectively. In the first, he thinks of the goddess as having a hook in her hand, as adorned with all ornaments, having a gracious countenance, and as seated on a serpent in the *padmāsana* position, *i. e.*, with the legs crossed, one hand on the hearth with the thumb up, the other on the left thigh, and the eyes fixed on a point near the end of the nose. The second is similar, but she has a sword in the hand and a half moon crest. In the third, she has a 'red-stone' in the hand, sits on a dead man, and wears garlands of *muṇḍā*-plant. A second version, given elsewhere in the MS., puts in a discus, a trident, a white serpent, a white bull, etc., etc., and describes the goddess as having three eyes, four mouths, a string of pearls in the nose, and so on. *

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cheek, Em to the upper lip, Aim to the lower lip, Om to the upper teeth, Aum to the lower teeth, Am to the cerebrum, Ah to the right shoulder-blade, Kam to the elbow, Kham to the wrist, Gam to the roots of the phalanges, Gham to the phalanges, Nam to the nails, Cham to the left shoulder-blade, Chham to the left elbow, Jam to the left wrist, Jham to the roots of the left phalanges, Nam to the left nails. Tam to the right heels, Tham to the right knee-bone, Dam to the right ankle, Dham to the roots of the phalanges, Lam to the tarsals. Similarly Tam, Tham, Dam, Dham and Nam to the several parts of the left leg. Pam to the right side, Pham to the left, Bam to the back, Bham to the navel, Mam to the stomach, Yam to the heart, Ram to the right shoulders, Lam to the neck bone, Vam to the left shoulders, Sam from the heart to the right hand, Sham from the heart to the left hand, Sam from the heart to the right leg, Ham from the heart to the left leg, Lam from the heart to the belly Ksham from the heart to the mouth.

* A meditation or prayer of the Durga Puja may be compared, with this, which reads as follows:—Om with locks of hair, braided and flowing, and the forehead ornamented by the crescent moon, with three eyes, with a face equal to the full moon in brightness, with a complexion of molten gold, well-formed and lovely eyed, full of the fresh-

After this contemplation comes the chief practice of the ceremony, the object of which, in the words of the MS., is the desire to destroy, the destruction of an enemy, the slaying, stupefying, making submissive and fixing like a post. The person takes leaves and seeds of the *āsurī** plant (black mustard) and grinds them into meal. An image of the person to be subdued was then made from this meal with melted butter, sour-milk, sugar (or honey), salt, or mustard-oil, according to the caste of the victim—the mustard-oil was for an enemy—after which the image was chopped up with some kind of a weapon and burned in the fire-pot with kindlings of different woods according to caste.

This practice was repeated one hundred and eight times—eighteen per day—beginning with six in the morning, continuing with six at noon, and ending with six at about sundown. The proper meditation was used in each case. The morning form helped in making submissive; that used at noon, in "fixing like

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ness of youth, decorated with all kinds of ornaments, with a set of pearly teeth, Devi, with a breast compact and full, gracefully bent at three places, Destroyer of the Buffalo Demon, with ten arms as soft and well rounded as the stalks of lotus, holding a trident on the right, a sword and a discus from upwards, a sharp arrow, and a dart in the right hand, a shield, a bent bow, a noose, a goad, and a bell or an axe. Under her lies a headless Buffalo carcass, whence rises the demon with a sword in hand, pierced in his heart by the Devi's trident, his lungs drawn out, his body besmeared with blood, and eye-balls distended encircled by the serpent noose, presenting a face made terrific by contracted eyes, brows, and frowns, the noose with the forelocks of the demon held by Durga, in her left hand, while the Devi's lion is described with a mouth tainted with red, the right foot of the Goddess resting evenly on the back of the lion, and the toe of the left foot a little higher on the shoulders of the demon. Om serene-faced Goddess, subduer of the pride of Daityas and Danavas! Om this representation of the Goddess is worshipped and prayed [to ?] by the immortals. * * * Om contemplate the mother of the world for the attainment of virtue, all wishes, and beautitude.

**āsurī* literally means pertaining to demons and the pungency of the plant probably accounts for the name.

a post"; and the third and last, in slaying the foe. It will be seen that it took just six days to complete the task, and at the end of that time the person was supposed to succumb.

This use of an image for such a purpose is to be found with some modifications among all peoples who have practiced witchcraft. Among the Romans it was an image of wax slowly melted, and in some parts of Europe to-day such an image is still used stuck full of pins. The Ojibway Indian medicine-men make a wooden image, put powders of some sort in a hole in the breast, mutter imprecations over it, and pretend by this means to transfer diseases from one person to another. The negroes of Virginia when suffering from rheumatism, or any malady that draws up the limbs, go to or send after a "Conjur Doctor," who is always a very old man. The "doctor" draws a picture of the supposed witch, pins it to an oak tree, loads his carbine with hair from the afflicted person's head, frog's legs, broken glass, meal, pepper, salt, and divers other things, and finally shoots the picture through the heart. This is supposed to kill the witch and at the same time heal the sick person. There are said to be cases in which the suspected witch has really died from fright, knowing that the picture was intended for her. A trick somewhat similar to those already given was even tried on Henry VI, of England, and early in the present century on the N i z a m of the Deccan.

The fact that images have been so universally used in witchcraft practices is no more remarkable than the fact that all nations have used them in religious worship. There is no discoverable connection between the Ojibway's wooden image and the Hindu's effigy of dough, other than the mere fact that each is the outcome of a desire to injure. Nature teaches both to think of what is practically the same expedient.

The *āsurī* rites do not end with this main practice, but there are several other similar ones with various objects, such as the production of epilepsy, boils, loss of sense, family discord, fever, eye-twitching, foolish or ridiculous action, a running behind the

witch's back by the person conjured, a running after senseless people, invincibleness, success in love, power to charm with a glance of the eye, also to obtain great treasure, to secure the growing up of one's sons, to make a kingdom submissive, to release a person from certain of the ill-effects already mentioned as obtained by the rites, also from obstacles, ill-luck, and the seeing of ominous portents. The MS. ends with the words: 'He is neither devoid of power nor destitute of children in whose house the divine (the goddess) Asurī is.'

It is a deep and interesting problem to determine the relation between religion and superstition in its various manifestations. The Asurī-Kalpa shows that, in some cases at least, witchcraft borrows the outward forms and symbols of religious practices, though it must be admitted that in India, certainly, the religious rites cannot claim to be much above the practices copied from them in the matter of superstition. Witchcraft also bears testimony to the universal belief of mankind in powers outside of and above ourselves, though it tacitly denies any divinity to them and seeks to control and use them for evil ends.

It will perhaps be unnecessary to go further into a description of the rites practiced by these charlatans of the East. Suffice it to say that, like the fortune-tellers and spiritualists of America, their own personal gain enters largely into the account in every case, and, furthermore, that they take advantage of the universal belief of mankind in some supernatural power to induce skeptics in religious matters to come to them and pay them for gaining by magic blessings for their patrons, which others get by prayers, or for producing evils to their fellows which their religion would not countenance, save as a means to do good for their gods or the Brahmans.

PROTECTION OF CONGRESSIONAL MINORITIES.

(Abstract of a Paper read before the Society.)

BY W. M. HALL.

I.

Abuse of the technical powers of majority and of minority has not in any state legislature gone permanently beyond the control of public opinion. But the House of Representatives at Washington is notorious for frequent straining of such powers, and there is little reason to expect a return to moderation without material changes in parliamentary law. Blame of one party or the other is useless towards finding a remedy. The trouble is old, and its marked increase in the present Congress is due chiefly to the possession by one party of all three legislative elements. Before, when the President or the Senate was hostile, the majority in the House had less temptation to ride rough-shod, and the minority could leave the defense of its interests to its allies beyond; repressive or obstructive abuses, when they did occur, were often caused by the formation, on particular measures not strictly partisan, of temporary majorities alike in both houses, either in sympathy with the President or thinking themselves strong enough to over-ride his veto. But now the motives for parliamentary misbehavior on both sides are kept permanently alive, and the descent is rapid.

When troublesome obstruction has become frequent, the majority always does something to preserve its right to legislate; but always hitherto by partial destruction of rights valuable to the country—the minority rights to debate, to offer amendments and have them put to vote, to make a conspicuous

protest, and sometimes to delay action till public opinion can be roused. Unquestionably the majority right to legislate is still more valuable. Is there no way to save it but by cutting down the other set of rights?

The present agitation over breaking and counting quorums is important only in so far as the minority has been driven from other fields of dilatory action, and that subject may be neglected if means can be found of restoring the more legitimate minority rights.

II.

The demoralization of the House, extending through many years, has been accomplished by a series of retaliations. Whether it be held that the minority first abused its rights of debate or of using parliamentary motions, (even dilatory use is fair, when not too frequent), or that the majority began to shut off debate and motions not meant to be dilatory, each development of obstruction has been met by new repressive rules or by more merciless use of old ones, and this in turn by more frequent exercise of such powers of obstruction as remained. The result is that the minority is disabled from giving the valuable services of a minority; for the more doubtful the merits of the legislation in hand, the more eagerly the majority uses its silencing powers, which are now great and effectual. We have the absurdity of a system that makes an important right impossible when it is most needed; and the absurdity will doubtless last as long as the majority can decide, from day to day, how often and on what subjects the minority can use its nominal privileges.

It is pointless to say that the minority deserves its punishment. The minority is not continuous. In each Congress it suffers for the deeds of former minorities of both parties. It is not the right of the minority as such, but the right of the whole people to have the proper services of a minority, that needs protection. It is good for the state that the minority should in some principal cases (1) make a conspicuous protest, and rouse public opinion for expression at the next election; or, (2) gain

time for contemporary opinion to work upon the majority and perhaps check its action. It is no small matter to lose these benefits. Yet under the present system the incompatibility of majority and minority rights is real, in a demoralized House. There must be some limit of frequency to the obstructive powers of the minority; and if the limit is applied by the majority, it will be used too often, and most sharply used at just the times when it ought not to be used at all.

The root of the trouble is that the majority controls the limit of frequency. If the control can be lodged elsewhere, the minority can perhaps be restored to usefulness without recovering power for mischief. To give control to the Speaker is a mere formal change, unless the speakership were made non-partisan. To create a separate officer of the House, to decide when the minority can properly use dilatory powers, is similarly impracticable. But there is another resource—the rules themselves. If the minority can be given some effectual right, to be used only a certain number of times, the limit is assured, and the minority itself can be left to choose the critical moments for using the right.

III.

Suppose a rule of the House giving to six or seven members of the minority each an absolute right to demand, twice during his term, four hours' speaking for his own side, with the privilege of naming the members to use the time, while the Speaker retains power to lengthen the debate by recognizing other members. A right to propose three amendments and require a vote on them might well be added. The members to be armed with these exceptional rights could be chosen by mere written nomination, (any twenty members naming one), or they could be officers of the House elected under a minority-representation rule of ballot. In either case the majority would have the same privilege, in name, without motive to use it.

The minority would then have half a dozen responsible men, chosen by their own caucus, able to secure in all, during the two

years, about fifty hours of debate at the most useful times; an allowance small enough to discourage waste of it, and large enough to give opportunity for a conspicuous protest against a dozen of the bills most objectionable to the minority. Each use of such a power would itself call public attention to the protest. At other times the majority would retain its present powers of forcing business forward rapidly. The right to legislate and the right to protest would no longer be incompatible.

It remains to provide means for saving the other function of a minority, of forcing the majority sometimes to halt long enough for adverse public opinion to act. Most of the bills beaten in that way are really bad. Mere tiring out the majority has no like claim to approval, and we need not regret seeing it made difficult. But most of the true virtue of delay by obstruction can be kept when general obstruction is abolished. Suppose, again, that each member who carries the special right to claim debate is further privileged once in his term to postpone arbitrarily for two weeks the final vote on passage of a bill (or of a resolution changing the rules or unseating a member); no repetition of the postponement admissible. The chance of an exercise of this right in the last two weeks of a session would put no unjust hardship on the majority; they must bring partisan measures to a vote before the last fortnight, or obtain a two-thirds vote to suspend the rules afterwards. The postponed bill could meanwhile (by a slight change of Senate rules) be considered by the Senate, go through conference committee, be passed by the Senate, and return to the House to await its fate on the postponed vote. But this is not an essential change; the bill could be left absolutely suspended through the fortnight.

The minority could thus take a fortnight's grace on six or seven important measures within two years, for the sake of bringing outside pressure to bear on the majority; and would do it without wasting any time of the House.

IV.

Such innovations need careful adjustment to the existing rules of the House; but they seem to carry no inherent contradiction such as has been developed into mischief by the working of the present system. One considerable objection is visible: the majority might divide into several short bills any measure on which they expected resistance, or bring in several entire bills with minor variations, hoping thus to exhaust rapidly the limited minority privileges of debate and postponement. There is perhaps no direct preventive of that trick, but in practice it might disappoint the majority. They would be crowding their own calendar and obstructing their own business; foreseeing that, they would usually bring in these families of bills early in the session, to insure their progress through committee. The plan being thus exposed, the minority could choose one bill, the worst of the group or the first one coming to a vote, and make their stand on that. The privileged debate would, as protest, be effectual in the public mind on the whole series, and if the privileged postponement killed the bill selected for protest, the same public opinion which drove the majority to retreat would go far towards stopping the other fractional bills.

It is not claimed that these changes in the rules would create a perfect relation between majority and minority. But the present system is full of abuses; abuses so connected that the abatement of one aggravates another. A new principle is needed, and apparently the only one that can loosen the deadlock is one which gives the minority some privilege, far-reaching when used, that can be used only a fixed number of times, the times being chosen by the minority.

There is more reason to look for a good working device, because the comparatively healthful condition of the Senate will not last if the House becomes a quick-cutting tool of the majority. Obstruction will be habitually transferred to the Senate; restraint of debate there will follow, and the downward course of

the House will be imitated. The process will doubtless be suspended when the President and majorities of both Houses are not of the same party; it may even be reversed, and the House itself brought back to moderation by some unexpected force of public opinion. But it is not prudent to stake the future of minority rights on such a chance.

PULSATIONS IN THE AORTIC ARCHES OF THE EARTHWORM.

BY M. R. MANN.

Anyone working with the common earthworm has probably noticed the extreme irritability of the animal at the touch of the hand, due to the rapid evaporation from the delicate cuticle surrounding the worm. Observing also that the pulsations in the aortic arches increased with the irritability, a few experiments were tried to note the effect of temperature upon the pulsating arches of the common worm.

In the lowest animals the nutrient material resulting from digestion is distributed in the same manner as in the cell. The simplest form of a vascular system is found in the Cœlenterates, in which the digestive cavity occupies nearly the entire space of the animal, and thus distributes the nutrient material.

When a distinct alimentary tract is developed, the nutrient fluid passes through the walls of the tract, into the cœlum, and thus gives rise to the fluid, the blood. In this space the blood circulates, the movement being effected chiefly by the muscular actions of the body walls.

At a higher stage, a rudimentary central organ of circulation appears, and the nutrient fluid is confined in special pulsating vascular trunks, by means of which the fluid is forced through the system.

In the common earthworm a completely closed vascular system is present, provided with pulsating trunks. A dorsal longitudinal trunk, and a central longitudinal trunk can be distinguished, the two being connected in the anterior end by

large pulsating aortic arches. These arches are sometimes called "primitive hearts."

These experiments were performed with large strong worms designated as A. B. C. The temperature was raised or lowered as seen by the table with the following results:

1. Change of temperature causes great irritability.
2. After a short time the worm becomes accustomed to change of temperature, and the aortic arches tend to revert to the original beat.

See specimen A, temperature 19°; beats 14 16 14.
" A, " 10°; " 8 11 8.
" B, " 10°; " 8 10 9½.

3. The pulse beats with greater force at a low tempearture.
4. Tendency to same rate of beat in same temperature.
5. The upper limit is reached at 35°, when the pulsations cease.

TABLE SHOWING PULSATIONS IN THE AORTIC ARCHES OF THE EARTHWORM.

Time.	Temperature C.	Beats per minute.		
		A.	B.	C.
A. M.				
10.40	19	11	18	20
10.45	19	16	16	14
10.52	19	14	14	14
11.15	10	8	8	10
11.20	10	11	10	9
11.30	10°	8	9½	9½
11.45	5	7	6	6
11.50	5	7	6	6
11.55	5	7	6	6
12.15	12	11+	12	10
12.35	22	26	22	19
12.50	29	28	26	30
P. M.				
1.07	35	0	0	0

DIALECTICAL STUDIES IN WEST VIRGINIA.

BY SYLVESTER PRIMER.

In studying the language or pronunciation of any section of the country, it is necessary first of all to trace back the history of the people inhabiting it to the earliest beginnings in order to explain understandingly the dialectical peculiarities of its grammar or pronunciation. I shall, therefore, preface my remarks on the linguistical peculiarities of this region with a brief sketch of its earliest settlement and later development.

The early history of Western Virginia, now known as West Virginia, begins a century later than that of Eastern Virginia, or Virginia proper. In 1710 Alexander Spotswood, a Scotchman, was the deputy-governor of the Colony of Virginia. In 1716 he "gathered a party of the choice spirits of the Old Dominion, and set out on an exploration of the country beyond the Blue Ridge and Alleghanies, advancing as far as the fertile fields of Kentucky." As far as we know, he was "the first white man to enter the Great Valley, which was soon thereafter occupied by large numbers of Scottish, and some German and English settlers." In 1774 Virginia purchased from the Indians the right to make settlements to the Ohio, and built a fort where Pittsburg now stands. In 1752 Robert Dinwiddie, then deputy-governor of Virginia, began active relations with the great western country. "He encouraged trade and exploration with this region, and the Virginia traders swarmed across the mountains to traffic with the Indians, and there met with the French, which finally led, as you remember, to the attack on Fort Duquesne (now Pittsburg) and Braddock's defeat. At about this time the Ohio Company of Virginia began to take

steps to settle the western region, and encouragement was given both before and after the revolutionary war to settlers in this region." In 1738 Augusta was the frontier county, and then extended westward indefinitely. To the north lie Rockingham, Shenandoah and Frederick counties. Nearly all this region was settled by Germans and Swedes. "A Swedish congregation was here collected, and the Rev. Peter Muhlenburg, son of the Rev. Mr. Muhlenburg, father of the Lutheran Church in America, was sent to take charge of it." To the south lie Rockbridge, Botetourt and Montgomery counties. Botetourt became in 1769 the western frontier. From here the emigrants pushed forward to the westward, and soon settled the whole of what is now West Virginia.

History tells us that the Presbyterians were first in this field, and the fact that the people are of Scotch and Scotch-Irish descent would lead to the same conclusion. But it must not be forgotten that a greater part of the population of West Virginia were emigrants from Maryland and Virginia. A comparison of the names also will indicate in a general way the national characteristics of the inhabitants, and show whether there has been an intermixture of outside elements with the original settlers. However, too great stress should not be placed on mere names, as they might not always show the true nationality of the section from which their bearers came. Yet they are not to be discarded altogether, but must be taken into account in dialectical studies. They will show approximately whether the population has remained pure from the earliest times. On examining the names of the vestrymen of the earliest Episcopalian churches of this region, my only source of information, I find among the settlers from Eastern Virginia such names as Ballenger, Maury, Burton, Scott, Rucker, Godwin, Taliafero, Cabell, etc. Among the Scotch and Scotch-Irish names can be mentioned Balmaire, Quarrier, Dunlap, etc., while Bittenger, Swearingen, Muhlenburg, show a sprinkling of German and Swedish names. The names of the present day afford the same testimony and

show that the earliest settlers are fairly well represented by the present inhabitants. I am well aware, as above remarked, that this is not always a safe guide, but may, like tradition, sometimes mislead; still, in lieu of a better one it renders tolerably efficient service.

In the earlier days of these settlements the educational advantages were naturally slight, but later the conditions for education were about the same as those described in my article on the Pronunciation of Fredericksburg, Va., printed in the Publications of the Modern Language Association of America, Vol. 5, p. 188ff. In the same article (pp. 192–195) I have compiled two Vowel Tables, one representing the vowel sounds of the Virginia English of the seventeenth century, and the other the vowel sounds of to-day. In the present article I shall constantly refer to them.

I will here begin with the vowel *a* as heard in *father* (Sweet's mid-back-wide).* Here, as in Fredericksburg, Va., we find the clearer, lighter sound of *a* as in *calm*, *psalm*, *palm*, *half*, etc. The other sound of these words (*i. e.*, kœœm, sœœm, pœœm, hœœf, etc., that is, Sweet's low-front-wide) is heard, though less frequently than in Fredericksburg, Va., or in Charleston, S. C. This sound of *a* in *father* (mid-back-wide) is also very common in words like *ask*, *demand*, *pass*, *trespass*, etc., though the tendency to the palatal *a* is strong. The two words *ant* and *aunt* are both generally pronounced (œnt, that is, low-front-wide), though the latter is often pronounced (aant, that is, mid-back-wide). The same remark applies to words like *gaunt*, *haunt*, *jaunt*, etc., where Sweet's low-front-wide (= *a* in *man*) is commonly heard, thus (gœœnt, Hœœnt, Dzhœœnt, etc). Occasionally one hears the mid-back-wide (= *a* in *father*); that is phonetically represented (gaant, Haant, Dzhaant), but I have never heard Sweet's low-back-narrow-round (= *a* in *law*;

* The signs of Sweet's Primer of Phonetics are used in this article, except in quotations.

gAAnt, HAAnt, DzhAAnt, etc.) Among all classes here, and especially among the uneducated, the mid-back-wide sound of *a* is retained in a large class of words where it either reflects the older pronunciation or shows the influence of the negro element. The negro is very fond of this *a*-sound, but I am in doubt whether it is natural to him or whether he may not have acquired it in early times from the whites themselves and retained it pure and uninfluenced by the change which this vowel has undergone in the progress of the language, just as the Irish have retained the older pronunciation of English. According to Elllis, E. E. P., this was the usual sound of the vowel *a* in the sixteenth century. In this list we find words like *clear*, *pair*, *there*, *where*, *fair*, *learn*, *prepare*, *queer*, *bear*, *square*, *were*, *rearguard*, *search*, *swear*, etc., in all of which the mid-back-wide is heard (klaar, paar, dhaar, whaar, etc.); we seldom hear the low-front-narrow (dhær, etc.) as in Charleston, S. C., but more usually the low-front-wide (= *a* in *man*). I am inclined to think that this sound is midway between the low-front-wide and the low-front-narrow. The negro pronunciation of *here* is (Hj'ar). Among the white population two pronunciations obtain: both ('ji'r) and ('ja'r) are common.

Under Sweet's mid-front-wide (our *e* in *met*, either long or short) we must class one peculiarity not yet noticed elsewhere, though found in England, viz., the pronunciation of the word *make* as mek, that is, mid-front-wide instead of mid-front-narrow. In the 17th century we find the same sound in England in the words *main* (meen), *major* (meedzhär), *mayor* (meer), *naked* (neeked), *nature* (neetɒr). Dryden has *pains of hell* (peenz of Hel) and (mee) for *may*. Garth has *distress* rhyming with *place* (plees). In the 16th century this word *make* was pronounced (maak), that is, Sweet's mid-back-wide

as we see in Shakespeare's Henry VIII., in the speech of Cardinal Woolsey to Cromwell:

> Neglect him not; make use now and provide
> For thine own future safety.

which according to Ellis, E. E. P., 3,991, is to be pronounced

> Neglekt H*i*m nɔt; maak yys nau and provaid
> For dhain *oo*un fyytyyr saaft*i*.

The Anglo-Saxon form is *macian* where this vowel has the short sound of *a* in *father*, that is, Sweet's mid-back-wide. In Mid. Eng. the form is still *makien* and retains this mid-back-wide sound. But in 1766 Buchanan in his conjectured pronunciation of Shakespeare has: "M*ee*d tu H*i*z m*i*str*i*s aibrau, and Kenrick, 1773, giving the pronunciation of the same passage has: "M*ee*d too H*i*z mistris aibrau." Both agree in the pronunciation of (meed) for *made*, so that this mid-front-wide sound of *a* in the verb *make* must have obtained in England to some extent in the 18th century. Benjamin Franklin, also, in his remarks on pronunciation in 1768 indicates the pronunciation of *makes* as (me*e*ks). I am also informed that in at least two counties of England, Lancashire and Derby, the pronunciation (me*e*k) is still heard.

The character *e* in the word *well* has a sound between Sweet's mid-back-wide (*father*) and low-front-wide (*man*), but inclining to the latter (wɒl), possibly Sweet's low-back-wide, Swed. m*a*t. The word *ancient* belongs also to this class as it is here often pronounced (*a*nshent) Sweet's mid-back-wide again. In addition to the two pronunciations (*a*gen, *a*genst) and (*a*geen, *a*geenst) we find the pronunciation (*a*g*i*n, *a*g*i*nst), though only as vulgarisms. *Fornent* (*fornenst*, pr. frn*e*nt) may have led to the pronunciation of (b*i*-j*e*nt).

Sweet's mid-back-narrow (*but*) is very common to this region. It is what Sweet and Ellis call the American sound of the *u* in *but*, not the English. Here belong words like *took*, *look*, *cook*, *shook*, *book* (?), *put*, and others, all of which have nearly the sound of our *u* in *but*. A sound between Sweet's mid-mixed-

wide-round (ö, Fr. homme) and his high-mixed-wide-round (ü. Swed. *upp*) is heard in *could, would, should;* in *cool, good, school, whŏ*, though preceded by the i-umschlag (k*iud*, w*iud*, sh*iud*; k*iul*, sk*iul*, H*iu*). I hardly know whether this sound is to be attributed to the Scotch element among the early settlers or not. It certainly comes very near the Scotch sound as heard in guide (g*iud*). It is, however, possible that it developed on this soil independently of the Scotch influence. This pronunciation is also peculiar to Fredericksburg, Va., and is heard in various places in the state of South Carolina. In the Upland region we have also a Scotch influence to some extent, so that even here it may be due to this element. I have noticed it moreover in other localities in the pronunciation of individual people. This sound of *u* in *but* is often heard in the pronunciation of *careless*, thus (karl*e*s). *Care* itself is often pronounced either (k*i*r) but more often however (k*e*ar). Here belongs also that peculiar pronunciation of *room* (r*ium*) and *tomb* in rhyme with *perfume* (t*ium*, pärf*ium*).

In studying the pronunciation of Fredericksburg, Va., I noticed two cases of the short (i) in the words (hill) and (mill). In West Virginia I found another example of the same sound in an individual pronunciation of the preposition *In* (iin). The word *ear* is often pronounced (jiir or even (jär). *Mischief* is accented on the ultima and pronounced (mistshiif). The past participle of *hear* is pronounced either (Hiirn or Härn; or Hiird or Härd), according to the form used. The sound (*i*) and (*e*) are often interchangeable, as (l*e*d) for *lid*, (r*e*d) for *rid*, (*e*f) for *if*; (g*i*t) for *get*, (j*i*t) for *yet*, (jistärde*i*) for *yesterday*, (k*i*tl) for *kettle*. To these we may add (dzhinëreshan) for *generation*, (sper*i*t) for *spirit*, (resër*i*kshan) for *reserection*. The word *muskmelon* is here often pronounced (maskm*i*ljan), which pronunciation goes back as far as 1685; for Cooper, in his list of words like and unlike, gives *melon*, melo, *million*, 1,000,000 sive centum myriades, which would indicate that the two words were nearly alike in sound. In *miracle* the *i*-sound often follows the

analogy of vowels before *r* and we hear not infrequently (mar*i*kl).

I feel convinced that we hear the open *o*-sound (Sweet's mid-mixed-wide-round=ö, Fr. encore) in the word *poor* (pr. pöör), and we also hear the long *o*-sound (Sweet's mid-back-narrow-round=o, Germ. so; pr. poor). The former is the same sound we often hear in the last syllable of *fellow* and *follow*, though the latter is the more common sound. The two words *born* and *borne* are both pronounced alike (bɔrn). *Forward* is frequently pronounced (färärd). The words *only* and *onhitch* (*unhitch*) belong here, as they are often pronounced (*ö*n-l*i*, *ö*n-h*i*tsh).

I can here repeat my remarks on the diphthongs in my article on the pronunciation of Fredericksburg, Va. "The sound (au, as in German Haus) is heard among a select few in *house*, *now*, etc., though the usual pronunciation is here (eu), never (əu). This latter diphthong (eu) is long (eeu) in *town*, *cow* and some other words, and short (eu) in most words, as *house*, *out*, *about*, *south*, *pound*, etc. Often (EEə) is heard instead of long (eeu), and (Eə) instead of short (eu). The diphthong (iu) is very common and the first element is often lengthened (iiu). Sometimes, however, the vanish is prolonged (iu^{u}). Instead of (iu), (*i*u) is often heard, especially among the lower classes. *Fruit* may be classed here also, or the sound often comes nearer the Swedish *u* in *hus* (frUt), or (yw). The same sound seems to be peculiar to people from the middle and upper parts of South Carolina." The diphthong (ai) is often changed to (oi), as in *title* (toitl). On the other hand (oi) often becomes (ai), as *boil* (bail), *joint* (dzhaint), etc., but this is a vulgarism common to all parts of the country. The word *ewe* (jiu) has frequently the pronunciation (joo), a pronunciation very common in Western New York. We also find (rai-at) for (rait), that is, *right*.

The consonants offer a few peculiarities. The *h* is often followed by the *j*-sound in the word *here* (Hj*ee*r); in this case the *h* frequently becomes silent, or rather a mere breathing ('j*ee*r).

W is exchanged for *v* in *very* (weri) and a few other words. The *r* is heard here more than in other parts of Virginia, but is often silent when final. We occasionally hear (kjart, gjardn, etc.), but not so commonly as in Virginia proper or in Charleston, S. C. After *s* a *t* is often added, as *close*, pr. *clost*. The *g* disappears in words like *length*, *strength*, etc., which are pronounced (lenth and strenth, etc.). The *t* between the *s* and *l* of words like *apostle*, *epistle*, etc., is sounded.

The accent of words is often changed, either as a general rule or by individuals. *Idea* frequently has the accent on the first syllable (aídi, or aídië). *Mischief* often transfers the accent to the last syllable (mistschiif), though this is considered a vulgarism. *Difficulty* sometimes has the accent on the antepenult (di-fik-al-ti). *Trespasses* sometimes takes the accent on the penultimate (tres-paś-sez). *Contrary*, when it means perverse, froward, wayward, always has the accent on the penultimate (k,)ntreéri). *Elizabeth* often has the principal accent on the ultima (IlizEbéTH). *Gethsemane* is often accented (Geth-se-meén).

Turning to the grammatical peculiarities we find the greatest variety in the verb. The tendency here is to form peculiar past tenses and past participles. Often one is exchanged for the other without any apparent reason. This is especially the case with the irregular verbs. The following list contains all those which I have observed:

1. Blow	blowd	blowd
2. Climb	climm or clomb	climm or clomb
3. Fight	fit	fit
4. Freeze	frozed	frozed
5. Hear	(Hiird), (Hiirn), (Harn)	(Hiird) heard
6. Heat	het	het
7. Help	helped (holp, pr. Hop)	helped (holp)
8. Know	knowd	knowd
9. Ride	rid	rid
10. See	saw (seen, see, seed)	seen (seed, saw)
11. Take	took (taken)	taken (took)

I find also a few lexicographical peculiarities which I shall give promiscuously, as it is not possible to arrange them in any definite order. *Reverent* is used in the sense of *genuine, thorough,* as *a reverent scolding,* that is *a thorough scolding. Satisfactual* is a vulgarism for *satisfactory. Shoot* is very common for *shot,* as "he made a good shoot." *Arter* for *after* is common all over the land. *Bold* is used in the sense of *strong, vigorous,* as *a bold spring* is one whose waters bubble up strongly. *A bunch of cattle,* is the only proper expression here in the West, but I never heard it in the East except in West Virginia. Webster's International, and the other dictionaries do not give this meaning for the word, though the Century gives *a bunch of ducks.* May not this meaning have started in West Virginia and passed to the West? *Gradjate* and *sosation* are vulgarisms.

An amusing popular etymology is found in the name of one of the valleys on Indian Greek. It is known as the Tuckahoe valley, and takes its name from the Indian tribe of that name, or it is at least an Indian name. The people living in this valley are of the lowest class, and have a peculiar dialect of which I have already noticed the most prominent features. Not being able to explain the word Tuckahoe they have based the derivation on the peculiar pronunciation of the past participle of *take* (took, p. tak), and *ahoe* is then made to mean *a hoe,* "he took a hoe." To this derivation the following legend has been attached: An inhabitant of this valley once became so poor that he was at last reduced to stealing, and *he took a hoe,* that is, stole a hoe. This will compare favorably with the English corruption of the name of the ship *Hirondelle* into *Iron Devil,* or the *route du roi* into *Rotten Row,* or *Bellerophon* into *Bully Ruffian,* etc.

Kittering means toppling, *afore* stands for before, *transits* means, not *transits of the planets,* but *transient guests* at a hotel. One minister, a hardshell baptist, or Ironsides as they call this sect there, spoke of the *texes* from which he preached

his sermon. I need not add that he was from Tuckahoe valley. *Slick* is used in the sense of *slippery*, *beegum* is used for *beehive.* The Century Dictionary gives the word; it was at first the body of the gum tree hollowed out and used for bees. A larger section, hollowed out in the same way, is used for a grain receptacle, and is called a *gum.* *A band of music* is called *musicioners.* In Fredericksburg, Va., I found this summer the word *burr* meaning gherkin, and have not yet been able to discover any explanation of it. The name may have some connection with *burr-weed*, though I doubt it. *Optionary* is an individualism for *optional*, though having the force of *capricious. Pert* (pr. piirt) is used in the sense of *well*, as *I am feeling right pert to-day.*

The region is full of peculiar expressions, and the careful collector would be well repaid for his trouble. Like the proverbs, the quaint sayings, the peculiar expressions of a nation form an interesting chapter in its history, and give a better insight into the distinguishing characteristics of a people than long years of its civil history. Often thoughts of ages are crystallized in such expressions and the study of intellectual growth and civilization of a nation cannot be pursued more effectively than by collecting and classifying its apothegms. But it is not my intention to enter so deeply into the subject, as I have given most of my time to the peculiarities of pronunciation and grammar. Besides, it would require years of study to collect all these expressions and put them in their proper order. I shall here select only a few of the most amusing and peculiar ones to show you what a rich field for such researches this section of the country affords.

A right smart little bit is extremely common, and *right smart seemingly* may find a place beside it. *I had laid out to go to the Dunkards to-night* is a not infrequent expression; of the same signification are *to go to do* and *to aim to do.* *Let on* is common nearly in the whole country; and so is *to get shet* (*shut*) *of.* *I feel like she did not do it* meaning *I think she did not do it* is

often heard among all classes of people. Some of the more amusing ones are: *I feel rather dauncy* meaning *I feel rather poorly.* Again we have a popular derivation for *dauncy*, which is rather more expressive than elegant. I have been told soberly by different ones that it is a contraction of *damn sick.* The dictionaries give no information on the subject. As an explanation of this word I would suggest, though with great hesitation, the French word *dancetté*, or the more common form is *danché* (*denché*), from which we have in English the two words *dancetté* and *dancy.* The great trouble is with the meaning. Both words are terms of heraldry, descriptive of *escutcheons* having the edge or outline broken into large and wide zigzags. The real meaning of the word is *indented*, and it probably stands for *denté.* Possibly the idea of being *broken* or *notched* like the teeth of a saw may have been applied figuratively to physical nature and would certainly have as much sense as the *broke bone* ever so common in malarial districts, and so dreaded. The most amusing expressions, however, were those heard in an Ironside sermon. But it would be impossible to reproduce them as they would lose their flavor if not delivered by the minister in person.

Often these expressions are of grammatical nature and deserve a careful study. Here are several of that character: *Would you rather have this as that?* though I have heard *as* used for *that* after the conjunction *than* in various parts of the country. A good old construction is retained in the following: *With the blood a-runnin' down his side.* This corresponds to, *he lay a-dying*, etc. Another (bible) construction is retained in this: *They looked for to see him die every day.* *For to* instead of *to* is now obselete. The double comparative is not infrequent here. I noted *more pleasanter* on several occasions. In expressions of weather I find the following in my note-book: *To fair off* meaning *to clear off; to have a cloud* meaning *to have a shower.*

THE STUDY OF DIOPHANTINE ANALYSIS IN THE UNITED STATES.

By Florian Cajori.

I. *Introduction.*

The term "number" was used by the Greeks in a restricted sense. With early mathematicians, as the Pythagoreans, it was never used except for integers. With them even unity was no number: a collection of units only was designated by that name. Fractions were looked upon as merely ratios of numbers. Later Greek authors, particularly Diophantus, enlarged the use of the term so as to include fractional numbers; but never were irrationals classified as numbers by any Greek mathematician.

The oldest extant work on algebra is the one of Diophantus. He took pains to exclude from his book the methods and conceptions of geometry which before his time had been extended by the Greeks even to arithmetic. Since irrational quantities were not looked upon by the Greeks as numbers and since, to their minds, they represented only lines, surfaces, and solids of definite dimensions, they could not be admitted into the algebra of Diophantus. According to him, algebra could be built up independently of geometry only by the exclusion of irrational solutions. Hence arose the condition, imposed by him, that solutions should be given in numbers.

The term "Diophantine Analysis" is sometimes used in a wrong sense. An equation, $ax + by = c$, in which only integral values of x and y are to be found, is frequently called "Diophantine." But the solution of this equation was unknown to Diophantus; nor did he ever propose to himself such a problem. He always imposed the condition that the results be rational,

but never that they be exclusively integral. The distinction between the modern Theory of Numbers and Diophantine Analysis has not been observed by American and many European writers; and we find it therefore most convenient, in this article, not to insist upon this distinction between the two.

The leading characteristics of Diophantus's solutions of indeterminate problems is the startling ingenuity and dexterity displayed, the lack of general methods, and the failure to detect multiple values in the answers.

II. Work in the United States.

The study of Diophantine Analysis was introduced into the United States by Robert Adrain, a mathematician whose work in one department of mathematics has not been sufficiently appreciated. He contributed to the *Mathematical Correspondent*, the earliest American journal of mathematics, started in 1804 in New York city, an essay, entitled, "A View of the Diophantine Algebra," in which he gives a general discussion of principles. He contributed later on the same subject to the *Analyst*, of which he himself was editor.

One of Professor Adrain's pupils, William Lenhart (died in 1840 at Frederick, Maryland), devoted much of his time, year after year, to the study of Diophantine Analysis. He was a cripple, and pursued mathematics for pastime. Frequent contributions were made by him to the *Mathematical Diary* and to the *Mathematical Miscellany*. In the latter were published "Useful Tables Relating to Cube Numbers." The editor states that besides these tables, the manuscript compiled by Mr. Lenhart with so much labor and care included a table containing a variety of numbers between 1 and 100,000 and the roots, not exceeding two places of figures, of two cubes, to whose difference the numbers are respectively equal; together with another table, no less curious, exhibiting the roots of three cubes to satisfy the indeterminate equation, $x^3 + y^3 + z^3 = A$, for all values of A from 1 to 50 inclusive. These enabled him to solve

many difficult problems relating to cube numbers. Thus, it can be found that 4 is composed of three cubes whose roots are greater than unity. This gives an answer at once to the old puzzle, "to divide unity into three such positive parts that, if each part be increased by unity, the sums shall be three rational cubes." "And if the division of unity," says David Engel in *Our Schoolday Visitor*, 1871, p. 36, "into three such parts as above stated has beaten the mathematicians of two continents, what would be the effect of dividing unity into 12 such parts, which (by means of these rules) can be done with about as much ease and facility in one case as in the other?" Mr. C. Gill, the editor of the *Mathematical Miscellany*, who furnished an abstract of Lenhart's speculations, points out the great defect in them, which robs them of any scientific value. It lies in their "tentative character." As it nowhere appears that a given number A, is necessarily capable of being decomposed into three cubes, in the manner proposed, "there is nothing to insure us that, by successive trials, we shall at last arrive at a number which is the sum of two cubes, much less that we shall arrive at one already tabulated."

Some attention to our subject was paid by John D. Williams, the author of works on elementary mathematics. In 1828 he started the *Mathematical Companion* as a rival, it would seem, to the *Mathematical Diary*. Williams had many opponents, and a bitter contest was carried on between the two parties. He finally issued his 14 famous "challenge problems," directed against all the mathematicians in America, excepting three.* The fact that all 14 problems were on Diophantine Analysis tends to show to what great extent that subject was engrossing the attention of a certain class of mathematicians

* To illustrate the bitterness with which the contest was carried on, we quote from a communication to a newspaper made by Williams in 1832 a passage which has reference to one of his rivals. "I beg leave to state that......I received from this gentleman correct solutions to questions 1, 2, 3, 4, 5, 10, 13 and 14. This, I suppose, is about his *ne plus ultra* —beyond which I defy him to advance 'till riper age shall with maturer force burnish his mind.'"

here. In the newspaper communication referred to in the foot-note, the first problem and its answer are stated as follows:

$$x^2 + y^2 = a^2 = z^2 + w^2 = \square$$

and $$x^2 - w^2 = z^2 - y^2 = \square$$

ANSWER, If $a = 7585$, then $x = 7400$. $y = 1665$. $z = 6273$, $w = 4264$.

The answers are, apparently, expected to be in integers. If a general solution is to be given, then the above problem is not so very easy, but Diophantists care but little for general solutions. Assuming a particular number for a and then finding one set of values for an answer gives them perfect satisfaction How many other values, if any, really exist, and what they are, does not concern them much.

Among the writers in the *Mathematical Companion* interested in Diophantine Analysis, appear the names of Dr. O'Riordon, Wm. Wright, T. Beverly, and Cunliffe.

One of the best Diophantine scholars in this country was Charles Gill, editor of the *Mathematical Miscellany* and professor of mathematics at the St. Paul's Collegiate Institute at Flushing, Long Island. He published, in 1848, a book of 90 pages, entitled, "Application of the Angular Analysis to the solution of Indeterminate Problems of the Second Degree." * This is a charming little work. Of all publications on Indeterminate Analysis that we have seen, this least deserves the adverse criticisms we shall make on the work done in this country, taken as a whole. Here an effort is made to introduce *method* and to exhibit *general* solutions. The novelty of the book consists in the application of the notation of Trigonometry to Diophantine Analysis. So far as we know Prof. Gill is the first one to make this application. If the sine and cosine of an angle can be expressed as rational numbers, we may assume $\sin A = \frac{2mn}{m^2+n^2}$ and $\cos A = \frac{m^2-n^2}{m^2+n^2}$, and then all other functions

* A copy of this now rare book was lent the writer by Dr. Artemas Martin, of Washington.

can likewise be represented by rational numbers. He then includes all equations for the six trigonometric functions in the symbolical form $A=\phi\left(\frac{m}{n}\right)$, $\frac{m}{n}$ being called the "root of the function." He then solves the three problems: (1) To find the root of the function in terms of the angle; given the roots of the function, (2) to find the root of their sum and (3) of their difference. This occupies eleven pages in the book. The rest of the work is devoted to the application of this notation to indeterminate problems and to geometry. The thirtieth and last problem on the former subject is "To find n square numbers such that the sum of every $n-1$ of them may be square numbers." As an example of the geometric problems solved we quote this: "To find the sides of right angled triangles in rational numbers, which have equal areas."

The solution of a few difficult Diophantine problems was communicated to *Silliman's American Journal of Science* by Theodore Strong, Professor at Rutgers College (died 1869). He was one of the few men in this country who possessed some knowledge of Gaussian methods in the theory of numbers, which he employed in published articles, once or twice.

All the mathematical periodicals, except two of our present journals which are devoted to advanced mathematics exclusively, have given a good share of their space to Diophantine analysis. Men of a mathematical turn of mind liked to exercise their ingenuity in this field. Rev. A. D. Wheeler discoursed "On the Diophantine Analysis" in the *Mathematical Monthly* of August, 1861. Professor George R. Perkins wrote on it for the *Analyst* of J. E. Hendricks. Dr. David S. Hart, of Stonington, Conn., was a faithful follower of Diophantus. He wrote an outline history of indeterminate analysis for the *Mathematical Magazine* (Vol. I, No. 10), also articles entitled "Square Numbers Whose Sum is a Square" (Vol. I, No. 10), "Diophantine Solutions" (Vol. 1, No. 3), "Consecutive Square Numbers Whose Sum is a Square" (Vol. I, No. 8), "Cube Numbers whose Sum is

a Cube" (Vol. I, No. 11). In *Our Schoolday Visitor* of 1871 is an article on our topic. In the *Mathematical Visitor* he solved Diophantine problems numbered 21, 65, 112, 129, 197. Other solutions of his are given in Dr. Matteson's collection. The *Mathematical Visitor* contains Diophantine solutions by Asher B. Evans (114), Rev. U. Jesse Knisely (123), Reuben Davis (126, 129), Josiah H. Drummond (114), George Eastwood (177), Sylvester Robins (179), Samuel Roberts (185), William Hoover (191).

In Vol. I, No. 5, of the *Mathematical Magazine* Dr. Martin quotes from sheets bound in a copy of J. R. Young's algebra, once owned by Abijah McClean, of New Lisbon, Ohio, some curious properties of numbers. Among other curiosities are given 27 numbers, all squares, composed of the nine digits, each digit being used once and but once. Dr. James Matteson was able, however, to add another number to the 27 of McClean, possessing the above property. In discovering these numbers Barlow's Table of Square Numbers was made use of. We are told that in 1835 an anonymous writer in the Stanford Sentinel challenged the entire faculty of Yale College to arrange the nine digits in such order as to form a perfect square. It will be seen that this is the same problem as the one given by McClean. A. D. Stanley, who then occupied the mathematical chair at Yale, a few days after the challenge, published one solution and called upon the proposer to produce other answers, as there was more than one. The opponent made an evasive reply, stating that there were nine solutions, but Stanley found twenty-eight different ones.

Neat solutions of Diophantine questions were published by Dr. Artemas Martin. In 1875 and '76 he communicated a series of sixteen articles to the *Normal Monthly;* in 1874 he wrote for the Analyst on the difficult problem to divide unity in three such parts that if each part be increased by unity the sum shall be three rational cubes. Solutions of the same problem were given by William Lenhart, by Reuben Davis, of Bradford, Ill.,

in the *Mathmatical Visitor* of 1880, and by A. L. Foot in the *School Messenger* (now *Mathmatical Messenger*), Vol. III, No. 8. Dr. Martin published solutions of his own also in the *Mathmatical Visitor*. In 1887 he read before the Philosophical Society of Washington a paper "On *n*th Power Numbers Whose Sum is an *n*th Power."

In the *Mathmatical Messenger* published by G. H. Harvill, of Louisiana, indeterminate problems are solved by A. L. Foote, B. F. Finkle, B. F. Burleson. In the *School Visitor*, published by John S. Royer, we have seen contributions of B. F. Finkel and D. H. Davison.

A most interesting "Collection of Diophantine Problems with Solutions, compiled by James Matteson, M. D., of De Kalb Centre, Illinois," and published by Dr. Martin, 1888, exhibits the results of Diophantine studies in this country at their most advanced stage. The solutions of problems 1 and 4 in this collection are by the late Abijah McClean, of New Lisbon, Ohio; 5, 7, 9, 22, and 24 are from the pen of Dr. D. S. Hart; 6, 8, and 12 are by Dr. Matteson. The remaining solutions are chiefly the work of Reuben Davis, of Clifton, Kansas. A small pamphlet giving "A Discussion of the equation $h^2 = p^2 + b^2$, when h, p, b are Integer Numbers," by Josiah Scott, was published at Bucyrus, Ohio, in 1871.

Diophantine problems were such a favorits study that at one time they found their way into some of our school-books. This may have been done in imitation of English writers, such as Bonnycastle, whose little algebra was republished in this country. Charles W. Hackley, in his "Treatise on Algebra," 1846, gives eleven pages to Diophantine Analysis and thirty-five pages to the Theory of Numbers in which he introduces the Gaussian notation for the treatment of congruent numbers. Enoch Lewis has a chapter on indeterminate analysis in his algebra. The key to Ray's Algebra devotes some space to our topic.

III. Estimate of the Work Done.

In the examination of the more difficult Diophantine problems solved by the men above referred to, we are often struck by the great ingenuity displayed. In following the intricate paths, we often wonder how the patience of the solver endured to accomplish it. We are sometimes tempted to exclaim, "Here we have indeed a true disciple of Diophantus," or "This solution is after Diophantus's own heart." Some of the work is, in point of difficulty, in advance of anything we have of Diophantus himself. And indeed it ought to be, for the ancient algebraist worked under the enormous disadvantage of having but few algebraical symbols and of not possessing the Arabic notation of numbers. And yet, our ingenious solutions, however dazzling, fail to satisfy us. They fail to teach us general methods. The solution of one problem may give us no clue whatever as to how the next one shonld be solved. Our Diophantists are dexterous and indefatigable; they lead us by intricate and laborious processes to an answer which may consist of rational fractions with numerators and denominators of 30 to 50 digits, or which may have integral values of upward to 150 digits. But little or no effort is made to determine whether all possible solutions have been found, and to show how the remaining answer can be obtained. The problems are, after all, treated very superficially. Their inner secrets remain, to a great extent, unraveled. The lack of general methods by which full and complete solutions can be obtained, renders the work done of little or no scientfic value. That the discovery of powerful methods is more valuable than the obtaining of an answer to some particular problem will be conceded by all. To illustrate: When Nathaniel Bowditch, in the *Analyst* of Robert Adrain, succeeded in adjusting discordant observations obtained from a survey of a certain piece of land, so as to obtain the most probable values in that particular case, he displayed cleverness; but when Robert Adrain worked out a general method by which

all such problems could be solved, and thus elaborated the all-important law of the Probability of Errors,—he did work of immeasurably greater value to science. When the future historian writes the history of indeterminate analysis, we fear that work in Diophantine Analysis done in this country will count for little or nothing. It is to be regretted that American mathematicians did not fall in line with leading foreigners and follow the path marked out by the illustrious Gauss in the Theory of Numbers. The modern theory of numbers is a field, wonderfully beautiful, and adorned by most elegant and powerful general methods. In a few mathematical centres of the United States the theory of numbers is now taught, but original contributions are still rare. In the past, Professors Benjamin Peirce and Theodore Strong are the only ones who by their publications in journals exhibited a knowledge of Gaussian methods. A quotation from an article by Dr. J. W. L. Glaisher (*Nature*, September 11, 1890) will apply to the United States much more forcibly than to England: "It is much to be regretted that this great theory, perhaps the greatest and most perfect of all the mathematical theories, should have been so little cultivated in this country. * * * From the moment that Gauss, in his wonderful treatise of 1801, laid down the true lines of the theory, it entered upon a new day, and *no one is likely to be able to do useful work in any part of the subject who is unacquainted with the principles and conceptions with which he endowed it.*"

THE ELLIPTIC FUNCTIONS DEFINED INDEPENDENTLY OF THE CALCULUS.

BY FRANK H. LOUD.

Among the minor works of the eminent mathematician, C. G. J. Jacobi, is one containing a geometrical construction for the addition-theorem of elliptic integrals,—a construction, as the author remarks, which is not without advantage over that given by Lagrange by the use of the spherical triangle. The suggestion of Jacobi has been followed by several writers of text-books upon the elliptic functions: it appears, *e. g.*, in brief on pp. 28 and 29 of Cayley's "*Elementary Treatise*," and is much more fully handled by Durège, "*Theorie der Elliptischen Functionen*," pp. 168–190. The object which I propose at present is to apply the Jacobian construction to a still more primary use; viz., to produce from it a definition of the elliptic functions which shall be independent of differentiation and integration, and being based on the methods of elementary mathematics alone, or those anterior to the calculus, shall enable the student to form a preliminary conception of the elliptic functions analogous to that which he acquires of the sine, cosine, etc., in elementary trigonometry.

The memoir of Jacobi here cited appeared in the third volume of Crelle's Journal; and is to be found in his collected works, (Berlin, 1881,) on pp. 277–293. It bears the date of April 1st, 1828, and is entitled, "Upon the Application of the Elliptic Transcendents to a known Problem of Elementary Geometry; viz., to find the Relation between the Radii and the Distance between the Centers of two Circles, one of which is inscribed in an Irregular Polygon, and the other is circumscribed about the

same." Among the historical references which it contains to researches bearing upon this problem, there are several to theorems of Poncelet, which, as Jacobi shows, may easily be demonstrated by the use of elliptic functions. In reversing the procedure of Jacobi, and attempting an application of elementary geometry to the elliptic transcendents, I make one of these theorems of Poncelet my first objective point; but I propose to reach it, not by the projective method of Poncelet himself, but by the less elegant but more generally familiar processes of analytical geometry. For this purpose I require a short series of introductory propositions, as follows:

Problem.—When two sides of a triangle inscribed in a circle are represented by a single quadratic equation, to derive thence the equation for the third side.

The general equation of a pair of lines intersecting at the origin is

$$Ax^2 + 2Hxy + By^2 = 0;$$

and to transfer the intersection to any point whose coördinates are c, s, we must write

$$A\,(x - c)^2 + 2H\,(x - c)\,(y - s) + B\,(y - s)^2 = 0;$$

which becomes, on expansion,

$$\begin{aligned} Ax^2 + 2Hxy + By^2 - 2\,(Ac + Hs)\,x - 2\,(Bs + Hc)\,y +& \\ (Ac^2 + 2Hcs + Bs^2) = 0;& \end{aligned} \qquad (1$$

This equation will represent two sides of a triangle inscribed in the circle

$$x^2 + y^2 = R^2, \qquad (2$$

provided we take for R^2 the value $c^2 + s^2$.

The loci of equations (1) and (2) have two coincident intersections at the point c, s; our problem is to write the equation of the line joining the two remaining intersections.

It is a familiar principle that, if $P = 0$ and $Q = 0$ are the equations of two conics, then an equation of the form $P + kQ = 0$, where k is an arbitrary constant, represents a conic passing through the four points in which the first two conics meet; and

conversely that by properly choosing the value of k, any conic through these four points may be thus represented. Any of the three conics may be a pair of lines. In the present case, if $P = 0$ and $Q = 0$ are our equations (1) and (2), it must be possible to give k such a value that the equation $P + kQ = 0$ shall represent the pair of lines, one of which is the line whose equation we seek, while the other is the line joining the remaining intersections of the given loci. But these two remaining intersections are the two consecutive points on the circle in which it is met by the two lines at c, s; accordingly the line joining these points is simply the tangent to the circle at the point c, s; that is, its equation is

$$cx + sy = R^2.$$

And if we represent the line whose equation we seek under the form

$$lx + my + n = 0,$$

(where l, m, and n, are quantities at present undetermined,) then the form

$$(cx + sy - R^2)(lx + my + n) = 0,$$

or, as this becomes on expansion,

$$\begin{gathered} clx^2 + (ls + cm)\,xy + msy^2 + (cn - c^2l - ls^2)\,x \\ + (ns - c^2m - ms^2)\,y - (c^2n - ns^2) = 0 \end{gathered}$$

is a form with which the equation

$$\begin{gathered} Ax^2 + 2Hxy + By^2 - 2\,(Ac + Hs)\,x - 2\,(Bs + Hc)\,y \\ + (Ac^2 + 2Hcs + Bs^2) + k\,(x^2 + y^2 - c^2 - s^2) = 0 \end{gathered}$$

must become identical by giving a suitable value to k.

Assuming this identity we have for the determination of four unknown quantities, viz., k, l, m, and n, the six equations

$$\begin{aligned} &cl = A + k, \qquad ls + cm = 2H, \qquad ms = B + k, \\ &cn - c^2l - ls^2 = -2Ac - 2Hs, \\ &ns - c^2m - ms^2 = -2Bs - 2Hc, \\ &\quad -c^2n - ns^2 = Ac^2 + 2Hcs + Bs^2 - c^2k - ks^2. \end{aligned}$$

The solution of the first four of these equations readily gives

the desired quantities; and the two remaining equations may then be used to verify these results. We find

$$l = \frac{Ac - Bc + 2Hs}{c^2 + s^2}, \qquad m = \frac{-As + Bs + 2Hc}{c^2 + s^2},$$

$$n = -A - B, \qquad k = \frac{2Hcs - As^2 - Bc^2}{c^2 + s^2}.$$

If now the above values of l, m, and n be substituted in the equation

$$lx + my + n = 0$$

and the latter cleared of fractions by multiplying through by $c^2 + s^2$, we have the sought equation of the third side of the inscribed triangle, viz.,

$$(2Hs + Ac - Bc)\,x + (2Hc - As + Bs)\,y = (A + B)(c^2 + s^2). \quad (3$$

PROBLEM.—If two chords of a given circle meet on its circumference, and are both of them tangents to a second given circle, to write the equation of the line which joins the extremities of these chords.

This is the same as the foregoing problem, with the added condition that the two lines which meet in the point c, s, are to be tangent to a second given circle. Let the radius of this second circle be r, and its centre situated at a distance a from that of the former; but, for subsequent convenience, let the sign of a be taken opposite to that of the abscissa of the centre, which we assume to be situated on the axis of X. Then the equation of this second circle is

$$x^2 + y^2 + 2ax + a^2 - r^2 = 0; \quad (4$$

and that of the pair of tangents to it from the point c, s is

$$\begin{aligned}(r^2 - s^2)\,x^2 + 2\,(as + cs)\,xy + (r^2 - a^2 - 2ac - c^2)\,y^2 \\ - 2\,(as^2 + cr^2)\,x + 2\,(a^2s - r^2s + acs)\,y \\ + (c^2r^2 + r^2s^2 - a^2s^2) = 0.\end{aligned} \quad (5$$

(I assume the latter equation as known, since it is given in the text-books, but I may remark that a handy method of obtaining it is identical in principle with that employed in the former

problem. To wit, if $P = 0$ be the equation of the polar of c, s, in respect to the new circle, then $P^2 + k(x^2 + y^2 + 2ax + a^2 - r^2) = 0$ must be identical with equation (1). This furnishes, as before, six conditions to determine four unknowns, which in this case are A, H, B and k).

A comparison of equations (1) and (5) shows that the latter is derived from the former by giving to A, H, B, the values $r^2 - s^2$, $as + cs$, and $r^2 - a^2 - 2ac - c^2$ respectively. Then if the same letters be replaced by these values in equation (3), the result will be the equation demanded by the present problem, viz.,

$$[2a(c^2 + s^2) + c(c^2 + s^2 + a^2)]x + s(c^2 + s^2 - a^2)y + (c^2 + s^2)(a^2 + 2ac + c^2 + s^2 - 2r^2) = 0. \quad (6$$

It is desirable to notice at what point the line whose equation has just been written, intersects the line which joins the intersections of the two circles, *i. e.*, the axis of X. If for this purpose we put $y = 0$ in the equation, and solve for x, replacing at the same time $c^2 + s^2$ by R^2, we have, as the abscissa, m, of the point in question,

$$m = -\frac{R^2(a^2 + 2ac + R^2 - 2r^2)}{2aR^2 + c(R^2 + a^2)}. \quad (7$$

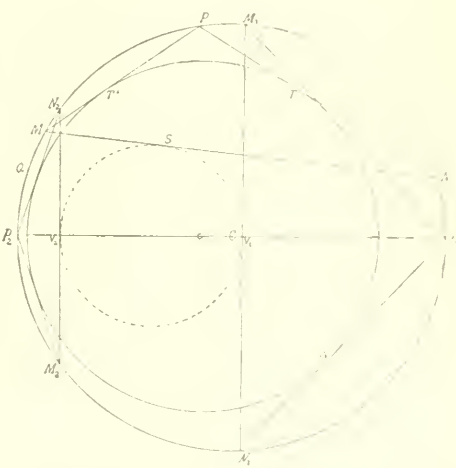
This expression does not contain s; hence if we regard the two circles as fixed in position but the point c, s as moving, (subject of course to the condition that it remain upon the circumference of the circle $x^2 + y^2 = R^2$), we may discuss the effect upon the position of the intersection m, 0, by supposing the value of c alone in the formula to be varied. The equation (7) is linear in c and in m, so that for any value which may be assigned to one of these quantities a single real value would be obtained for the other. But not all values of c are possible consistently with the restriction of the point c, s to the circumference of a circle; c, in fact, cannot be greater than R or less than $-R$. Hence the values of m which would be obtained by assigning to c any value transcending these limits are equally

impossible, and it follows that there is a region upon the line joining the centres which is never crossed by the line whose equation is the above equation (6); and moreover that the limits of this region or segment are the points which we obtain from (7) by giving to c the value R and $-R$ successively; viz., they are the points whose abscissas are

$$m' = \frac{R[2r^2 - (R+a)^2]}{(R+a)^2}, \text{ and } m'' = \frac{R[(R-a)^2 - 2r^2]}{(R-a)^2}. \quad (8$$

These results are such as would be expected from a purely geometrical point of view. If we construct a diagram, Fig. 1, in which two circles $P_1PP_2M_2$ and $TT'W$ are drawn in any relative positions, and assigning to a point P several different situations in succession upon the circumference of one of them, construct, for each position of P, the system of chords as described

Fig 1



in the foregoing, it will be evident that corresponding to the singly infinite succession of positions possible to P, there is a singly infinite succession of positions of the line MN; and accordingly it is to be expected that these successive lines envelope some curve, which, in consequence of the general symmetry of the figure, would be symmetrical to the axis PP_1. Moreover, when the point P is made to traverse the circumference P_1PMM_2, a fixed point Q on the latter becomes twice and only twice the extremity of one of the chords MN; that is, once when the point M and again when the point N arrives at Q. Hence it might be at least conjectured the that curve

enveloped by MN is of that class to which two and only two tangents can be drawn from a given point; that is, that it is a conic. The limiting positions found in (8) are the points V_1, V_2, on the axis of symmetry P_1P_2 at which tangents at right angles to this line meet the curve. We may so far avail ourselves of the analogy which the enveloped curve appears to bear to the conic, as to name the point midway between V_1 and V_2 the *centre*, and the distance of the centre from either of them the *semi-axis*. Then, if we denote by g the abscissa of the centre, taken with the contrary sign, and by p the semi-axis, without regard to sign, (since for the present we shall be concerned only with the absolute value,) we shall have

$$-g = \frac{m' + m''}{2}, \text{ and } p = \frac{m' - m''}{2};$$

or, substituting from (8) the values of m' and m'',

$$g = \frac{4ar^2R^2}{(R^2 - a^2)^2} \text{ and } p = \frac{R[2r^2(R^2 + a^2) - (R^2 - a^2)^2]}{(R^2 - a^2)^2}. \quad (9$$

We shall now prove analytically that the line MN does in fact envelope a curve, viz., the *circle*, whose centre is at the point $-g$, 0; and whose radius is p.

THOREM.—If a point move upon the circumference of a fixed circle, and if two chords of this circle, which meet in this point and hence move with it, continually touch a second fixed circle; then the third chord which joins the extremities of the two former, will also continually touch a fixed circle. Moreover, the three circles have a common radical axis.

When g and p represent any two quantities whatever, a circle with radius p and centre $-g$, 0 has the equation

$$x^2 + y^2 + 2gx + g^2 - p^2 = 0;$$

and the condition that this circle may be touched by any right line,

$$lx + my + n = 0,$$

is found by eliminating one of the variables, and forming the condition that the resulting equation may have equal roots. If

y be the variable eliminated, this condition appears at first in the form

$$(gm^2 + ln)^2 = (l^2 + m^2)(g^2m^2 - m^2p^2 + n^2) = 0;$$

but is easily reduced to the simpler statement

$$p\sqrt{l^2 + m^2} = gl - n.$$

And if in this formula we substitute for g and p the values given in the equations (9), and for l, m, and r, the coefficients of equation (6), remembering that $c^2 + s^2 = R^2$, we find that the formula is identically satisfied; and thus it is shown that however the point c, s, may be situated on the circumference $x^2 + y^2 = R^2$, the chord represented by (6) is tangent to the circle whose position is defined by (9). The equation of this circle, as written at the beginning of the present paragraph, becomes, when g and p are replaced by their values,

$$\begin{aligned}(R^2 - a^2)^2(x^2 + y^2) + 8ar^2R^2x - R^2(R^2 - a^2)^2 \\ + 4r^2R^2(R^2 + a^2) - 4r^4R^2 = 0.\end{aligned} \tag{10}$$

To prove the second part of the theorem, we have only to bear in mind that the equation of the radical axis of two circles is obtained by subtracting the equation of one circle from that of the other, having first multiplied either by such a factor as may be necessary to render the coefficients of x^2 and y^2 alike in both. Combining in this way the equation $x^2 + y^2 - R^2 = 0$ with the two equations (4) and (10) successively, we obtain in each case as the equation of the radical axis,

$$2ax + R^2 + a^2 - r^2 = 0. \tag{11}$$

Jacobi cites the foregoing theorem as due to J.-V. Poncelet, and occurring on page 326 of his celebrated work, "*Traité des Propriétés Projectives des Figures.*" In the edition of 1865 I find the theorem on page 315, in a form which may be translated as follows:

"If an angle, being at the same time inscribed in one circle and circumscribed about another, be made to move while subject continually to these same conditions, then the chord which

it subtends in the first of these circles will envelope a third circle which will pass through the points of intersection of the two former, or have in common with them the same secants, whether real or ideal."

By an "ideal secant" Poncelet means a real line, whose points of intersection with the curve are imaginary. The description "a common real or ideal secant" applies therefore to the radical axis of two circles, which in all cases fulfils the analytical condition for passing through the common finite points of the two curves; but, in the case of circles which do not meet in real points, can be called their common secant only in an "ideal" sense. Jacobi appears to prefer the designation for this line which was introduced by Steiner, viz., "the locus of equal tangents,"—a name given in reference to the well known fact that whether two circles meet in real points or not, the tangents drawn to both from any point of the radical axis which is exterior to them, are equal in length.

The proof given by Poncelet for the theorem depends upon the properties of conics having double contact with one another.

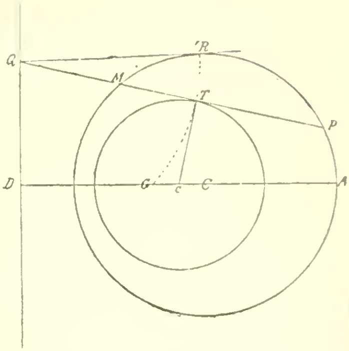
The property of the radical axis, to which reference has just been made, furnishes an easy geometrical solution of the following problem:

Having given a circle, a chord of the same, and a line as radical axis, to determine the circle which has, in common with the given circle, the given line as a radical axis, and which touches the given chord. (See Fig. 2).

From C the center of the given circle, draw a line CD perpendicular to the given radical axis. On this line, extended indefinitely, lie the centers of the whole system—or "pencil,"—of circles which have with the given circle the given radical axis. Produce, if necessary, the given chord PM to meet the radical axis in Q. Determine in the usual manner, a point R on the given circle so that QR would be a tangent. With Q as center and QR as radius strike an arc RTG meeting the given chord

PM in T. This is the point of contact. From T draw Tc perpendicular to PM, and meeting CD in c. The circle with center c and radius cT is the circle required. For it touches PM, while the tangents QR and QT are equal, so that QD is, as required, the locus of equal tangents of the two circles.

Fig 2



There are two solutions. For the arc RTG, extended, will meet PQ produced, if the produced part be made equal to TQ. We may state this as follows:

Of all the circles having a given radical axis in common, two and only two are tangent to any given line, and their points of contact lie on opposite sides of the radical axis.

In all of the foregoing the relative positions of the circles considered, or, in the analytical part, the relative values of R, r and a, have been subject to no limitation. We have now, however, to impose a restriction in this respect.

THEOREM.—When two chords of a circle, which meet on its circumference, are tangent to a second circle which lies wholly within the former, the third circle, which is touched by the line joining the extremities of these chords, from whatsoever point of the circumference they are drawn, lies also wholly within the first circle.

If this joining line be MN of Fig. 1, (supposed indefinitely produced) it is, in any given position, as we have just seen, tangent to *two* of the circles which have a common radical axis with the two given circles. If the point P be moved, and the line MN move in consequence, it cannot of course remain tangent to both; and the one to which it does remain tangent has been already found; being uniquely determined, from among all the circles having the before-mentioned radical axis, by either

one of the two elements given in equation (9). Now if the second given circle lie wholly within the first its radius r is less than $R - a$, that is,

$$r^2 < (R - a)^2.$$

But the distance g between the centres of the first and third circles, as given in (9) may be factored as follows:

$$g = \frac{r^2}{(R-a)^2} \cdot \frac{4aR}{(R+a)^2} \cdot R.$$

Of these factors, the first has just been shown to be less than unity; and the second is so likewise, since

$$\begin{array}{lrl} & (R - a)^2 = R^2 - 2aR + a^2 > & 0. \\ \text{Add} & 4aR & = 4aR \\ \hline & (R + a)^2 = R^2 + 2aR + a^2 > & 4aR. \end{array}$$

We may here assume a and R are taken in their positive values; hence, on the same supposition with regard to g, (which must be positive if a and R are so),

$$g < R.$$

Hence the centre of the third circle is within the first; therefore the third circle either lies wholly within the first or intersects it; but the latter alternative is not admissable, since, the three circles having a common radical axis, if two intersect, the other must pass through the points of intersection, which would be contrary to the hypothesis; hence the third circle lies wholly within the first.

It will accordingly be assumed in future that the circle whose radius is R is exterior to all other circles of the diagram which have a common radical axis with it: each of the latter forming a member of an infinite series of circles, the maximum of which coincides with the circle $x^2 + y^2 = R^2$, and each succeeding circle lies wholly within the preceding one, until the lower limit is reached in the point, or circle of zero radius. This point, marked G in Fig. 2, is there determined as the intersection of the arc RTG with the axis CD; and, by the application of the

same principle then employed, would be similarly found on a circle described from any other point of DQ as a centre, with radius equal to the tangent to the outer circle from that point. It is in fact a point common to all the circles that cut orthogonally the system whose radical axis is DQ, and is fixed in position as soon as the circle PRM and the line DQ are given; or conversely, this circle and the point G will determine the line DQ. This circle, line, and point will be assumed as given in position in subsequent constructions, and are to be regarded as remaining unchanged in passing from one construction to another.

When in addition we have given a chord, as PM in Fig. 1, in one of its positions, we are already able to construct the remainder of the triangle PMN, and the circle which, in the successive positions of the triangle, is enveloped by MN. For we may first construct the circle $TT'W$ as directed in the construction attached to Fig. 2; we may then locate T on its circumference by making $PT = PT'$; the line PT produced determines N and accordingly MN; and finally a second application of the construction of Fig. 2 gives the circle to which MN must remain tangent. As we have to attend only to circles interior to the given circle, no ambiguity is encountered at any stage of the process.

If on the other hand the chord MN is given, and it is required to complete the triangle by drawing PM and PN, we may first construct, as above, the circle tangent to MN, — in the figure, V_1SV_2 — and then draw a tangent to this circle at either of the points V_1 or V_2 where it meets the axis. If M_1N_1 is such a tangent, and P_1M_1 a line joining one of its extremities to an extremity of the diameter P_1P_2, we can construct that circle of the system which is tangent to P_1M_1; and the symmetry of the figure shows that the same circle will be tangent to P_1N_1 as well. Accordingly, if tangent lines are drawn to the circle from M and N, they must meet on the circumference of the given circle at a point P, and the triangle will have been constructed as required.

In this construction there is evidently an ambiguity. We may draw the tangent to the circle V_1SV_2 at either point V_1 or V_2, and join its terminal points upon the given circle with either P_1 or P_2. It is evident, however, that we do not thus obtain four results, but only two; since the two sides of the triangle $P_1M_1N_1$ will, on the movement of P, envelope the same circle as those of $P_2M_2N_2$, and those of $P_1M_2N_2$ the same as $P_2M_1N_1$.

The analytical parallel to the geometrical construction just given consists in determining a and r, the elements of the circle $TT'W$, in terms of those of V_1SV_2, that is, in terms of g and p. And the double solution of the geometrical problem answers to the ambiguity in the sign of p, of which, in equation (9), we fixed only the absolute value. We now notice that m' is the abscissa of the point of contact of the base of that triangle whose vertex is at P_1, where $c = R$; and m'' corresponds in the same way to the vertex at P_2, where $c = -R$. Hence if the base of the triangle is on the same side of the circle V_1SV_2 as its vertex is,—as in the figure,—we obtain the radius of the circle with its positive sign by the formula $p = \frac{1}{2}(m' - m'')$, but if the triangle were $P_1M_2N_2$ the radius would be of opposite sense and denoted by the formula $p = \frac{1}{2}(m'' - m')$. Assuming that the former is the case, we have

$$p + g = -m'' \quad \text{and} \quad p - g = m'$$

whence, by equation (8),

$$p + g = \frac{R}{(R-a)^2}\left[2r^2 - (R-a)^2\right]$$

and

$$p - g = \frac{R}{(R+a)^2}\left[2r^2 - (R+a)^2\right].$$

If we take from one of these the value of r^2 and substitute it in the other we obtain for the unknown quantity a a quadratic equation, whose roots are

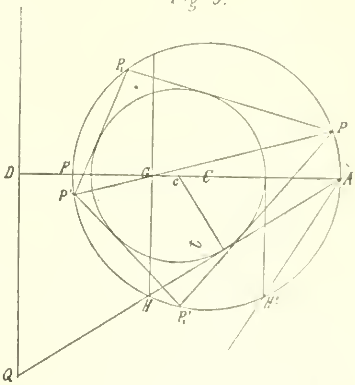
$$a = \frac{R}{g}\left[R + p \pm \sqrt{(R+p)^2 - g^2}\right]. \qquad (12$$

Here there appears to be a new ambiguity. Both of these roots are real, but one of them, as is obvious on inspection is greater than R; that is, the circle to which it belongs has its centre outside the circle $x^2 + y^2 = R^2$; hence it is not a member of the series of circles to which our attention is at present restricted. But the other root, while of the same sign as g is less than g; whence the circle corresponding to it lies between the circles whose radii are R and p, and so exterior to the latter. We learn accordingly, that starting from a given base MN we can always determine in one and only one way a triangle PMN whose vertex shall lie on the opposite side of MN from the circle to which the latter is tangent, and whose sides, meeting in P, shall touch a circle exterior to that touched by MN.

We can now solve, both analytically and geometrically, the following problem:

Within a given circle to determine the position of a second circle, having in common with the first a given line as radical axis, and such that a polygon of 2^n sides may be at once inscribed in the first circle and circumscribed about the second.

Fig 3.



The process and the result are entirely independent of the position of the first vertex P of the polygon, which may be assumed arbitrarily upon the circumference of the given circle.

The first polygon to be constructed is that of *two* sides, which consists of a chord extending from P to some other point of the circumference and thence back to P. As the two sides coincide, the inscribed circle is of zero radius, hence consists of the point G, geometrically determined as in Fig. 2. The second vertex of the polygon is therefore fixed at the point P', where PG meets the circumference.

The analytical determination of the point G presents no difficulty. If the length of CD be denoted by d, that of a tangent from D to the given circle will be $\sqrt{d^2 - R^2}$, since such a tangent forms a right triangle with CD and the radius to the point of contact. Therefore the distance DG is $\sqrt{d^2 - R^2}$, or CG is $d - \sqrt{d^2 - R^2}$.

We now proceed to the construction of the quadrilateral, and its inscribed circle.

The construction has already been in substance given. A chord at right angles to AD is drawn through G, and through H, its extremity, a line AH is drawn, and extended to meet the radical axis at Q. The distance QG, laid off on this line from the point Q, fixes t, the point of contact. whence both the centre c and radius ct of the circle become known. Tangents to this circle are drawn from P and from P', and these tangents, meeting at P_1 and P_1', compose the quadrilateral.

Analytically, we apply equation (12) to determine a, which is cC of the figure, having for g the known distance GC, and for p the radius of the circle at G, which is zero. Having determined a, we find r from equation (11) in which we replace x by its known value, $-d$. The formula then is

$$r^2 = R^2 + a^2 - 2ad.$$

Each process,—the geometrical and the analytical,—may now be applied anew in the case of the octagon. A tangent at right angles to AD is drawn to the circle whose centre is c, and the chord AH' is drawn to the point where this tangent meets the outer circle, etc. For the analytical computation the equations (12) and (11) are again employed, but now g and p have the values which were previously denoted by a and r, while a and r denote the elements of the new circle inscribed in the octagon. And thus the process may be carried on as many times as desired.

A practical difficulty occurs in the geometrical construction, on account of the use made of the point in which AH' meets

the radical axis, for as the number of sides of the polygon increases these lines approach parallelism. It is to be noted, however, that this part of the construction is merely the application of the method given for finding on a given line the point of contact of a tangent circle which has in common with a given circle a given radical axis. A second method for solving this subsidiary problem will be given later, and thus the above-noted mechanical difficulty will be obviated.

It seems well to notice here,—although it is aside from the immediate purpose,—that the equations thus far obtained serve also for the case of a *triangle* inscribed in one circle and circumscribed about another; and hence for the determination by the above process of the elements of the inscribed circle for a polygon of 3×2^n sides. In Fig. 1, PMN will be such a triangle as just described, provided the circle touched by the side MN is identical with that touched by PM and PN, that is, if g and p are equal to a and r respectively. Applying this condition in the value of g given in equation (9) we have.

$$a = \frac{4ar^2 R^2}{(R^2 - a^2)^2}.$$

Dividing through by a and extracting the square root we obtain

$$R^2 - a^2 = 2rR, \tag{13}$$

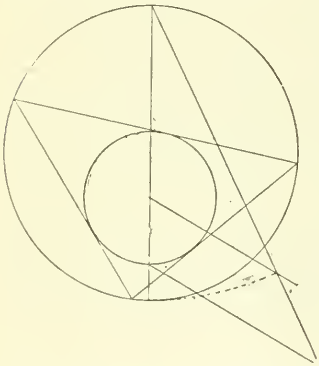
as the relation which must subsist between the radii of an exterior and an interior circle, and the distance between their centres, in order that a triangle inscribed in one may be circumscribed in the other. If R and a be regarded as known, r may be found geometrically by constructing the proposition

$$2R : R + a = R - a : r.$$

When the circles have been determined, the radical axis may be found either geometrically, by well-known constructions, or analytically by equation (11); and thenceforward the work proceeds as in the foregoing paragraph.

The form of the above equation (13), (which is ascribed to

Euler), is noteworthy as containing only the symbols R, r and a with no auxiliary quantities. The data of this and the preceding paragraph afford, of course, the material for writing similar equations for polygons of 2^n or 3×2^n sides, if, instead of introducing as known quantities in the equations the elements of previously solved problems, we should eliminate all such quantities. For polygons not belonging to these groups an independent investigation would be necessary for every polygon



of k sides (where k is an odd number) in order to apply the preceding method to extend the result to $k \times 2^n$ sides. Equations between R, r and a had been obtained before the date of Jacobi's memoir, cited at the beginning of the present paper, by Nicolaus Fuss and by J. Steiner, the former of whom obtained formulæ applicable to each of the polygons of sides not greater in number than eight, and the general method indicated by Jacobi has since been applied to polygons of still higher numbers of sides by F. J. Richelot and others; but, so far as I know, without appending methods of geometrical construction. *

It is especially to be noticed, in the problems of the two preceeding paragraphs, that when an initial vertex has been chosen, the remaining vertices are thereby fixed, not only for that polygon, but for all that are successively derived from it by doubling the number of sides; and so that if, in passing in a determinate direction around the polygon, the number of sides between the two vertices is to the whole number as m to n, this holds true for the same two points in all the polygons subsequently derived. These two points, then, always have between

* The memoirs of Richelot are to be found in Crelle's Journal vols. 5 and 38; there is also an article in vol. 81 of the same Journal (1875), by Max Simon; the latter paper succeeding a Latin dissertation by the same author which appeared in 1867. These writers have made extensions of the problem in several directions.

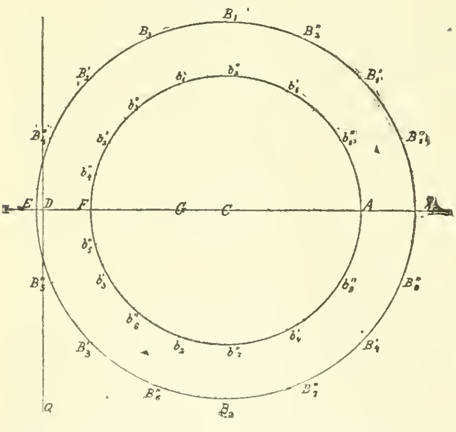
them m-n^{ths} of the whole perimeter of the polygon (when this is estimated by the *number*, not the *length* of the intervening sides,) however great the number of sides may be made.

Let us now return to the consideration of the former of these last two problems—viz., that relating to the polygons of 2^n sides,—and let us suppose that there is given, as in that and previous problems, the circle of radius R, also the right line which has been previously used as a radical axis; and we will define the position of the latter a little more strictly than heretofore by requiring that the diameter to which it is perpendicular, which may be called the *initial diameter*, shall be placed horizontally, while the radical axis itself stands on the *left* side of the circle. This circle we will name the *circle of the amplitude.* To avoid unnecessary repetitions of a statement already familiar, let it be understood in future that whenever a polygon is mentioned as inscribed in the circle of the amplitude, it is meant that the same polygon is also circumscribed about one of the circles which constitute what we shall henceforward call the *interior system*, *i. e.*, those which are within the circle of the amplitude, and have with it the given line as radical axis—the particular circle touched by any polygon of 2^n sides being always uniquely determined to each value of n, by the method already explained. Let us now suppose another circle to be drawn, exterior to the circle of the amplitude, and hence, of course, to the whole interior system, and not having with them the radical axis which they have in common, but being instead, concentric with the circle of the amplitude. We will call this new circle the *circle of the argument.* Its radius might, for our present purpose, be of any convenient length. If, after defining the elliptic functions, we pursue the consideration of them sufficiently far, we may discover a reason for choosing a particular length for this radius, but we will at present be content to regard it as arbitrarily taken of a length whose ratio to that of R we will denote by $2K : \pi$. The right hand extremity of the initial diameter, in

either of these circles, we will call the *origin of arcs*, for that circle.

Suppose now, that a polygon of 2^n sides, say a quadrilateral, is inscribed in the circle of the amplitude, having one vertex at the origin; and at the same time a *regular* polygon of the same number of sides—a square—is inscribed in the circle of the argument, this also having a vertex at the origin. All polygons inscribed in this circle are to be regular. In order to avoid confusing the diagram, the vertices only of either set of polygons are noted in the figure. (Fig. 4.) The trapezium inscribed in the circle of the amplitude is $Ab_1\,Fb_2$, and the square in the circle of the argument is $XB_1\,EB_2$. Next let an octagon be inscribed in each circle. A new vertex is now inserted between each of the former vertices and the succeeding. In the circle of the amplitude these new vertices are b_1', b_2', b_3', b_4'; and the corresponding new vertices in the other circle are B_1', B_2', B_3', B_4'. Next, the polygon of sixteen sides may be constructed, adding the new vertices b_1'', b_2'',... b_8'' in one circle and B_1'', B_2'',... B_8'' in the other. And so the process may be continued as far as desired, with the result of fixing—in a perfectly definite way in each case—as many points as we please on one circle, and the corresponding points on the other.

Fig. 4



We are now able to define, for any point on one circle, its corresponding point on the other. Let a point be given at pleasure on the circle of the argument, and also let an arc of the circle of the amplitude be stated in magnitude but not in

position;—merely a definite fraction of the circumference. We may then continue bisecting the arcs of the circle of the argument, at the same time fixing on the other circle the points which correspond to these points of bisection, until one of two things must take place. Either one of the points of bisection falls on the given point, in which case its corresponding point is unequivocally determined, or else two points of bisection will be found, containing the given point between them, and such that the arc of the circle of the amplitude, contained between their corresponding points, is less than the assigned arc. But this will be true, how small soever the assigned arc may have been. There is then a limiting position on the circle of the amplitude, corresponding to the given point on the circle of the argument.

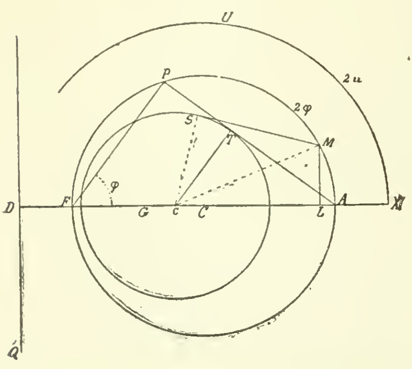
It may be remarked that to fix approximately the point corresponding to a given point, with sufficient accuracy for a geometrical illustration, will not in general require many bisections, for the circles within the circle of the amplitude, inscribed in the successive polygons, rapidly approach coincidence with the latter circle,—as a result of which fact, a point within an arc of this circle soon comes to divide the arc in sensibly the same ratio in which the corresponding arc of the circle of the argument is divided by the corresponding point.

Now let a point move on the circle of the argument with a uniform velocity, and in a positive direction of rotation, starting from the origin of arcs; and let a second point move on the circle of the amplitude, starting from its origin, and moving so that it always occupies the point corresponding to that occupied at the same instant by the former moving point; whence it is evident that its motion cannot be uniform, but will be more rapid in the first part of the semi-circumference than in the latter part. Then to any arc of the circle of the argument, beginning at the origin, corresponds an arc of the circle of the amplitude, beginning at its origin; viz.: they are the arcs traversed by the two points in the same interval of time. These arcs are denoted by $2u$ and $2x$ respectively, and the halves of

these arcs, viz., u and x, are named respectively the *argument* and the *amplitude*. So that to find the amplitude corresponding to any given argument, we have first to double the latter, then to find the point on the circle of the amplitude corresponding to the termination of the donbled arc, and finally to bisect the arc included between the point so found and the origin.

It is to be remarked that the argument, for which the corresponding amplitude is thus found, is given as an arc of a circle whose radius (if we make R unity) has been taken to be $\frac{2K}{\pi}$, hence its circumference is $4K$. In other words, the argument is supposed to be given, not in absolute value, but by its ratio to a constant, K; just as an arc of the circle of unit-radius is often most conveniently given by its ratio to π. When the point moving on the circle of the argument has described the whole circumference $4K$, so that $u = 2K$, the point on the circle of amplitude has also completed its circuit, and the relations of position between these points for all greater arcs are the same as for the arcs by which these exceed this respective circumferences, hence the amplitude of $2nK + u$ exceeds by $n\pi$ the amplitude of u. But as on the one hand it is totally unnecessary to the foregoing construction to know the value of K, so conversely, the construction affords no means of determining K. The considerations by which a definite value is assigned, though external to the main purpose of the present paper, will be indicated in a concluding paragraph.

Fig. 5



The fixed parts of the figure, whose relation of magnitude is material to the construction, are only the circle of the amplitude and the radi-

cal axis of the system; and accordingly, when the scale of the diagram has been fixed by choosing a value of R, there is but *one independent constant* which is required to determine its proportions that, namely, which fixes the position of the radical axis. It has been found most convenient to define this position by means of the ratio of the diameter of the circle of the amplitude to the distance of its origin from the radical axis. This ratio, which is $\frac{AF}{AD}$ in the figure, is denoted by k^2; and the quantity k is called the *modulus*.

To turn from the fixed to the variable elements of the figure it is manifest that whatever is dependent on the magnitude of φ may also be regarded as depending on that of u. Thus the trigonometric functions of φ are functions of u as well. In particular, the sine and cosine of φ, regarded as functions of the argument, u, are denominated respectively $sn\ u$ and $cn\ u$, (read "sine-amplitude u" and "cosine-amplitude u"). If P be the termination of the arc 2φ, (Fig. 5) these two functions are evidently geometrically represented, at least so far as regards their absolute value,—by the ratios which the distances of P from the two extremities of the horizontal diameter FA bear to the length of this diameter. If now we also draw that circle of the interior system which is tangent to the chord AP, the right triangles FPA and cTA are similar; and if r and a, as before, denote cT and cC, we easily find, to determine the length of AT, (which we will call t,) the equation

$$t^2 = (R + a)^2 - r^2. \tag{14}$$

Whence

$$sn\ u = \frac{AP}{FA} = \frac{AT}{cA} = \frac{1}{R+a}\sqrt{(R+a)^2 - r^2}; \tag{15}$$

and

$$cn\ u = \frac{FP}{FA} = \frac{cT}{cA} = \frac{r}{R+a}. \tag{16}$$

To these two functions is added a third, called the delta of φ or the delta-amplitude of u,—written $dn\ u$,—and defined as

the ratio in which the chord of 2φ is divided at the point of contact of its tangent circle of the interior system; or as the ratio in which the diameter of the circle of the amplitude is divided at the center of the same interior circle. That is,

$$dn\,u = \frac{TP}{AT} = \frac{cF}{Ac} = \frac{R-a}{R+a}. \qquad (17$$

We may also express k^2 in terms of the same letters,—not forgetting, however, that k^2 is constant, while a and r vary with u. The length of CD as found from equation (11), is

$$CD = \frac{R^2 + a^2 - r^2}{2a}.$$

For AD, we must add R, and then divide $2R$ by the result to obtain k^2, whence

$$k^2 = \frac{4aR}{(R+a)^2 - r^2}. \qquad (18$$

But from the above value of $dn\,u$, it appears that

$$(1 + dn\,u)\,(1 - dn\,u) = \frac{2R}{R+a} \cdot \frac{2a}{R+a} = \frac{4aR}{(R+a)^2}$$

whence

$$\frac{1 - dn^2u}{sn^2u} = \frac{4aR}{(R+a)^2 - r^2}.$$

Comparing this with the foregoing value of k^2 we obtain the equation

$$\frac{1 - dn^2u}{sn^2u} = k^2$$

or

$$dn^2u = 1 - k^2sn^2u. \qquad (19$$

The definitions of the three "*elliptic functions*," as stated in the analytic form, are accordingly as follows:

$$sn\,u = \sin\varphi; \quad cn\,u = \cos\varphi; \quad dn\,u = \sqrt{1 - k^2\sin^2\varphi}. \qquad (20$$

The first and second of these definitions determine the sign as well as the magnitude of $sn\,u$ and $cn\,u$: the sign of $dn\,u$ is taken always positive.

Another *geometrical* definition of $dn\, u$ may be obtained as follows. Let M be any point on the circle of the amplitude, the termination of an arc $2\varphi'$, and let its coordinates be x' and y'. The diameter FA is divided at L, the foot of the ordinate, into the segments $R + x'$ and $R - x'$, which are proportional to the squares of the chords FM and AM, and hence to the squares of $\cos\varphi$ and $\sin\varphi$ respectively; hence (if the arc $2u'$ correspond to $2\varphi'$) we have

$$cn^2\, u' = \frac{R + x'}{2R} \quad \text{and} \quad sn^2\, u' = \frac{R - x'}{2R} \tag{21}$$

and from the latter equation

$$x' = R\,(1 - 2sn^2\, u').$$

Now the square of the distance of any point x', y' from the point c is

$$y'^2 + x'^2 + 2ax' + a^2$$

whence, if t' denote the length of the tangent to the circle whose centre is c,

$$t'^2 = y'^2 + x'^2 + 2ax' + a^2 - r^2.$$

But the point being M, on the circle of the amplitude, we may put for $y'^2 + x'^2$ the equivalent R^2, and for x' in the succeeding term the value just obtained, and find

$$t'^4 = R^2 + 2aR\,(1 - 2sn^2\, u') + a^2 - r^2$$
$$t'^2 = R^2 + 2aR + a^2 - r^2 - 4aRsn^2\, u'$$

or finally

$$t'^2 = [\,(R + a)^2 - r'^2]\left(1 - \frac{4aR}{(R + a)^2 - r^2}\, sn^2\, u'\right).$$

The former factor is the square of the tangent AT, as given in a former equation, (14), and the latter is, — see equations (18) and (19),—

$$1 - k^2 sn^2\, u' \quad \text{or} \quad dn^2\, u.$$

Hence the value of t' is briefly expressed

$$t' = t\, dn\, u'. \tag{22}$$

Now as M was taken at pleasure on the circumference, it stands in no special relation to the particular circle whose centre is c, hence we may define the delta of an arc φ to be the ratio which the tangent drawn from the extremity of 2φ to *any* circle of the interior system bears to the tangent to the same circle drawn from the origin of amplitude.

If P and M be any two points of the circumference, as in Fig. 2,—the arcs AP and AM being respectively designated as $2\varphi'$ and $2\varphi''$, and the corresponding arcs of the circle of the argument as $2u'$ and $2u''$,— then the tangents from these points to any circle whatever of the interior system are in constant ratio, to wit, $dn\ u' : dn\ u''$. Two limiting cases are to be noted. First, the tangents in the case of one of the circles will form one right line, the chord PM, which is then divided, in the ratio just named, by the point of contact of the interior circle. Second, the circle to which the two tangents are drawn may vanish in the point G; it then appears that the two distances PG and MG are to each other in the same before-mentioned ratio. Combining these two statements, the triangle PGM has its base PM divided at the point T in the ratio of the sides $PG : MG$; hence it follows that a line joining G to T would bisect the angle PGM.

Two consequences follow immediately. The first is the promised second solution of the problem: On a given line to find, by a geometrical construction, the point at which it is tangent to one of a given system of non-intersecting circles which have a common radical axis. The solution is: Find, as in the previous construction, a point G, which represents a vanishing circle of the system; connect this with the two points P and M in which the given line meets any other circle of the system, (for in the general statement of the problem, just given, the circle of the amplitude has nothing to distinguish it from the others;) the point in which the given line is met by the bisector of the angle PGM is the required point of contact.

The second consequence is still another geometrical defin-

ition of the function $dn\,u$, which may now be defined to be the ratio which the distance of the termination of the arc 2φ from the fixed point G bears to the fixed distance GA.

We have hitherto regarded the polygons which have been inscribed in the circles of the argument and amplitude, as figures of the same character as the polygons of Euclid's Elements;—such, that is, that in passing along the sides from one vertex to the same vertex again, only one circuit is made about the center of the circumscribed circle. But this restriction is entirely unnecessary, and the geometrical constructions as well as the analysis can easily be applied to the case of polygons whose sides make any desired number of circuits. Suppose, *e. g.*, that in the construction of Fig. 3., we had chosen to regard the arc extending from P to P again as consisting of 6π instead of 2π. The polygon of two sides, PP', would have been constructed as before, but the distance on the circumference from P to P' requires a whole revolution and a half. So also the quadrilateral presents the same appearance as before, but the order of the vertices is $PP_1'\,P'\,P_1$. But coming to the octagon, we use the chord FH' instead of AH' as determining the tangent circle, and place the second vertex of one figure (the first being at P), on the arc P_1P'. Thence forward we proceed as before. In a similar manner we may form polygons which make five circuits, or any other number. In fact, if P be a point on the circle of the amplitude, the extremity of the mth side (counting from A) of a polygon of 2^n sides, then AP will be the side of a polygon of 2^n sides, all of which touch one circle of the interior system, and the last of which at the close of m circuits of the circumference, returns to A. And on the same principle by which we defined to every point on the circle of the argument its corresponding point on the circle of the amplitude, we may regard the chords of any two corresponding arcs, $2u$ and 2φ, measured from the origins of the two circles, as the initial sides of inscribed polygons.

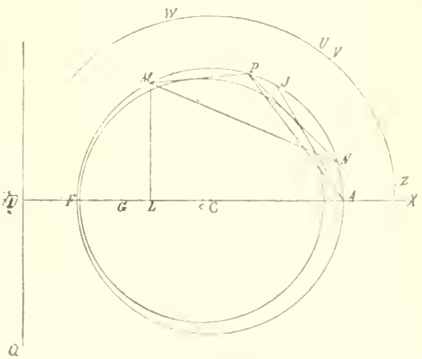
But instead of beginning at the origins, we may start from any two corresponding points; the interior tangent circles will be the same. Whence it appears that if α and α' be any two *equal* arcs of the circle of the argument, and β and β' the corresponding arcs of the circle of the amplitude,—*i. e.*, having both extremities of each at points corresponding to the extremities of α and α',—the chords of β and β' will be tangent to one and the same interior circle.

From this follows immediately,—at least for the case of the interior system of circles, and it is not difficult to see how this proof might be generalized,—the truth of another theorem of Poncelet, which, in the treatment of that author, precedes the one already quoted;—from which, indeed, he derives that one as a special case. It is as follows: (op. cit., p. 312.) "If three circles, situated in the same plane, have a common secant, real or ideal, and if there be inscribed in one of them a series of triangles ABC whose sides AB, AC touch respectively the two other circles, the third side BC of the triangle will continually remain tangent to a fourth circle, having the same secant with the foregoing three."

In the preceding paragraphs, the names belonging to the elliptic functions, which are usually defined by the aid of the calculus, have been applied to ratios presenting themselves in a geometrical figure. In order to show that these ratios really sustain such relations to one another as are known to characterize those functions, and hence are entitled to the names, two courses might be pursued. Either differential expressions might be deduced which should lead to the ordinary definitions, or some special property, which, when taken in connection with the relations already given, will distinctively characterize the functions, might be shown to hold true of the ratios in question. I prefer the latter course, and find the property suitable for the purpose in the "addition formula." This is a formula expressing the functions of the sum of two arguments in terms of the functions of the arguments themselves taken separately.

Let U and V, (Fig. 6) be the terminations of two arcs of the circle of the argument, of lengths respectively denoted by $2u$ and $2v$, and let P and J be the corresponding points of the circle of the amplitude. A circle of the interior system is drawn, tangent to the chord AJ. Then if W and Z are the terminations of arcs $2u + 2v$, and $2u - 2v$, they correspond respectively to M and N, the extremities of chords drawn from P, tangent to the same interior circle. I use, as before, r to denote the radius of this circle, and $-a$ for the abscissa of its center, and designate the co-ordinates of P by c and s. The line MN is then the same as that the equation of which was written, in equation 6. We have seen that we can express the functions $sn\,(u + v)$ and $cn\,(u + v)$ in terms of the abscissa of the point M. In obtaining the abscissa from the equation, we shall necessarily find at the same time that of the point N, and thus derive simultaneously the formulæ for addition and subtraction of two arguments.

Fig. 6



To eliminate y between the equation of MN and that of the circle of the amplitude we have to replace that number by $\sqrt{R^2 - x^2}$; and if at the same time we put $\sqrt{R^2 - c^2}$ for s we have:

$$(a^2c + 2aR^2 + cR^2)\,x + (R^2 - a^2)\sqrt{(R^2 - c^2)(R^2 - x^2)} + R^2(a^2 + 2ac + R^2 - 2r^2) = 0.$$

On clearing of radicals, collecting terms, and dividing by R^2, this equation is readily reduced to the form,

$$(a^2 + 2ac + R^2)^2 x^2 + 2x(a^2c + 2aR^2 + cR^2)(a^2 + 2ac + R^2 - 2r^2) + (a^2c + 2aR^2 + cR^2)^2 - 4r^2R^2(a^2 + 2ac + R^2) + 4r^4R^2 = 0.$$

In solving this, it is convenient to denote the quantities $a^2 + 2ac + R^2$ and $a^2c + 2aR^2 + cR^2$ by A and B respectively. The equation is then written

$$A^2x^2 + 2Bx\,(A - 2r^2) + B^2 - 4Ar^2R^2 + 4r^4R^2 = 0;$$

and is solved without difficulty, yielding

$$-x = \frac{B\,(A - 2r^2) \pm 2r\sqrt{(A - r^2)\,(A^2R^2 - B^2)}}{A^2};$$

whence we have, for the values of $sn^2\,(u + v)$ and $cn^2\,(u + v)$ respectively,

$$\frac{R - x}{2R} = \frac{A^2R + AB - 2Br^2 \pm 2r\sqrt{(A - r^2)\,(A^2R^2 - B^2)}}{2A^2R},$$

and

$$\frac{R + x}{2R} = \frac{A^2R - AB + 2Br^2 \mp 2r\sqrt{(A - r^2)\,(A^2R^2 - B^2)}}{2A^2R}.$$

As these are to be perfect squares, it is natural to inquire whether the common radical term may not be, in each numerator, the double product of two factors whose squares make up the remaining terms. To test this in the case of the first fraction, we may put

$$m^2 + n^2 = A^2R + AB - 2Br^2, \text{ and } mn = r\sqrt{(A - r^2)(A^2R^2 - B^2)},$$

and then forming $(m^2 + n^2)^2 - 4m^2n^2$, we obtain a perfect square, from which we have

$$m^2 - n^2 = A^2R + AB - 2Ar^2R.$$

Hence we have at once the separate values of m^2 and n^2, and may rewrite the value of $sn^2\,(u + v)$ as follows:

$$\frac{R-x}{2R} = \frac{r^2(AR-B) \pm 2r\sqrt{(A-r^2)(A^2R^2-B^2)} + (A-r^2)(AR+B)}{2A^2R}; \quad (23$$

and proceeding in a similar manner with the other fraction, we shall obtain

$$\frac{R+x}{2R} = \frac{r^2(AR+B) \mp 2r\sqrt{(A-r^2)(A^2R^2-B^2)} + (A-r^2)(AR-B)}{2A^2R}. \quad (24$$

We have now to express these fractions in terms of the func-

tions of u and v, for which the following values are furnished by the preceding equations, (15) to (18), and (21),

$$\left.\begin{array}{lll} sn^2u = \dfrac{R-c}{2R} & cn^2u = \dfrac{R+c}{2R} & \\ sn^2v = \dfrac{(R+a)^2-r^2}{(R+a)^2} & cn\, v = \dfrac{r}{R+a} & dn\, v = \dfrac{R-a}{R+a} \\ \text{also } k^2 = \dfrac{4aR}{(R+a)^2-r^2}. & & \end{array}\right\} \quad (25$$

No value has hitherto been found for $dn\, u$, in our present notation,—*i. e.*, where $2u$ corresponds to an arc of amplitude terminating at c, s;—hence we must compute one from the formula $dn^2u = 1 - k^2sn^2u$. We have

$$\begin{aligned} dn^2u &= 1 - \frac{4aR}{(R+a)^2-r^2}\cdot\frac{R-c}{2R} \\ &= 1 - \frac{2a\,(R-c)}{(R+a)^2-r^2} = \frac{a^2+2ac+R^2-r^2}{(R+a)^2-r^2}. \end{aligned} \quad (26$$

The substitution of these values in the foregoing fractions presents little difficulty. Beginning, in the first fraction, with the term $r^2(AR-B)$, we have to evaluate in the first place the factor $AR-B$, by substituting for A and B the polynomials which they replace. We thus find this factor equal to $(R-c)(R-a)^2$. We recognize in this the product of the numerators of the above values of sn^2u and dn^2v, and in the other factor, r^2, that of cn^2v. The denominators, all multiplied together, amount to $2R(R+a)^4$, and by this quantity, therefore, we will divide both the numerator and denominator of the value of $\dfrac{R-x}{2R}$. In the last term of the numerator we now have $\dfrac{(A-r^2)(AR+B)}{2R(R+a)^4}$. But $AR+B$ directly reduces, (as above, but with change of sign) to $(R+c)(R+a)^2$. Hence $\dfrac{AR+B}{2R(R+a)^2}$ is cn^2u, and there remains the factor $\dfrac{A-r^2}{(R+a)^2}$, or $\dfrac{a^2+2ac+R^2-r^2}{(R+a)^2}$; and this is evidently $sn^2v\, dn^2u$.

In the numerator of the second fraction the same factors occur, the arrangement alone being different. It only remains, therefore, to evaluate the denominator which is common to both, and which, when divided by the same quantity as the numerator, becomes $\frac{A^2}{(R+a)^4}$; or the square of $\frac{a^2+2ac+R^2}{(R+a)^2}$. But we have just found that

$$\frac{a^2+2ac+R^2-r^2}{(R+a)^2} = sn^2v\, dn^2u, \quad \text{and} \quad \frac{r^2}{(R+a)^2} = cn^2v,$$

hence the denominator must be the square of $sn^2v\, dn^2u + cn^2v$, and when we put for dn^2u and cn^2v their values $1 - k^2 sn^2u$ and $1 - sn^2v$, this becomes

$$(1 - k^2\, sn^2u\, sn^2v)^2.$$

When, now, all the factors of our two formulæ (23) and (24) are replaced by the equivalent expressions in terms of the functions of u and v, the two members of each are perfect squares, and by extraction of the square root we obtain

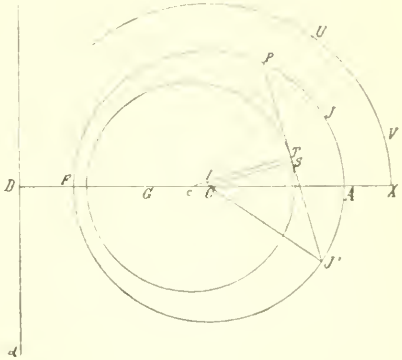
$$sn\,(u \pm v) = \frac{sn\,u\; cn\,v\; dn\,v \pm sn\,v\; cn\,u\; dn\,u}{1 - k^2\, sn^2u\, sn^2v}, \qquad (27$$

$$cn\,(u \pm v) = \frac{cn\,u\; cn\,v \mp sn\,u\; sn\,v\; dn\,u\; dn\,v}{1 - k^2\, sn^2u\, sn^2v}. \qquad (28$$

The extraction of the square root, both here and in the previous solution of a quadratic equation in x, yields a double-valued result. But we may remove all ambiguity by considering, in the first place, that these formulas cannot be affected in sign by a change in the value of k^2, as by making it approach zero; while secondly, for $k = 0$, the foregoing equations reduce to the trigonometric formulas for $sin\,(u+v)$ and $cos\,(u+v)$. The radical axis is in that case at an infinite distance, and all the circles are concentric. But the above arrangement of signs is that which occurs in the special case of the trigonometric formulas, and is therefore the right one in general. The same conclusion might of course be reached by an examination of the figures, in detail.

In order to deduce the formula for $dn\,(u+v)$, let us consider another diagram, (Fig. 7), in which, in so far as the same letters are used as in the preceding, they stand for the same things; viz., the double arguments $2u$ and $2v$ terminate at U and V respectively, and these points correspond to P and to J. Let us name the arcs AP and AJ, 2φ and 2ψ respectively, and let an arc AJ', of a length 2ψ, be measured from A in the negative direction. Join PJ', and draw the circle of the interior system tangent to this chord. As this circle is not the same as was drawn in Fig. 6, its centre is marked c' instead of c. We also denote the abscissa of c' by $-a'$, and the radius of the new cirle by r'. In the former figure a chord tangent to the interior circle would always terminate in points such that their corresponding points on the circle of the argument were separated by an arc of $2v$; here that constant arc is of length $2\,(u+v)$; so, while there we had

Fig 7



$\dfrac{r}{R+a}$ for $cn\,v$, and $\dfrac{R-a}{R+a}$ for $dn\,v$, here we shall have

$$\frac{r'}{R+a'} = cn\,(u+v), \quad \text{and} \quad \frac{R-a'}{R+a'} = dn\,(u+v). \qquad (29$$

Let S be the middle point of the chord PJ', and T its point of contact with the interior circle; draw CS and $c'T$, which will be perpendicular to PJ', and CI parallel thereto.

Now as $J'P = 2\varphi + 2\psi$, the angle $SCJ' = \varphi + \psi$; and if from this be taken ACJ' or 2ψ, the remainder, ACS, is $\varphi - \psi$. Now $c'T$ is r', but

$$c'T = c'I + IT, = c'I + CS.$$

$$c'I = a' \cos(\varphi - \psi) \quad \text{and} \quad CS = R\cos(\varphi + \psi);$$

$$\therefore \quad r' = a'(\cos\varphi\cos\psi + \sin\varphi\sin\psi) + R(\cos\varphi\cos\psi - \sin\varphi\sin\psi),$$

$$= (R + a')\cos\varphi\cos\psi - (R - a')\sin\varphi\sin\psi.$$

$$\therefore \qquad \frac{r'}{R + a'} = \cos\varphi\cos\psi - \frac{R - a'}{R + a'}\sin\varphi\sin\psi.$$

But φ and ψ are the amplitudes of u and v, whence,—comparing equation (29),—

$$cn\,(u + v) = cn\,u\;cn\,v - sn\,u\;sn\,v\;dn\,(u + v).$$

Now a similar formula, containing functions of $u - v$, may of course be obtained in like manner, using an interior circle that is tangent to PJ instead of to PJ'. And when the two are combined in one statement, we have

$$cn\,(u \pm v) = cn\,u\;cn\,v \mp sn\,u\;sn\,v\;dn\,(u \pm v). \tag{30}$$

This is the so-called "Formula of Lagrange." From it is derived directly

$$dn\,(u \pm v) = \frac{cn\,u\;cn\,v - cn\,(u \pm v)}{\pm\, sn\,u\;sn\,v},$$

and thence, by substituting the value of $cn\,(u \pm v)$, as previously found,

$$dn\,(u \pm v) = \frac{dn\,u\;dn\,v \mp k^2\,sn\,u\;sn\,v\;cn\,u\;cn\,v}{1 - k^2\,sn^2u\;sn^2v}. \tag{31}$$

It has already been noticed that when any argument u is increased by $2nK$, the corresponding amplitude φ is increased by $n\pi$. Moreover, when $u = 0$, then also $\varphi = 0$. And since the elliptic functions of u are defined by trigonometric functions of φ, it is at once evident that since the latter have a period of 2π, the elliptic functions must correspondingly repeat themselves in periods of $4K$. These inferences may be tested by the addition formulæ just derived, and in particular it may be shown that $dn\,u$ has in fact the shorter period $2K$.

I wish to indicate in a single paragraph, by way of appendix, how easily the definitions and formulæ of the present paper

may be connected with the methods of the calculus, and also the character of the assumption which has to be made, in order to give to the circle of the argument its appropriate relative magnitude, *i. e.*, to fix the value of K.

By subtracting, one from the other, the two formulæ included in equation (27) we have

$$sn\,(u+v) - sn\,(u-v) = \frac{2sn\,v\; cn\,u\; dn\,u}{1 - k^2\, sn^2 u\; sn^2 v}.$$

Let us replace u by $w + \frac{1}{2}h$ and v by $\frac{1}{2}h$, and then divide each side of the equation by h; and we obtain

$$\frac{sn\,(w+h) - sn\,w}{h} = \frac{2sn\,(\frac{1}{2}h)\; cn\,(w+\frac{1}{2}h)\; dn\,(w+\frac{1}{2}h)}{[1 - k^2\, sn^2\,(w+\frac{1}{2}h)\; sn^2\,(\frac{1}{2}h)]\,h}$$

$$= \frac{cn\,(w+\frac{1}{2}h)\; dn\,(w+\frac{1}{2}h)}{1 - k^2 sn^2(w+\frac{1}{2}h)\, sn^2(\frac{1}{2}h)} \cdot \frac{sn\,(\frac{1}{2}h)}{\frac{1}{2}h}.$$

We have here separated the result into two factors. When h is made to approach zero, the denominator of the first factor approaches unity, because the second term of that denominator contains the vanishing factor $sn^2(\frac{1}{2}h)$. The numerator of the same factor approaches $cn\,w\; dn\,w$. But in the second factor we have the ratio of the chord of a small arc of the circle of the amplitude divided by the corresponding arc in the circle of the argument. That this factor may approach unity as h approaches zero, it is necessary that corresponding arcs, measured from the origins in the two circles, shall approach equality as their length is diminished. Or if corresponding arcs are traced simultaneously by two points, one on each circle, these points must set out from the origins with equal initial velocities. This condition will determine the relative magnitude of the circles; and when it is fulfilled, the limit, for $h = 0$, of $\frac{sn\,(w+h) - sn\,w}{h}$ is $cn\,w\; dn\,w$.

ON TWO PASSAGES IN THE CRITO.

BY H. W. MAGOUN.

ΣΩ. Σκοπῶμεν, ὦ 'γαθέ, κοινῇ, καί εἴ πῃ ἔχεις ἀντιλέγειν ἐμοῦ λέγοντος, ἀντίλεγε, καί σοι πείσομαι· εἰ δὲ μή, παῦσαι ἤδη, ὦ μακάριε, πολλάκις μοι λέγων τὸν αὐτὸν λόγον, ὡς χρὴ ἐνθένδε ἀκόντων Ἀθηναίων ἐμὲ ἀπιέναι· ὡς ἐγὼ περὶ πολλοῦ ποιοῦμαι πεῖσαί σε ταῦτα πράττειν, ἀλλὰ μὴ ἄκοντος.

The above passage occurs in the Crito of Plato, 48 D. and E. The works *πεῖσαί σε*, near the end, seem to have proved a veritable *crux* to the grammarians and the opinions concerning them have been various. They are spoken by Socrates in his argument with Crito, in which he tries to show him the fallacy of urging an escape from the prison, to avoid the fatal hemlock, so soon to be administered to himself in accordance with the sentence pronounced against him by the Athenians. A brief review of the various views held, so far as the limited library resources at my command has supplied them, will be stated first, after which will be given what seems to me to be the meaning of the passage.

The MSS. without exception, so far as I can discover, read as above, *πεῖσαί σε*. . . . Schleiermacher, Stallbaum, Elberling, and many other commentators have adopted this reading, taken *σε* as the subject of *πεῖσαι*, supplied *ἐμέ* as its object, and understood *ἐμοῦ* with *ἄκοντος*. . . . Jowett evidently agrees with this, since he renders; "for I am extremely desirous to be persuaded by you, but not against my own better judgment." . . . Bekker and some others take *σε* as the object of *πεῖσαι* and supply *σου* with *ἄκοντος*, rendering: "As I esteem it of great importance to persuade you to do this, but not to do it against

your will." . . . W. S. Tyler, Professor of Greek at Amherst, in his edition of 1876, adopts this rendering and explains it to mean: "I am exceedingly desirous to pursue the course I am pursuing *with your consent* and *not against your will.*" In a later edition, which was prepared with the assistance of Professor H. M. Tyler, he reads πείσας σε, in accordance with the emendation given just below, and renders: "*As I esteem it of great importance to do this with your consent.*" . . . Buttmann amended to πείσας σε and the emendation has been accepted by a number of scholars. The meaning, of course, being that just given. . . . Goebel suggests that the reading should be παῦσαί σε, 'I am anxious to stop you,' *i. e.*, from bringing forward the same old arguments. . . . Meiser proposes a change of order; πεῖσαί σε, ἀλλὰ μὴ ἄκοντος ταῦτα πράττειν. . . . Professor Dyer, of Harvard, in his edition based on that of Cron, adopts the emendation of Buttman as Tyler does in his latter edition. . . . Wagner reads πείσας, "with your approval," and puts it as antithetical with ἄκοντος, "without your approval."

Before considering these various renderings it may be well to have the rest of the passage translated that the meaning of the whole may be before us. Jowett's rendering, which with the exception of the last clause, can hardly be improved upon, is as follows: "Let us consider the matter together, and do you either refute me if you can, and I will be convinced; or else cease, my dear friend, from repeating to me that I ought to escape against the wishes of the Athenians: for" etc., as above.

The position of Schleiermacker, which is followed by Jowett and others, is met by Professor W. S. Tyler in his old edition, where he says: "But besides the improbability of πεῖσαι being *followed* by its subject, and *omitting* its object, it does not accord with the sentiments and character of Socrates that he should say, I deem it of great importance that you should persuade me to *leave the prison,* which would then be the meaning of the passage." This may be regarded as a sufficient answer to this rendering. . . . The emendation of Buttmann may

be considered next. It is evidently intended, as is brought out by Wagner, to make the two expressions, πείσας and ἄκοντος antithetical, as though the craze for antithesis which reaches its akme in Isocrates had actually taken possession of plain and rugged Socrates who was by no means in the habit of amplifying a statement for the sake of making it well balanced. Leaving out of the question the harshness of the three sigmas in the emendation, πείσας σε, which is alone sufficient to condemn it from a Greek standpoint, the question remains as to what he is anxious to do when, according to the emendation, he says, in the words of Tyler, "I esteem it of great importance to do this with your consent." Remain in the prison, say the commentators. Then in accordance with Greek usage, we may expect to find a statement to that effect in the context immediately preceding; but Socrates has nowhere uttered any such sentiment nor is it at all likely that he would say that he was anxious to stay there. The meaning of the ταῦτα must be looked for in the context, not explained by dragging in some outside idea. On this basis, according to the emendation, Socrates would say: I am exceedingly anxious to do this, *i. e..* examine this question with you (σκοπῶμεν . . . κοινῇ), if you are willing. . . . Taking now the MS. reading, which is followed by Bekker and some others, and looking for the explanation of ταῦτα in the context, the passage means, not as Tyler explains it in his old edition, "I am exceedingly desirous to pursue the course I am pursuing;" but 'I am exceedingly desirous to induce you to do this,' *i. e.*, refute me (ἀντίλεγε) if you can (εἴ πῃ ἔχεις ἀντιλέγειν) or else stop (παῦσαι) bringing forward the same old argument (τὸν αὐτὸν λόγον); 'but I do not wish to force it upon you' (ἀλλὰ μὴ ἄκοντος). This is evidently the underlying thought in Goebel's emendation to παῦσαι which is, however, hardly tenable on other grounds, and the meaning still remains the same in Meiser's proposed change of order, which is unnecessary. This meaning tallies exactly with what has preceded and with all that follows. His whole object and aim has been to do just this, namely to force

Crito to either take new ground or let him alone. In his own mind, of course, the latter is the real goal and Tyler evidently recognized this in his explanation of the MS. reading; but it is only an inference from the whole passage, not a meaning to be taken directly from the Greek; indeed that this was his real aim become clear only as the argument proceeds. Every doubt of his object vanishes when the last words of the dialogue are reached, in which he tells Crito that the voice which hums in his ears prevents him from hearing any other and that it will be useless for him to say anything more.

μὴ οὐ δέῃ ὑπολογίζεσθαι οὔτ᾽ εἰ ἀποθνῄσκειν δεῖ παραμένοντας καὶ ἡσυχίαν ἄγοντας, οὔτε ἄλλο ὁτιοῦν πάσχειν πρὸ τοῦ ἀδικεῖν.

This passage occurs in 48 D, just above the one already considered. Most of the editors with Stallbaum place a comma after ἄγοντας which has the effect of making πρὸ τοῦ ἀδικεῖν depend upon πάσχειν. Some of the recent editors omit the comma and thus allow the πρὸ τοῦ ἀδικεῖν to go back in thought to the ὑπολογίζεσθαι where it logically belongs as can readily be seen from the context. The burden of his entire argument is that they must consider first the question as to whether they are doing right or wrong and everything else must give way to this. We have here then, as often in Plato, a condensed expression such as is quite characteristic of Socrates who seems to delight in "cutting across lots." It stands for something like this; μὴ οὐ δέῃ ὑπολογίζεσθαι οὔτ᾽ εἰ ἀποθνῄσκειν δεῖ παραμένοντας καὶ ἡσυχίαν ἄγοντας οὔτε ἄλλο ὁτιοῦν πάσχειν πρὸ τοῦ ὑπολογίζεσθαι εἰ ἀδικεῖν δεῖ ἢ μή.

The commentators, so far as I have been able to consult them, seem to regard the πρὸ τοῦ ἀδικεῖν as depending upon πάσχειν, which involves a logical contradiction, perhaps too subtle to be noticed by the casual observer; but still a clear logical contradiction of his position; for it makes his say; "we must not consider the question, whether we must die, if we remain in prison and keep quiet, nor whether we must suffer any thing

else, however dreadful, rather than do wrong," *i. e.*, we must not consider the question of suffering anything whatever rather than do wrong, whereas his real position is the exact reverse of this, namely, we m u s t consider the question of suffering anything whatever rather than do wrong and must make up our minds to suffer anything or everything rather than do wrong. In effect he puts before Crito this question: Shall we suffer anything whatever rather than do wrong? and asks him to consider it with him. His reply to the question is; Yes. While the commentators give renderings similar to the one cited near the beginning of this paragraph, they do not seem to realize that they put the "suffering anything whatever rather than do wrong" together as a single thought and then make Socrates say that it must not be considered; for they all agree that the passage means as Tyler puts it, "we must not take the *consequences* into the account at all, but only the question of right and wrong."* That this is the meaning there can be no shadow of a doubt; but how to get it without carrying the πρὸ τοῦ ἀδικεῖν back to the ὑπολογίζεσθαι, which none of them seem to do in their translations certainly, is a puzzle. That this is its real dependence and that the sentence is a condensed one for something like the expanded form suggested above seems very likely, since it removes every difficulty in the way of a clear understanding of the passage. †

* Jowett dodges the difficulty while giving the sense perfectly. He renders: "and if the latter [if we shall do rightly in escaping], then death or any other calamity which may ensue on my remaining here must not be allowed to enter into the calculation."

† Cf. the similar passage, A p o l o g y, 28 D.; ἐνταῦθα δεῖ, ὡς ἐμοὶ δοκεῖ μένοντα κινδυνεύειν, μηδὲν ὑπολογιζόμενον μήτε θάνατον μήτε ἄλλο μηδὲν πρὸ τοῦ αἰσχροῦ.

THE CALIBRATION OF BURETTES.

BY DOUGLAS CARNEGIE.

The balance is the supreme tribunal of the chemist. No matter how simple and brief, or how complicated and prolonged, a chemical operation may be, its alpha and omega is a weighing.

Oftentimes, however, the conditions of the investigation are such that direct weighing cannot be carried out with the exactitude demanded. In such cases, recourse is had to a method of indirect weighing which I venture to call volumetric weighing.

It is the object of this note to show how this volumetric weighing may be carried out with accuracy.

Suppose that we wish to ascertain the exact quantity of silver nitrate that is necessary to completely precipitate 1 gramme of common salt in a solution colored yellow by potassium chromate. It is well known that if we add the silver nitrate in small quantities to such a solution, the latter will finally change from yellow to red, when just a little more silver nitrate has been added than is necessary to precipitate the whole of the salt. Suppose that in this investigation we add the silver nitrate in small pinches from a quantity originally weighed, and that as soon as the color change appears we weigh the quantity of silver salt remaining. It is obvious from what precedes that the quantity of silver nitrate thus deduced will be two great by a pinch, or some fraction of a pinch. If the average weight of the pinches was, say, 1 m. g.: and the whole experiment turns on $\frac{1}{1000}$th of this quantity, it is clear that the investigation does not lend itself to a method involving direct weighings.

The recipes of chemistry are not like those of the cuisine; the latter deal with a palate which is insensible between certain wide limits—the range embraced between the extremes "too salt," and "not salt enough," being a fairly wide one, and easily hit off. But the recipes of chemistry have to do with indicators which are so fastidious that there is always either too much or too little salt for their taste; and the object of indirect or volumetric weighing, is to give them a little too much salt, but at the same time to contrive that the excess is infinitesimal in comparison with the degree of accuracy aimed at.

If a known weight w of silver nitrate be dissolved in a known volume (a litre, say) of water, so that one drop of the solution contains $\frac{1}{1000}$ m. g. of dissolved silver salt, then it is clear that by adding such a solution, drop by drop, from a burette to the common salt solution, a very much more accurate and quicker result will be obtained than by the first procedure. If x c. c. of the silver solution, as read off from the burette graduations, have been used, then we know that 1 gramme of salt requires exactly $\frac{x \times w}{1000}$ grs. of silver nitrate to precipitate it, *provided only that the graduations of the burette used to measure both the x c. c. and the litre are consistent among themselves.* The determination whether or no this is the case constitutes the "calibration of the burette."

The method of calibration given in all text books is roughly as follows. The burette is filled with water, and successive portions, corresponding to *5 c. c.* or *so*, are run out into dry beakers. From the weights of these successive portions a table of corrections can easily be deduced. This method in the case of an ordinary *50 c. c.* burette and with *5 c. c.* tests, involves at least 11 weighings and 10 level-readings.

The method of calibration I would suggest is, I believe, more accurate than the above, and involves no weighing whatever; for it is quite a matter of indifference to the chemist, whose measurements are all relative, whether he adopts as his unit the true cubic centimetre, or an arbitrary one.

The principle of the method consists in causing a constant volume of liquid to slide in the burette (so that it may be made to assume any desired position therein) and then reading off its length in terms of the burette divisions.

In Fig. 1 the burette B is attached by means of caoutchouc tubing C, to a reservoir R, which is filled with water, and which can easily be raised or lowered by sliding the clamp K along S. On the surface of the water in the burette there is a small column of carbon bisulphide W, which obviously, can be made to assume any desired position in the burette by either raising or lowering R, and then turning the stop-cock n very gently.

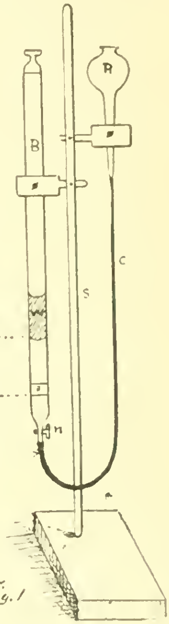


Fig. 1

In my first experiments I used mercury in place of water in R, and water in place of carbon bisulphide as a movable index. But this combination was unsatisfactory; I found it exceedingly difficult to read the level l, both on account of the convexity of the mercury surface, and also on account of the peculiarly disturbing light reflexion from the bounding water-mercury surface.

The combination, water and carbon bisulphide works admirably if the following precautions be taken.

The burette must be *thoroughly cleaned* from grease and dust before calibration. The resevoir R is then filled with freshly distilled water, and matters are so arranged that the level of the water stands at g, the *lowest* graduation of the burette. A long thistle funnel is then inserted into the burette, and a small quantity (say *5c. c.*) of recently distilled CS_2 is poured gently on the surface of the water. Under these circumstances the CS_2, in spite of its higher specific gravity, floats on the water, and exhibits two easily localized concave meniscuses. It is well to shake the CS_2 up with distilled water, so as to thoroughly satur-

ate it, and to tint it slightly with iodine before pouring it into the burette.

The method of procedure is obvious. I will merely add that in altering the position of the CS_2 index, it is well to let it rise a little higher in the burette than the required graduation, and then lower it by careful manipulation of the stop-cock *n*. The burette should, of course, be kept stoppered as much as possible during the calibration.

I would also particularly emphasize the fact that the method works satisfactorily *only* when the calibration is effected as described—from below, upwards.

The readings obtained can be used for calibration as described in detail in Bunsen's "Gasometry."

In order to avoid errors of parallax in reading off the levels of the CS_2 column, I silver a strip on the back of the burette This mirror strip, which does away with all floats and similar devices for avoiding errors of parallax in the subsequent use of the burette, is easily made as follows. The burette having been

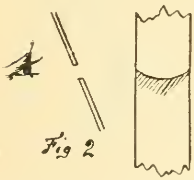


Fig 2

thoroughly cleaned externally, is corked air-tight, and then inverted in a tall cylinder containing an alkaline silvering fluid* After some time it is withdrawn and allowed to become *thoroughly air-dried*. The silver can then be easily removed from the front and sides by scraping, followed by a final rubbing with a rag moistened with dilute nitric acid. The slip of silver backing the burette is then fixed by one or two coats of varnish.

In reading such a burette it will be found advantageous to look at the meniscus through a slit cut in a sheet of white glazed paper, held at an angle to the vertical as shown in Fig. 2. The burette graduations are also rendered very much more distinct by rubbing the front of the burette first with a piece of flannel on which a paste of turpentine and mercuric iodide has been placed, and then with a dry cloth.

*Suitable fluids are described in Roscoe & Schorlemmer's Treatise on Chemistry Vol. II., part I., p. 363.

ON A PASSAGE IN THE FROGS.

BY H. W. MAGOUN.

In the Frogs of Aristophenes in a speech of Dionysus, line 268, occur the words:

ἔμελλον ἄρα παύσειν ποθ' ὑμᾶς τοῦ κοάξ,

'I thought I should make you stop your croaking sometime.' The passage seems to have troubled some of the commentators, notably Fritzsche who thinks that the commentators as a whole have quite failed to understand the passage. He takes it in perfect seriousness and supposes that Dionysus must mean all that he says.

He infers from line 257,

οἰμώξετ'· οὐ γάρ μοι μέλει,

'Go howl; for I don't care a fig,' which he renders 'be cudgeled,' (*vapulate*) etc., that Dionysus takes his oar and beats the frogs; but he fails to explain what Charon was doing all this time that he allowed such proceedings, or how the boat could come to land with one oar only in use for rowing. The simple fact is that Dionysus is merely an overgrown boy in his actions and words, in fact a kind of clown almost, and he must be judged on this basis. The Greeks as a whole were "but children of a large growth" at this time, and in their comedies especially were like a crowd of boys. The frogs began their croaking only when Dionysus embarked and began to row, lines 206–7,

ἀκούσει γαρ μέλη
κάλλιστ', ἐπειδὰν ἐμβάλῃς ἅπαξ.

It is natural then to expect them to stop when he ceases rowing as the boat comes to land. This is exactly what happens and Dionysus, who has been bawling at them to stop their noise, in

true boyish fashion now takes all the credit of their silence to himself. It may be added that Charon had told him that the singing would make his rowing easy, line 206, ῥᾷστ' ἀκούσει etc., as above, although he was, as he himself said, a land-lubber, ἀθαλάττευτος, line 204. Human nature is all that is needed to explain the passage, or more accurately youthful nature indulging itself in boyish pranks and words.

A NOTE ON THE HADLEY-ALLEN GRAMMAR

By H. W. Magoun.

While Professor Allen has made an excellent revision of Hadley's Greek Grammar, it has happened in a few instances that the improvement intended has failed to be realized. Such a case appears in § 199, where he says of feminines in *-ώ* of the third declension: "These stems seem to have formerly ended in *-oϝ-ι*: hence the voc. sing. in *-οῖ* an older form of the nom. in *-ῴ*: *Σαπφῴ*." This explanation might help to make clear the voc. sing., which in the third declension is prevailingly like the stem; but it must fail to account for the nom., since the question arises at once how it happens that the nom. did not take the regular ending *-ς* precisely as other iota-stems do, such as *πόλι-ς*, *δύναμι-ς*, etc., which it will be observed are also feminines, and it further produces the grave difficulty of accounting for the dropping of both the digamma and the iota in the oblique cases where such a stem ought to give us a diphthong *-οῖ-*. It seems very likely that the real explanation may be found in supposing that these are really "yod"-stems, that is, that the real final letter is the so-called iota which appears under a changed form in the so-called iota class of Greek Verbs, where the old Greek *-ιω* appears in Sanskrit as *-yā-*. If this is the case, it may be possible to explain the whole paradigm. In the oblique cases this "yod" simply dropped, exactly as it did in *τρεῖς*,* Sanskrit *tráy-as*, and contraction took place precisely as

* See Brugmann, *Grundriss der Vergleichenden Grammatik der Indogermanischen Sprachen*, Erster Band, §§ 129 and 130 Cf. also §§ 117, 118, 119, etc. For verb forms see Müller's *Handbuch der Klassischen Altertums-Wissenschaft*, Zweiter Band, §§ 123 and 124.

in similar cases. For the voc. the following equation may be ventured upon:

Voc. σαπφοῖ : stem of σαπφώ : : voc. -βοῦ : stem of βοῦ-ς

that is, in each case the stem form is used for the voc. according to the regular rule; but the final elements (digamma* and "yod") are forced to pass into the corresponding vowels and so become part of a diphthong by the Greek law which prevents any consonant from being final, save ν, ρ, ς. This rule is plain for the digamma: it may not be so clear for the "yod," as will be shown below. However, analogy is quite sufficient to establish the equation and make this explanation hold good. The accent in both cases can be readily accounted for by the well-known Indogermanic law that the accent of vocatives is recessive.† The nom. remains te be accounted for. Professor Allen mentions the fact that the old form ended in -ῴ. That this was the original form has been made clear by certain inscriptions, for example, those of the Corinthian dialect, of which a few have been found. In these inscriptions names of women appear in the nom., ending, as Cauer ‡ writes them in -ό with ι-subscript, which means of course that they were written in the old alphabet and ended in *OI*. Just here it may be noted that all other nouns of the third declension in Greek, except neuters, end in a consonant, either the regular ending -ς or the final consonant of the stem with a lengthening of the stem vowel. If, then, the final iota of these old nominatives can be shown to be a "yod" or y, the form is a perfectly normal one ending in the consonant of the stem with the regular vowel lengthening to -ω-. This would account for the missing -ς and make the form

* That the stem of βοῦ-ς is βοϝ-, in spite of Sanskrit *gāú-s*, appears from Lat. *bō-s*, gen. *bov-is*. Cf. Muller's *Handbuch*, Zweiter Band § 24[5]. The fact that the dialects show ἱερής for ἱερεύς may be taken as evidence that βασιλεύς, etc., ought also possibly to be classed as digamma stems. See Gustav Meyer, *Griechische Grammatik*, Zweite Auflage, § 323.

† See *American Journal of Philology*, Vol. IX, p. 16, Bloomfield, *The Recessive Accent in Greek*.

‡ See Cauer, *Delectus Inscriptionum Græcarum*, p. 52.

plain. Later on, when *-ωι* became *-ῳ* in dative forms, etc., this "yod" would naturally drop altogether; for, while graphically the two endings were the same, phonetically they may be represented as follows:

Final *-ωι* of *Σαπφῴ*: final *-ωι* of dat. *ὁδῷ* : : final *οι* of *οἴκοι* : final *-οι* of *οἶκοι*.

That is, the final letter of *Σαπφῴ* is probably the weaker of the two,* being about equivalent to an English -y, while in the second case the *-ι* is really a vowel. This explanation is suggested merely as a possible one which seems to meet the pecular difficulties of the case. These nouns were treated a few years ago in one of the German periodicals; but the writer of this note has never seen the article and it is not at present available. It only remains to be said that while such nouns as Sanskrit *sénā*, voc. *séne* may throw some light on these formations, it seems rather doubtful on the whole.

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* See *American Journal of Philology*, IX, 25. If it be true that the final *-οι* of *οἶκοι* is really -oy, the reason for its being short in determining accent becomes clear at once. Moreover, in such lines as Hom. Il. B. 136

αἱ δέ που ἡμέτεραί τ' ἄλοχοι καὶ νήπια τέκνα

it will not be necessary to fall back upon the *ictus* as a reason for the long final *-αί* and *-οι*; but the final *-ι* plus the following consonant being equal to *-y* plus a consonant will make position and so cause a long syllable.

HISTORICAL NOTE ON THE DIFFERENTIATION OF A LOGARITHM.

BY FLORIAN CAJORI.

The preface of Olney's Calculus contains the following passage: "In conclusion I must do myself the pleasure to acknowledge my indebtedness to my accomplished colleague and friend, Prof. J. C. Watson, Ph. D., for the original, direct, and simple method for demonstrating the rule for differentiating a logarithm . . . . *which banishes from the Calculus the last necessity for resort to series to establish any of its fundamental operations.*" Statements to this effect are also found in a few more recent American works on the Calculus.

Dr. E. W. Davis has called my attention to the fact that a method of proving the rule for differentiating logarithmic expressions without resorting to infinite series is given in De Morgan's Calculus. I see that such a method is found also in John Rowe's "Introduction to the Doctrine of Fluxions," Fourth Edition, London, 1809. The first edition was printed in 1751. Doubtless other old books with similar methods of demonstration which do not involve infinite series can be found. The credit of first banishing from the Calculus "the last necessity of resort to series to establish any of its fundamental operations" can, therefore, not be ascribed to Watson.

A MATHEMATICAL ERROR IN THE CENTURY DICTIONARY.

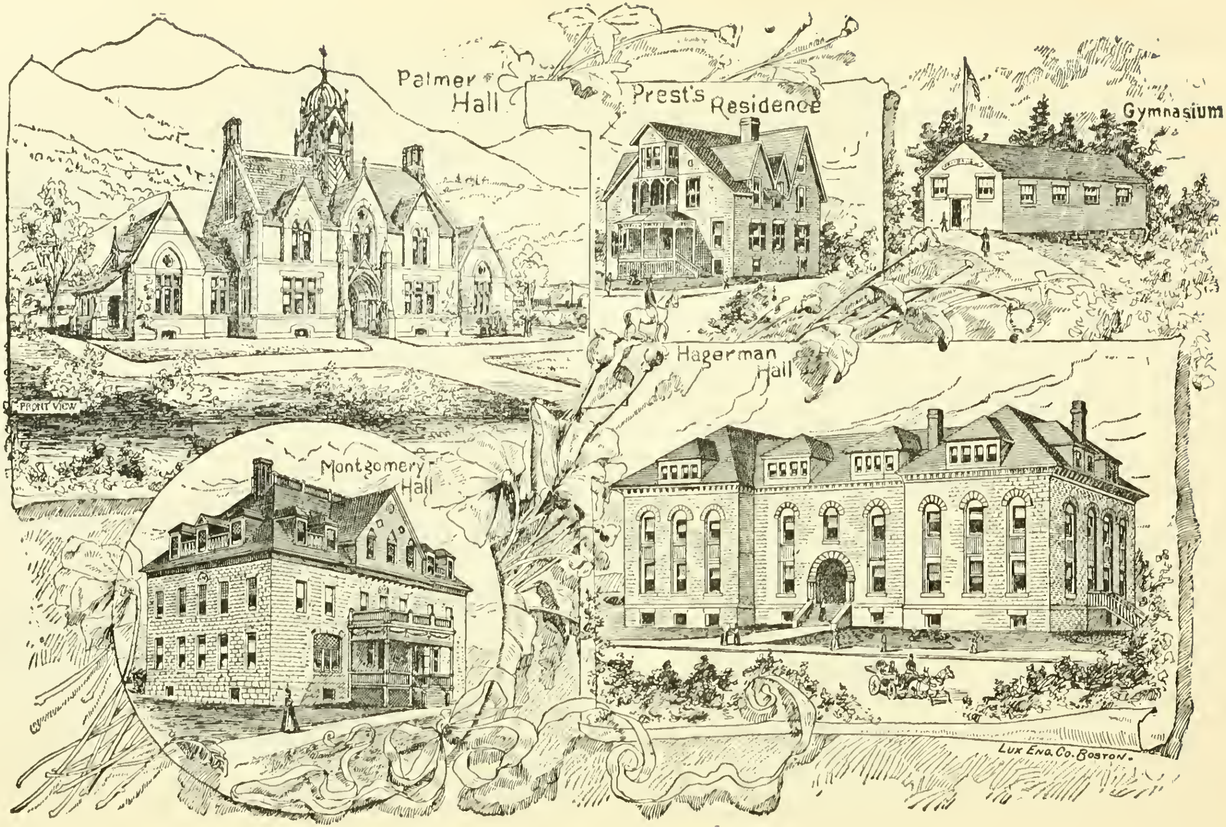
BY FLORIAN CAJORI.

The Century Dictionary, under the head of "Logarithm," gives a table in which are found the two left hand columns of numbers given below. The second column is wrong, all the numbers being too small by 61180956. The relation between Napier's and natural logarithms is expressed by the well-known formula,

$$\log_N a = 10^7 \log_e \frac{10^7}{a}.$$

Calculating by this formula the logarithms of .1, 1, etc., we get the numbers in the columns on the right, which are the true Napier's logarithms. Napier's original publication, the *Descriptio canonis* of 1614, contained only logarithms of *sines*. The logarithms of the particular numbers given below can therefore not be found in the table. But by our formula we are able to reproduce the logarithms in Napier's tables, and we are, therefore, sure that our calculated logarithmic figures in the column on the right are correct. For example, the sine of 44° 19′ is given in Napier's table as 6986235 and its logarithm, 3586432. Quite the same logarithm is obtained by calculation by our formula. Again, the Century Dictionary gives a *minus* logarithm for 10^7, while Napier's logarithms of all numbers below 10^7 are positive.

Natural Numbers	Napier's Logarithms, as given in C. D.	The true Napier's Logarithms.
0.1	123025851	184206807
1	10000000	161180956
10	76971149	138155105
100	53948298	115129254
1000	30922147	92103103
10000	7896596	69077552
100000	—15129255	46051701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Prest's Residence
Gymnasium
Front View
Hagerman Hall
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THIRD ANNUAL PUBLICATION.

Colorado College Studies.

PAPERS READ BEFORE THE COLORADO COLLEGE SCIENTIFIC SOCIETY.

COLORADO SPRINGS, COLO.

1892.

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ANNOUNCEMENT.

The *Colorado College Scientific Society* has entered upon the third year of its existence, and hereby issues its third annual publication. It is the intention of the Society to include in the publication for next year meteorological records obtained from self-registering instruments in the Physical Laboratory of Colorado College.

The following is a complete list of the papers read at the monthly meetings of the Society during the past year. Several of the papers are printed in full in this pamphlet, while others have been or will be published elsewhere.

October 13, 1891 -

The German Conditional, ... Dr. Sylvester Primer

Historical Sketch of the Study of Mathematics in the United States (published in *Bibliotheca Mathematica*, No. 3. 1891, *Stockholm, Sweden*) . . . . Florian Cajori.

November 10, 1891—

Notes on Jefferson's Draft of the Ordinance of 1784, ... W. M. HALL,
The Delphin Classics, ... DR. W. P. MUSTARD.

December .5, 1891—

Dust, ... DOUGLAS J. CARNEGIE.
The Australian Ballot System, ... W. M. HALL.

January 19, 1892—

A Review of Weissman's Theory of Heredity, ... PRES. WM. F. SLOCUM.

February 15, 1892—

Some Notes on Blaydes' Nubes, . . . DR. A. T. MURRAY.
Helmholtz's and Koenig's Theories of Harmony, ... DOUGLAS J. CARNEGIE.

March 29, 1892—

Cold Winds of the Plains (Extract from a Paper prepared for the *U. S. Weather Bureau*), ... FRANK H. LOUD.
Multiplication of Series (published in the *Bulletin of the N. Y. Mathematical Society*), ... FLORIAN CAJORI.
Draper's Barograph, ... FLORIAN CAJORI.

April 26, 1892—

Hypnotism, ... MARION M. NOYES.
The Detection of Gas Leakage in House Plumbing, ... WILLIAM STRIEBY.
On a Passage in Euripides' Iphigenia Taurica, ... DR. A. T. MURRAY.
The Etymologies in the Servian Commentary to Vergil, ... DR. W. P. MUSTARD.

May 16, 1892—

The Rise of the New Psychology, . . . F. R. HASTINGS.
Actinomycosis or Lumpy Jaw, . . . H. C. CROUCH, M. D.
The Evolution of Criteria of Convergency (to be published in the *Bulletin of the N. Y. Mathematical Society*), . . . FLORIAN CAJORI.

THE ETYMOLOGIES IN THE SERVIAN COMMENTARY TO VERGIL.

By WILFRED P. MUSTARD.

I.—Servius the Etymologist.

The Roman philologists, from Aelius Stilo down, were much given to etymologizing. Some interesting specimens of early effort in this line are cited by Quintilian, Inst. or. I 6. 32-38, where some of the results of the principles recognized and employed by his predecessors are characterized in the words '*inde pravis ingeniis ad foedissima usque ludibria labuntur.*' The vast number of etymological notes in the commentary to Vergil shows what a charm this fascinating study had for the grammarians of a later day.* In one or two points Servius shows a distinct improvement upon the methods of his predecessors, but he has not escaped all the faults of his age. His character as an etymologist may be inferred from the following summary.

I. He makes free use of the principle that objects may be named from their contraries, κατ' ἀντίφρασιν, applying it to the explanation of the following words: ardea, A. 7, 412; bellum, A. 1, 22; Charon, A. 6, 299; Eumenides, A. 3, 63; 6, 250; 6, 375; G. 1, 278; lucus, A. 1, 22; 1, 441; lustrum, A. 1, 607; mactare, A. 4, 57; manes, A. 1, 139; 3, 63; Parcae, A. 1, 22; *G. 1, 278.* This convenient principle, which Voss in his Etymologicon, *v. lucus*, called '*inane Grammaticorum commentum*,' had long been employed by the Greeks. That it was recognized from an early period at Rome is evident from Paulus, p. 88†: *Militem Aelius a mollitia* κατὰ ἀντίφρασιν

* Servius offers or quotes etymologies for almost a thousand words.

† References to Festus and Paulus in this paper are to the pages of Ponor's edition, 1889.

dictum putat, eo, quod nihil molle, sed potius asperum quid gerat.

II. He is very fond of deriving Latin words from Greek. On Aen. 1. 184. he says: Sciendum autem est etiam Latina nomina Graecam plerumque etymologiam recipere. Naturally enough he goes to an extreme in this direction, and we find him offering a Greek etymology not only to Latin words, which are at most cognate with Greek. but to others where the connection he assumes is absolutely wrong. A few examples will suffice to show some of the results of this tendency.* animus. A. 1. 57. and anima, A. 8. 403. ἀπὸ τῶν ἀνέμων. antes. G. 2. 417, ἀπὸ τοῦ ἀντιστήκειν. aptum, A. 4. 482; 11, 202. ἀπὸ τοῦ ἅπτεσθαι. ars, A. 5, 705. ἀπὸ τῆς ἀρετῆς. inclita, A. 6, 781. Graecum est; nam κλυτὸν gloriosum dicunt. ara, A. 2. 515. a precibus. quas Graeci ἀρὰς dicunt. rura. A. 1. 430. Graece ἄρουρα dicuntur. Aphaeresis ergo sermonem fecit Latinum. telum. A. 2, 468; 8, 249; 9. 507; 9. 744, ἀπὸ τοῦ τηλόθεν. ulna, B. 3. 105. ἀπὸ τῶν ὠλενῶν. ululae. B. 8. 55, ἀπὸ τοῦ ὀλολύζειν. uri. G. 2. 374, ἀπὸ τῶν ὀρέων. Frequently we have two or more derivations offered for the same word. and in several cases one of these optional etymologies is from the Greek. For examples see the explanations of Acidalia, A. 1. 720: ancile, A. 8. 664; annus, A. 1. 269; clarigatio. A. 10, 14; Quirinus, A. 1, 292; scopulus, A. 1, 45; Vesta. A. 1. 292.

Nettleship, *Lectures and Essays, p. 212*, notices a remarkable difference between Varro and Verrius Flaccus in the matter of etymology. Varro, if we may judge by the De Lingua Latina, preferred to explain Latin words by assuming for them a Latin origin: Verrius, to judge from the epitome of Paulus, certainly seems to have a predilection for deriving Latin words from Greek. *Possibly Servius' fondness for Graecizing etymologies is one of the results of his indebtedness, direct or indirect, to Verrius' great work.*

III. The Roman philologists usually confined their inquiries very strictly to their own and the Greek language, paying little attention to other languages or even to the

* All quotations from Servius in this paper are from the edition of Thilo and Hagen, 1878-1887.

Italian dialects. It may be noted that Servius mentions not only several Greek words which are not Attic but about thirty words which are neither Latin nor Greek. These he assigns to various languages and dialects as follows: *Punic:* Bal. A. 1, 729; caesa, A. 1. 286; Carthago, A. 1. 366; Dido, A. 4, 36; *4, 335*, and 4, 674; magar, A. 1. 421. *Sabine:* cupeneum, A. 12. 538; curis, A. 1, 292; dira, A. 3. 235; hernae, A. 7, 684; hirpi, A. 11. 785; Loebasius, *G. 1, 17*; nar, A. 7. 517. *Gallic:* Alpes, A. 4. 442; caesar, A. 11. 743; gaesa, A. 7. 664; gaesos, A. 8. 660; virga, A. 8. 660; volema, G. 2. 88. *Etruscan:* arimos, A. 9. 712; Camillus, A. 11. 558; capys, A. 10, 145; Mantus, *A. 10, 198*. *Egyptian:* Isis, A. 8. 696; θΕΥθ, A. 4. 577. *Macedonian:* phalanx, A. 11, 92; sarissa, A. 7. 664. *Doric:* 'Ωρίων, A. 1. 535; Paean, A. 7. 769. *Laconum lingua:* tityrus, Proem. ad Buc. *Cretan:* sminthicem, *A. 3. 108*. *Phrygian:* sminthos, *A. 3, 108*. *Aeolic:* σιῶς, A. 3. 445; 6. 12. *Libyan:* Ammon, *A. 4, 196*. *Assyrian:* El, A. 1. 612. *Persian:* gaza, A. 1. 119; 1. 359. *Oscan:* Lucetius, A. 9. 567. *Umbrian:* dira, A. 3. 235. *Tyrian:* sar, G. 2. 506. *Illyrian:* Varro, *A. 11, 743*. *Lingua Theotisca:* cateia, A. 7. 741.

IV. Servius distinctly forbids the derivation of Greek words from Latin. On Aen. 11, 31 (Parrhasio), and G. 2. 4 (Lenaeus) he says 'nam Graecum nomen etymologiam Latinam non recipit,' in each case rejecting the explanation of Donatus. And yet we find him deriving castor, G. 1. 58, 'a castrando'; pausia, G. 2. 86, 'a paviendo'; tus, G. 1. 57, 'a tundendo'.

V. He lays down the important rule that a word should agree in quantity with the word from which it is derived. This principle is emphasized several times: Aen. 1. 498, (Diana); A. 1, 535 (Orion); A. 1, 185 (totus); A. 1. 726 (lucerna); A. 2. 557 (litus); A. 8. 51 (Pallanteum). On Aen. 1. 726, *e. g.*, he says: "a lychno autem lucerna dicta est, unde et brevis est 'lu' . . . , si enim a luce diceretur, non staret versus." On Aen. 2, 557 he rejects the derivation of 'litus' (from 'litare', or from 'lituus') offered by Donatus — the same Donatus to whom he says 'latebat', A. 3, 636, suggested 'late patebat', and 'exilio', A. 2. 798, seemed a metrical equivalent for 'ex Ilio'. M. Thomas remarks, *Essai sur Servius, p. 224*.

that Servius himself forgets this rule in explaining 'curulis'. A. 11, 334 ('a curru'. *sc.*). It is not hard to find much more striking violations of it. There are several passages in which words of different quantity are connected without comment: G. 2. 97, aminneum (quasi sine minio); A. 6, 4, anchora (ἄγκυρα); A. 8, 190, Cacus (κακός); A. 6, 299. Charon (quasi ἀχαίρων); G. 2. 93. defrutum (defraudatur . . . . fraudem)*; A. 3. 35, Gradivus (gradior)*; A. 6. 180, cedria (quasi καιομένης δρυὸς ὑγρόν); B. 4. 35. heroas (terra ἔρα dicta); A. 1. 292. securis (quasi semicuris); A. 1, 688: 4, 2, venenum (quod per venas eat); and the implied etymologies of Acheron, A. 6, 107, (quasi sine gaudio). and irritum, A. 7, 421 (a retibus).

VI. M. Thomas, *Essai sur Servius, pp. 208-210*, discusses Servius' attitude towards the fables which proved so attractive to the grammarians of Quintilian's day (see Inst. or. I 8, 19). Our commentator remarks more than once (on Aen. 6, 74, and 6. 617.) that Vergil, and the poets generally, are apt to vary the forms of these stories. Frequently he mentions a fable only to reject it: ad Aen. 3, 73, 'veritas longe alia est'; ad Aen. 6, 134, 'ratio autem haec est'; ad Aen. 2, 7, 'sed hoc fabulae est'; ad Aen. 6, 14. etc. Such stories are always quoted as fabulous, and are usually prefaced by some such words as 'fabula autem talis est', yet they prove a convenient resource, especially in Daniel's Scholia, for the explanation of several words. Accounts of people changed into animals, birds or plants, of implements named after their inventors, etc., are given under the following words: ἀετός, *A. 1, 394*†; amaracus, A 1, 693; anethus. B. 2, 47; χελώνη, *A. 1, 505;* χίονες, A. 4, 250; circinus, A. 6. 14; δάφνη, *A. 3, 91;* hyacinthus,

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* Vergil, G. 4, 269, has defrutum (*u short*); Plautus, Pseudol. II 4, 51, defrutum(*u long*)—'Murrinam passum defrutum mellam mel quoiuismodi,' (Ussing's reading). Minton Warren, *Amer. Journ. of Phil., Vol. IV, p. 73,* has found four instances of Gradivus (*a short*) out of fifty-three where the word occurs in Latin poetry. These are Ov. M. 6, 427; Val. Fl. 5, 651; Sil. 15, 15; 15, 337. In each case Gradivus is at the end of a hexameter.

† In quoting from the Servian commentary I have everywhere distinguished between the 'vulgate' and the additional notes found in the fuller version. Thus an italicized reference such as *ad Aen. 4, 255* means that the note which follows is found only in Daniel's Scholia; when part of a note is printed in italics as 'Amazon, quasi ἄνευ μαζοῦ, *sine mamma*', the italicized words are added in the fuller version, the rest is in the vulgate. Such references as Africa, Aen. 6, 312; *5, 128,* quasi ἄτερ φρίκης,' are meant to imply that the same or a similar etymology is given in each division.

A. 11, 69; B. 3, 106; lynx. A. 1. 323; myrtus. *A. 3. 23;* 5. 72: narcissus, B. 2, 47; G. 4. 160; *οἶνον, G. 1. 8;* palaestra, *A. 8. 138;* papaver, B. 2. 47; philyra, *G. 3. 93; φύλλα*, B. 5, 10; *σταφυλήν*, *G. 1*, 8; thorax, A. 9. 503. With the exception of palaestra (A. 6. 642; G. 2. 531) no other explanation of any of these words is offered in any part of the commentary.

VII. The phonetic possibilities recognized in Servius' etymological notes may be grouped as follows:

(*a*) 'nomina corrupta' or 'aetate corrupta': Arpi. A. 11, 246 (Argyrippa); Casperuli, *A. 8. 638* (Caspiri): Crustumerium, A. 7, 631 (Clytemestra).

(*b*) 'in diminutione plerumque multa mutantur': ofella, A. 6, 420 (offa).

(*c*) Vowel-changes 'in derivatione': caelata, A. 1. 640 (celum . . . in derivatione mutatur). There is a similar note on G. 2, 291 (aesculus ab esu). In two cases. however. the diphthongs apparently presented no difficulty and the derivations of Maenala, G. 1. 17 (*ἀπὸ τῶν μήλων*). and cedria. A. 6. 180 (quasi *καιομένης δρυὸς ὑγρόν*) are given without comment.

(*d*) There are seven words distinctly put down for 'per antistoechon' formations: curculio. G. 1. 186 (quasi gurgulio); laquearium, A. 1. 726 (lacunarium): magalia. A. 1. 421 (magar); meditor, B. 1. 2 (*μελετῶ*, 'l' enim et 'd' interdum sibi invicem cedunt.); Poeni. A. 1. 301 (quasi phoeni); sella. A. 7. 169; B. 1, 2 (quasi sedda): solium. A. 7, 169 (quasi sodium a sedendo).

(*e*) et amurca per 'c' scribitur et per 'g' pronuntiatur, ut 'C.' Gaius, 'Cn.' Gnaeus. G. 1. 194.

(*f*) Implied similarity of certain sounds. Tumultus. A. 2, 486; 8. 1, quasi timor multus. Mercurius. *A. 8, 138. alii Mercurium quasi Medicurrium a Latinis dictum volunt.* cadaver, A. 8, 264, cadaver est corpus nondum sepultum. dictum *cadaver* quod careat honore sepulturae. asylum. Aen. 2. 761. dictum 'asylum' quasi 'asyrum.'

(*g*) Synaeresis. saltem. A. 4, 327 (salutem . . . per synaeresin); conpostus, A. 3, 152 (pro conpositus).

(*h*) Aphaeresis. A. 1, 430, rura Graece ἄρουρα dicuntur. aphaeresis ergo sermonem fecit Latinum. With this compare the following etymologies which are given without comment: Boaulia, A. 6, 107; 7, 662 (caulam bubus fecit); carceres, A. 1, 54; 5. 145; G. 3, 104 (quasi arcer ab arcendo); caulae, A. 9, 59 (Graecum nomen 'c' detracto); Caulon, A. 3, 553 (Aulon mons est etc.); cortina, *A. 3, 92 (alii . . . quasi ortina . . . quod inde vox oriatur);* Segesta, A. 1, 550 (Egesta); A. 5, 718 (Acestes).

(*i*) Rhotacism. *A 4, 219, aras (asas), Valerios (Valesios), Furios (Fusios), etc.*

(*k*) Representation of certain Greek sounds in Latin.

i. φ. apricus, A. 5, 128; 6, 312 (quasi ἄτερ φρίκης).
Africa, *A. 5. 128;* 6. 312 (quasi ἄτερ φρίκης).
herba, G. 1, 120 (φορβή).

ii. Spiritus asper. Formiae, A. 7, 695 (inmutato H in F . . . ἀπὸ τῆς ὁρμῆς).
Septem, B. 2, 11 (in multis enim nominibus, quae in Graeco aspirationem habent, nos pro aspiratione 's' ponimus: inde est . . . pro hepta 'septem').
aptum. A. 4, 482; 11, 202 ἀπὸ τοῦ ἅπτεσθαι).

iii. Digamma. Belus, A. 1, 642 (El . . . addita digammo . . .).
Velia, A. 6, 359 (Elia . . . accepit digammon . . .).

In a number of etymologies the recognition of any unusual phonetic possibility is so doubtful that they have not been included under this head. If Servius derives 'Africa' from ἄνευ φρίκης in one part of his commentary, from ἄτερ φρίκης in another, he probably means in each case only 'a privative.' So with the explanations of 'Amazon' (quasi ἄνευ μαζοῦ) and 'apricus' (quasi ἄτερ φρίκης). In other notes 'a privative' is indicated by the word 'non' or 'sine': Acheron, A. 6, 107 (quasi sine gaudio); Alexis, B. 2, 1 (quasi sine responsione); aminneum, G. 2, 97 (quasi sine minio); atomos, B. 6, 31 (quia τομήν non recipiunt). The 'sine' which appears in another group of etymologies perhaps means only 'se', although Servius makes no such definite statement on

this point as we find in Isidore. or. X 247: segnis. id est sine igne, ingenio carens. Se autem sine significat. ut sedulus sine dolo: securus, quasi sine cura, etc. In two other passages Isidore omits this explanation: orig. X 244. sedulus familiare verbum Terentii: hoc est sine dolo: orig. X 262. sepultus, sine pulsu. id est sine motu. Accordingly, when Servius explains 'securus', *A. 2, 374*, as '*sine cura*', sedulus, *A. 2, 374*, as '*sine dolo*', segnis, A. 1, 423; *2. 374*, as 'sine igne', and 'sepultus', A. 3, 41; 6, 424, 'quasi sine pulsu', it seems safer to suppose that he really means 'se dolo', 'se igne', etc., than to infer that he allows the 'n' of 'sine' to disappear.

VIII. The additional notes of Daniel's Servius are sometimes supplementary to those of the vulgate, sometimes repetitions of them, sometimes inconsistent with them. Examples of inconsistency in the matter of etymology may be seen under the following words*: ara, A. 2, 515; *4. 219*; bruma, A. 2, 472; *G. 1, 211*; feretrum, A. 11, 64; insertas, A. 3, 152; jubar, A. 4, 130; latrones, A. 12, 7; palaestra, A. 6, 642; *8, 138*; G. 2, 351; Praeneste, A. 7, 682; vitula, A. 1, 533; B. 3, 30.

One of the chief arguments relied on by Thomas (p. 49) and Thilo (Praef. XIII) as showing that these 'additional' notes did not form part of the original commentary of Servius is the following: the additional notes quote a variety of opinions upon disputed points without deciding upon any one in particular, while the vulgate usually does so only to adopt one in preference to the others. As far as etymological notes are concerned this statement seems to require some modification. Omitting the explanations of proper names, we have in the vulgate several cases where one etymology out of two or more offered or quoted is distinctly preferred*: cortina, A. 6, 347; 3, 92; delubrum, A. 2, 225; 4, 56; fur, G. 3, 407; harena, A. 1, 178; indigetes, A. 12, 79; G. 1, 498; latrones, A. 12, 7; lucerna, A. 1, 726; manes, A. 3, 63; tus, G. 1, 57; vestibulum, A. 2, 469; 6, 273 and perhaps circenses, A. 8, 636; G. 3, 18. In an equal number of cases, however, no such preference is manifested: amoenus, A. 6, 638; ancile, A. 8, 664; annus, A. 1, 269; castra, A. 3, 519; clarigatio, A. 10, 14; cuna-

* These etymologies are quoted in the second part of this paper

bula, B. 4, 23; foedus, A. 1, 62; palaestra, G. 2. 531; scopulus, A. 1, 45; senatores, A. 5, 758; sparus, A. 11, 682; urbs, A. 1, 12. As far as the etymologies in the additional notes are concerned, Thomas' statement seems to be strictly correct.

In this connection may be pointed out a few inconsistencies which are found in the vulgate itself. For the words cadaver, A. 6, 481; 8, 264; cortina, A. 3. 92; 6, 347; delubrum, A. 2, 225; 4, 56; fur, A. 9, 348; G. 3, 407, and Segesta, A. 1, 550; 5, 718 different etymologies are preferred in different parts of the commentary. For Carthago, A. 1, 343; 1. 366; 4, 670; cedria, A. 6, 180; 7, 178, and formosus, A. 1, 359; 8, 453, we have, if not different etymologies, at least a more general and a more exact explanation of each word in different places. For several words two etymologies are offered or quoted in one place, only one in another: amoenus, A. 5, 734; 6, 638; clarigatio, A. 9, 52; 10. 14; delubrum, A. 2, 225; 4, 56; fur, A. 9, 348; G. 3, 407; scopulus, A. 1, 45; 1. 180; solium, A. 1. 506; 7, 169. For scopulus the two derivations are offered earlier in the commentary than the one which is apparently preferred; for each of the other five words the two optional etymologies are not mentioned until after one of them has been given.

IX. Sometimes etymologies are only implied: Acheron, A. 6, 107 (quasi sine gaudio); dolones, A. 7, 664 (a fallendo dicti); hydra, A. 6, 287 (ab aqua dicta); etc. Even when they are more definitely stated it is sometimes difficult to determine the exact meaning. For example, on the passage 'saepe volutabris pulsos silvestribus apros latratu turbabis agens', G. 3, 411. we have the comment 'volutabra loca sunt, in quibus se apri volvunt.' Is 'apri' part of the etymology, and not due merely to the 'apros' of Vergil's line? Such an explanation would not be too bad for Isidore, who has, or. XVI 1, 5, 'volutabra appellata quod ibi apri volutentur', and the notes on the following words suggest that such an explanation was good enough for Servius: antarium, A. 11, 156 (quasi ante aras); circenses, A. 8, 636; G. 3, 18 (vel a circuitu, vel . . . ab ensibus circa quos currebant); fatiscunt, A. 1, 123 ('fatim' abundanter, hiscere aperiri); fortuitus,

6, 179 (ab eundo et a fortuna); furcillae, G. 2. 389 (quibus frumenta cillentur). A similar case is the explanation of 'legumen'. On G. 1, 74, we read 'dicitur quod manu legatur nec sectionem requirat'; on G. 1. 199, "'manu legeret'; hinc quidam volunt dictum legumen". The fact that 'legumina' is represented in an old glossary by χεδροπά seems to confirm the suspicion that, if Servius himself did not derive the word from legere — manus, he at least refers to such a derivation in his note on G. 1, 199*. A third instance is the note on 'lugentes campi', A. 6. 441: "lugentes, quasi 'lucis egentes'". This is probably meant for an etymology, not merely the explanation of a metaphor. Voss in his Etymologicon mentions a derivation, 'ingeniosius quam verius', of 'lugere' from 'λύγη, i. e. σκοτία tenebrae'.

X. The difficult question of the sources of Servius' etymological notes must be reserved for a separate paper. At present, however, two general statements may be made. About thirty derivations are attributed to Varro, but he undoubtedly was the ultimate authority for a good many more. Verrius Flaccus is only once mentioned by name, but there is much in these notes which must have come directly or indirectly from the great work De Verborum Significatu. This might have been inferred from the papers in which Nettleship has pointed out some of the parallels between Servius and Festus or Paulus. See his *Ancient Commentators to Vergil*, prefixed to the fourth edition of Conington's commentary, and the papers on Verrius Flaccus, Nonius Marcellus and Thilo's Servius in his *Lectures and Essays*, *Oxford*, *1885*.

Many of the etymologies quoted in our commentary are introduced by such general phrases as 'quidam . . .', 'alii . . .', 'fabula est . . .', etc., or by others which are almost as indefinite: 'ut lectum est in historia Poenorum' (Carthago, A. 1, 313); 'lectum tamen est in philologis' (Caieta, A. 7, 1); 'prudentiores tamen dicunt' (Silvanus, A. 8, 601). On Aen.

*It may be noted that the latest etymology offered for this word is also due to the feeling that 'legere' in itself is not sufficient to explain 'legumina'. Stowasser, *Dunkle Woerter*, *p. 29*, proposes *legere* + **umina*. "*Umen* von *uere*, wie *flumen* von *fluere*, *numen* von *nuere*, *acumen* von *acuere* gebildet, bedeutet ersichtlich Huelle, Huelse."

7, 678, we have the following general statement: 'de civitatibus totius orbis multi quidem ex parte scripserunt, ad plenum tamen Ptolomaeus graece, latine Plinius. de Italicis etiam urbibus Hyginus plenissime scripsit. et Cato in originibus'. Omitting the cases in which Vergil's own explanations of words are discussed, we have etymologies distinctly referred to the following authors:—

Aeschylus, pergama, A. 1. 95.
Alexarchus, Campania, *A. 3, 334.*
Marcus Antonius, Umbros, *A. 12, 753.*
Aristonicus, Campania, *A. 3, 334.*
Asper, solium, A. 7, 169.
Ateius, Roma, *A. 1, 273.*
Carminius, amoenus, A. 5. 374.
Cassius, fana, *G. 1, 10.*
Cassius Hemina, Crustumerium, A. 7, 631; magalia, *A. 1, 421.*
Cato, Graviscae, A. 10. 184: intempestae, A. 10, 184; Iulus, A. 1, 267; magalia, *A. 1, 421;* Praeneste, *A. 7, 682;* Sabini, *A. 8, 638.*
Cicero, foedus, *A. 8, 641;* seditio, A. 1. 149; Tenedos, *A. 2, 21.*
Cincius, delubrum, *A. 2, 225;* fana, *G. 1, 10.*
Clinias, Roma, *A. 1, 273.*
Clodius Tuscus, mussare, A. 12, 667.
Conon, Sarrastras, *A. 7, 738.*
Donatus, latebat, A. 3, 636; Lenaeus, G. 2, 4; litus, A. 2, 557; Parrhasio, A. 11, 31.
Ennius, Roma, *A. 1, 273.*
Eratosthenes, Myrmidones, A. 2. 7; Roma, *A. 1, 273.*
Gellius, Sabini, *A. 8, 638.*
Heraclides, Roma, *A. 1, 273.*
Hyginus, Ardea, A. 7. 412; Caere, A. 8, 597; Casperuli, *A. 8, 638;* Hesperia, A. 1, 530; Sabini, *A. 8, 638.*
Livius, Campania, *A. 3, 334;* Carthago, A. 1, 343; 1, 366.
Lutatius, Baiae, *A. 9, 707.*
Masurius Sabinus, delubrum, A. 2, 225.
Naevius, Roma, *A. 1, 273.*
Nigidius, herba, *G. 1, 120.*
Ovidius, Ardea, A. 7, 412; names of months, G. 1, 43.

Philochorus, Pelasgi, *A. 8. 600.*

Philostephanus, Trinacia. *A. 1, 196.*

Piso, Pilumnus, *A. 10, 76.*

Plinius, intempestae, A. 10, 184: junior. A. 6. 304; lyciscae, B. 3, 18: sucinum. A. 8. 402.

Postumius, Baiae, *A. 9. 707.*

Sallustius, magalia, *A. 1, 421:* senatores. A. 5, 758.

Saufeius, Aborigines. Cascei. Latium, *A. 1, 6.*

Seneca, Abatos. Philos, A. 6, 154.

Statius Tullianus, Camilla, *A. 11, 543.*

Thukydides, Italia, A. 8, 328.

Varro, amoenus, A. 5, 374: ara, 4, 219; arena. A. 1. 172; Aventinus, A. 7, 657; Campania, *A. 3, 334;* candelabrum, *A. 2, 225;* carceres, A. 5, 145; cernuli. A. 10. 894: curculio, G. 1, 186; delubrum. *A. 2, 225;* faunos, *G. 1, 11;* focus, *A. 11, 21;* frenos. *A. 8, 230;* germanus. A. 5. 412; harena, A. 1. 172; junior, A. 5, 409; 6. 304; Latium, A. 8, 322; latrones. A. 12. 7: Libya, A. 1, 22; Oenotria, A. 1. 532; Palatinus, A. 8, 51;. palla, A. 1, 648; proceres. A. 1. 740; pronuba, *A. 4. 166;* senior, A. 5, 409; sparus, *A. 11, 682;* testudo. A. 1, 505: valvae. A. 1, 449; Varro, *A. 11, 473;* vates, *A. 3, 443;* vestibulum, A. 6. 273.

Verrius Flaccus, juniperi, *B. 7. 53.*

Vitruvius, aditus. ostium, A. 6, 43.

XI. Sometimes we find Servius attributing to earlier writers opinions which they apparently did not hold. Unless it be supposed that Varro in his voluminous writings may have preferred different etymologies for the same word in different places, we must infer that he has been misrepresented at least two or three times.

amoenus.—Servius says, ad Aen. 6, 638, that Varro explained amoena 'quasi amunia.'

Isidore, or. XIV 9, 33, makes Varro derive amoena from amare.

Latium.—Serv. ad Aen. 8, 322, Varro autem Latium dici putat, quod latet Italia inter praecipitia Alpium et Apennini.

Varro, L. L. V 32. says Apulia and Latium are named 'ab hominibus.'

vates.—Serv. ad Aen. 3, 443, vates a vi mentis appellatos, Varro auctor est.

Varro, L. L. VII 36, antiqui poetas Vates appellabant a versibus viendis.

ostium.—Serv. ad Aen. 6, 43, nam Vitruvius qui de architectonica scripsit, ostium dicit per quod ab aliquo arcemur ingressu ab obstando dictum.

To use Thilo's words, falso Servius 'ostium, etc.' Vitruvio tribuit.

II.—FALSE AND POPULAR ETYMOLOGIES.

The great value of Servius' etymological notes is apparent to every reader of his commentary. There are, of course, many derivations offered which are no longer accepted, but many of the words which he attempts to explain still defy certain analysis, and in some of the cases where he offers two optional etymologies for the same word each of these has its supporters at the present day. In the following pages an attempt is made to collect all his etymologies which may be confidently rejected. It is hardly necessary to disclaim any pretence to completeness of treatment where completeness of treatment is practically impossible. At the same time this paper professes to contain all Servius' etymologies (excluding proper names) which, according to the highest and most recent authorities, are no longer tenable. One word of explanation should be added. In cases where Servius offers two optional etymologies of the same word both are regularly quoted, often without comment. Large as the following list is, it might have been much larger, had it included all the unnecessary derivations of Latin words from Greek, cases, *i. e.*, where the Latin word is at most cognate with the Greek, not derived from it. In many cases Servius' statements as to the exact relation between two cognate Latin words do not agree with modern views. These too are regularly omitted, though one or two extreme examples have been retained: fores, Aen. 1, 449 (quae foras aperiuntur); fundus, G. 2, 468 (rerum omnium fundamentum).

As often as one of Servius' more remarkable etymologies has been found in an earlier writer the passage is added below,

but in no case is it definitely stated that he borrowed his note from this source. It is somewhat surprising to find how few of these appear in the extant works of his predecessors and how many of them recur in Isidore, for whom no etymology was too grotesque. It is hard to imagine that Servius is himself responsible for all these ingenious explanations. Whether Isidore drew directly upon Servius, as Thilo thinks, or upon Servius' sources, as Nettleship maintains, he doubtless preserves many derivations that were offered by Servius' predecessors.

Such works as Palmer's *Folk Etymology*, Andresen's *Deutsche Volksetymologie*, and Keller's *Lateinische Volksetymologie* deal chiefly with Folk Etymology in the narrower sense of the term: where the form of a word is affected by false derivation or mistaken analogy, or where the signification is warped and perverted from a false relationship being assumed. Even in this narrow sense of the term our commentary furnishes several examples of Folk Etymology. Thus Servius, on Aen. 1. 172, prefers the derivation 'arena ab ariditate' to 'harena ab haerendo', where Varro left an option and an optional spelling. The Sabine 'fasena' shows that the initial 'h' is etymologically correct*. On G. 1. 57, he says that the old derivation of 'tus' (ἀπὸ τοῦ θύειν) led to the spelling 'thus'. Cases of perverted meaning due to mistaken analogy are more numerous: see especially the notes on gurgulio, indigetes, latrones, orichalcum, and postumus, quoted in the following pages. For the words indigetes and latrones Servius himself distinctly mentions popular etymologies: with these may be compared the comment on Aen 6, 392: sane Alciden volunt quidam ἀπὸ τῆς ἀλκῆς dictum, id est a virtute: quod non procedit, quia a prima aetate hoc nomen habuit ab Alcaeo, patre Amphitryonis, et scimus agnomina ab accidentibus dari.

ADOREA, *Aen. 10, 677*, '*Turnus adoro*' *id est juxta veteres, qui adorare adloqui dicebant: nam ideo et adorea laus bellica, quod omnes eum cum gratulatione adloquebantur, qui in bellis fortiter fecit.*

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*Corssen, Vol. I p. 102.

AESCULUS, G. 2, 291, ab esu dicta.

AMELLUS, G. 4, 278, Mella fluvius Galliae est, juxta quem haec herba plurima nascitur; unde et amella dicitur.

Servius' explanation seems to have been suggested by Vergil's line, 'et curva legunt prope flumina Mellae'. See Wharton, *Etyma Latina, p. 4,* "amellus starwort: Gaulish for *ampellos 'loved by bees', fr. *ampis bee (M spelt am, and p disappearing in Celtic: see Stokes, B.B. 9, 194). cf. OHG. impi and ἐμπίς gnat?"

AMINNEUM, G. 2, 97, aminneum vinum dictum est quasi sine minio, id est rubore; nam album est. *et aliter: Amineos Aristoteles in politiis hoc scribit Thessalos fuisse, qui suae regionis vites in Italiam transtulerint, atque illis inde nomen inpositum.*

AMOENUS, Aen. 6, 638, amoena autem quae solum amorem praestant, vel ut supra (ad Aen. 5, 374, *sc.*) diximus, quasi amunia, hoc est sine fructu, ut Varro et Carminius docent.

Paulus, p. 2, says 'amoena dicta sunt loca, quae ad se amanda adliciant'. Isidore makes Verrius Flaccus derive 'amoenus' from 'munus', Varro from 'amare'; orig. XIV 9. 33: 'amoena loca dicta Varro ait eo quod solum amorem praestent et ad amanda adliciant: Verrius Flaccus, quod sine munere sint, nec quicquam in his officii, quasi amunia, id est sine fructu etc'.

ANCILE, Aen. 8, 664, ancile autem dicitur aut quasi undique circumcisum, aut quasi ἀμφίχειλον, id est undique labrum habens.

ANNUS, Aen. 1, 269, annus autem dictus quasi anus, id est anulus, quod in se redeat, . . . vel ἀπὸ τοῦ ἀνανεοῦσθαι, id est ab innovatione.

ANTARIUM, Aen. 11, 156, sane hoc bellum 'antarium' vocari solitum, quod sit ante urbem, quasi ante aras.

APRICUS, Aen. 6, 312. quasi ἄτερ φρίκης, id est sine frigore ut diximus supra (ad Aen. 5, 128, *sc.*)

cf. Paul. p. 2, apricum locum a sole apertum a Graeco vocabulo φρίκη appellatum, quasi ἀφρικής, id est sine horrore, videlicet frigoris, unde etiam putatur et Africa appellari.

ARA, Aen. 2, 515, dicitur a precibus, quas Graeci ἀρὰς dicunt.

Aen. 4, 219, veteres aras 'asas' dicebant; postea inmutata littera 's' in 'r' 'aras' dixerunt, sicut Valesios Valerios, Fusios Furios: quod Varro rerum divinarum in libro quinto plenius narrat.

ARDEA, Aen. 7, 412. sciendum tamen ardeam *avem* κατ᾽ ἀντίφρασιν dictam, quod brevitate pennarum altius non volat.

G. 1, 364, ardea dicta quasi ardua.

ASYLUM. Aen. 2, 761. dictum 'asylum' quasi 'asyrum'. *alii 'asylum' ideo dictum, quod nullus inde tolleretur, id est quod* συλᾶσθαι, *hoc est abripi, nullus inde poterat.*

Aen. 8, 342, templum misericordiae, . . . unde nullus posset abduci.

ATRIUM, Aen. 1, 726, ibi et culina erat: unde et atrium dictum est; atrum enim erat ex fumo. *alii dicunt Atriam Etruriae civitatem fuisse, quae domos amplis vestibulis habebat: quae cum Romani imitarentur atria appellarunt.*

AUGURIUM, Aen. 5, 523, dictum quasi 'avigerium', id est quod aves gerunt.

The same derivation is given ad Aen. 1, 393; 1, 397; *1, 398, 2, 702; 3, 89; 6, 198.*

cf. Paul. p. 2, augur ab avibus gerendoque dictus, quia per eum avium gestus edicitur: sive ab avium garritu, unde et augurium.

BELLUM, Aen. 1, 22, κατὰ ἀντίφρασιν, a nulla re bella.

cf. Charis. p. 276, 15 K. antiphrasis . . . ut bellum dicitur, quod minime sit bellum.

Donat. ars gram. p. 402, 4 K. ut bellum, hoc est minime bellum.

BIDENTES, Aen. 4, 57. 'bidentes' autem dictae sunt quasi biennes, quia neque minores, neque maiores licebat hostias dare. sunt etiam in ovibus duo eminentiores dentes inter octo, qui non nisi circa bimatum apparent.

Aen. 6, 39, 'bidentes' autem ut diximus supra oves sunt circa bimatum, habentes duos dentes eminentiores.

Servius here combines two different derivations. The derivation from 'annus' is mentioned by Aulus Gellius, who

devotes a whole chapter to the word 'bidentes', and approved by Nonius Marcellus.

cf. Gell. XVI 6, 13, scriptum invenimus in commentariis quibusdam ad jus pontificum pertinentibus, 'bidennes' primo dictas, 'd' littera inmissa, quasi 'biennes', tum longo usu loquendi corruptam vocem esse et ex 'bidennibus' 'bidentes' factum, quoniam id videbatur esse dictu facilius leniusque.

Non. Marc. Lib. 1, p. 53 M. bidentes qui existimant ob eam causam oves a Vergilio dictas quod duos dentes habeant, pessime a vitio intellegunt. . . . . et melius intellegi potest, si *bidennis quasi* biennis dixeris auctoritate.

BRUMA, Aen. 2, 472, dicta . . . quasi βραχὺ ἦμαρ, id est brevis dies.

G. 1, 211, bruma dicta a brevioribus diebus.

BURIS, G. 1, 170, quasi βοὸς οὐρά, quod sit in similitudinem caudae bovis. *alii . . . : buris enim ut curvetur, ante igni domatur, id est amburitur. . . . . Varro ait† totum burim indici ab urbe.*

CADAVER, Aen. 6, 481, caduci . . . a cadendo; unde et cadavera dicta.

Aen. 8, 264, cadaver est corpus nondum sepultum, dictum cadaver quod careat honore sepulturae.

Aen. 11, 143, sane haec corpora sive proici jubebantur a cadendo, sive quod sepultura carebant 'cadavera' dicta.

CAESARIES, Aen. 1, 590; *8, 659*, a caedendo dicta.

CALAMISTRUM, Aen. 12, 100, acus maior, quae calefacta et adhibita intorqueat capillos.

cf. Varr. L.L. V 129, calamistrum, quod his calfactis in cinere capillus ornatur.

CALCULUS, G. 2, 180, dictus, quod sine molestia sui brevitate calcetur.

CARCER, Aen. 1, 54, dictus quasi arcer ab arcendo. G. 3, 104, ab arcendo dictus.

Aen. 5, 145, 'carceres' quasi 'arceres' secundum Varronem.

cf. Varr. L.L. V 151, carcer a coercendo. V 153, carceres dicti, quod coercentur equi.

Varro at least attempted to account for the initial 'c'. To Servius no such explanation would seem necessary. See CAULAE.

CASSES, CASSUS. *Aen.* 2, 85, *cassum est quasi quassum et nihil continens; nam et cas quassum, quod humorem in se non continet et est vacuum. unde et retia casses, quod multum in se vacui habeant.* Aen. 11, 104, 'cassis' vacuis: unde et retia casses dicimus, et vestimenta araneorum casses dicuntur.

cf. Non. p. 45, 9 M. cassum veteres inane posuerunt. Et arbitrandum est eius verbi proprietatem magis ab aranearum cassibus dictam, quod sint leves et nullius ponderis, non, ut quibusdam videtur, quasi quassum.

CASTOR, G. 1, 58, castores autem a castrando dicti sunt.

O. Keller, *Lateinische Volksetymologie und Verwandtes*, *p.* 285, derives *castrare* from *castor*, citing an ancient belief (Cic. Ovid. Plin. Juven. Apulej.) concerning the beaver. 'Also *castrare aliquem* heisst einen nach Biberart behandeln, seine ὄρχεις zerstoeren'.

CASTRA, Aen. 3, 519, dicta autem 'castra' quasi casta, vel quod illic castraretur libido: nam nunquam his intererat mulier.

CATUS, Aen. 1, 423, id est ingeniosus ἀπὸ τοῦ καίεσθαι.

cf. Donat. ad Ter. Andr. V 2, 14, catus: callidus, doctus, ardens, παρὰ τὸ καίειν, unde Cato dictus, ingeniorum enim igneus vigor esse videtur.

CAULAE, Aen. 9, 59, 'caulas' munimenta et saepta ovium. est enim Graecum nomen 'c' detracto: nam Graeci αὐλὰς vocant animalium receptacula.

Servius evidently means to connect 'caulas' and αὐλάς etymologically, not merely to point out an interesting coincidence in form. That the initial 'c' in the Latin word presented no serious difficulty to him may be inferred from his explanation of CAULON, Aen. 3, 553, 'Aulon mons est Calabriae, etc.', and the comments on CORTINA, *Aen.* 3, 92, and CARCER, Aen. 1, 54; 5, 145.

CEDRIA, Aen. 6. 180. cedria dicta est quasi καιομένης δρυὸς ὑγρὸν. id est arboris umor ardentis.

Aen. 7. 178. 'e cedro'. unde et cedria.

CERNULUS, CERNUUS, Aen. 10, 894. cernuus equus dicitur, qui cadit in faciem. quasi in eam partem cadens qua cernimus: unde et pueri quos in ludis videmus ea parte, qua cernunt, stantes, cernuli vocantur, ut etiam Varro in ludis theatralibus docet.

cf. Non. p. 21, 2 M., cernuus dicitur proprie inclinatus, quasi quod terram cernat.

CHELYDRUS, G. 3, 415, 'chelydri' dicti quasi chersydri, qui et in aquis et in terris morantur: nam χέρσον dicimus terram, aquam vero ὕδωρ.

CIRCENSES. Aen. 8. 636. circenses dicti vel a circuitu, vel quod ubi nunc metae sunt, olim gladii ponebantur, quos circumibant. dicti autem circenses ab ensibus, circa quos currebant.

G. 3, 18, circenses dicti sunt, quia exhibebantur in circuitu ensibus positis; licet alii a circumeundo dicant circenses vocari.

CIRCUMVOLAT, Aen. 3. 233, aut circum praedam volat uncis pedibus: aut intra volam amplectitur praedam: unde et involare etc. See VOLANDO.

CLARIGATIO. A. 9. 52, a claritate vocis.

A. 10, 14. aut a clara voce qua utebatur pater patratus, aut a κλήρῳ, hoc est sorte.

CLASSIS, A. 1. 39, dicta est ἀπὸ τῶν κάλων. id est a lignis. So A. 6, 1.

CLIENS, A. 6, 609, si enim clientes quasi colentes sunt etc.

CORTINA, A. 3, 92, dicitur autem cortina, vel quod Apollinis tripos corio Pythonis tectus est, vel quod certa illinc responsa funduntur, quasi certina. vel quod est verius, quia cor illic vatis tenetur. *alii cortinam quasi ortinam tradunt, quod inde vox oriatur.*

A. 6. 347. cortina dicta est aut quod cor teneat, aut quod tripus saeptus erat corio serpentis, ut diximus supra: aut certe secundum Graecam etymologiam ὅτι τὴν κόρην τείνει ἤτοι τινάσσει, id est quod extendit puellam, ut 'maiorque videri'.

CUNABULA, Buc. 4, 23, lectuli in quibus infantes jacere consueverunt: vel loca, in quibus nascuntur, quasi cynabula: nam κύειν est Graece niti.

CURA, A. 1, 208; 4, 1. cura dicta ab eo quod cor urat.

cf. Varr. L.L. VI 46, cura quod cor urat.

Paul. p. 35, cura dicta est, quasi coreda, vel quia cor urat.

CURCULIO, G. 1, 186, Varro ait hoc nomen per antistoechon dictum, quasi gurgulio, quoniam paene nihil est nisi guttur.

DEFRUTUM, G. 2, 93, dictum, quod defraudatur et quasi fraudem patitur.

DELUBRUM, A. 2. 225, delubrum dicitur quod uno tecto plura complectitur numina, quia uno tecto diluitur. . . . alii, *ut Cincius*, dicunt, delubrum esse locum ante templum, ubi aqua currit, a diluendo (*a deluendo*, Nettleship, Contrib. p. 429.)

On A. 4. 56. a similar explanation is given with this addition: aut certe simulacrum ligneum delubrum dicimus, a libro, hoc est raso ligno factum, quod Graece ξόανον dicitur.

Daniel's Servius on A. 2. 225. attributes this explanation 'a delibratione corticis' to Masurius Sabinus, and quotes another from Varro, '*rerum divinarum libro † XIX*': *aut in quo loco dei dicatum sit simulacrum, ut* (*sicut*) *in quo figunt candelam, candelabrum appellant, sic in quo deum ponunt, delubrum dicant.*

Nettleship, *Lectures and Essays*, *p. 239*, has compared two parallel passages in Macrobius and Paulus, and suggests a possible inference as to the source of Servius' information. "Macrobius (3, 4, 3) has one note, Paulus, p. 73, has another, on this word. But the substance of both notes is combined by Servius on Aen. 2, 225. Paulus says 'delubrum dicebant

fustem delibratum, hoc est decorticatum, quem venerabantur pro deo'. Servius not only gives this explanation, but also those quoted in Macrobius from Varro's Rerum Divinarum; and much the same comment recurs, with an addition, in Servius on Aen. 4. 56. The impression left is that both Macrobius and Servius were copying from an article in Verrius Flaccus, of which only a short extract has survived in the epitome of Paulus."

DEUS, A. 12, 139, nam quod graece δέος, latine timor vocatur, inde deus dictus est, quod omnis religio sit timoris.

cf. Paul. p. 50, deus dictus, quod ei nihil desit, . . . sive a Graeco δέος quod significat metum, eo quod hominibus metus sit.

DIRA, A. 4. 453, dira enim deorum ira est.

A. 3, 235, Sabini et Umbri, quae nos mala, dira appellant.

cf. Paul. p. 49, dirus, dei ira natus.

Non. Marc. p. 30. 14 M. dirum . . quasi deorum ira inmissum.

EBUR, A. 1, 592. ebur a barro dictum, id est elephanto.

EXTUDERAT, A. 8. 665, studiose fecerat.

FATISCUNT, A. 1, 123, fatiscunt abundanter aperiuntur; 'fatim' enim abundanter dicimus. unde et adfatim, hiscere autem aperiri.

FERA, A. 1, 215, feras dicimus aut quod omni corpore feruntur, aut quod naturali utuntur libertate et pro desiderio suo feruntur. So A. 2, 51.

FLAGELLA, G. 2, 299, dicuntur summae arborum partes, ab eo quod ventorum crebros sustinent flatus.

cf. Varr. R. R. 1. 31. 3. neque ex se potest eiicere vitem, quam vocant minorem flagellum, maiorem etiam unde uvae nascuntur, palmam. prior, litera una mutata, declinata a venti flatu, similiter flabellum ac flagellum.

FLAGRANTIA, A. 1. 436, quotiens incendium significatur, quod flatu alitur, per 'l' dicimus, quotiens odor, qui fracta specie major est, per 'r' dicimus.

FLAMINES. A. 8, 664; *10, 270*, a filo quo utebantur. flamines dicti sunt quasi filamines.

cf. Varr. L.L. V 84, quod . . . caput cinctum habebant filo, flamines dicti.

Paul. p. 62, Flamen Dialis dictus, quod filo assidue veletur; indeque appellatur flamen, quasi filamen.

Varro's explanation of this word is now generally rejected, though De Vit says it is confirmed by an inscription 'ap. Gruter. 227, 6'.

FOEDUS, A. 1. 62, dictum vel a fetialibus, id est sacerdotibus per quos fiunt foedera, vel a porca foede, hoc est lapidibus occisa, ut ipse 'et caesa jungebant foedera porca'.

The same explanation is given on Aen. 8, 641 and *Aen. 12, 109*. On *Aen. 8, 641* Daniel's Servius adds: *Cicero foedera a fide putat dicta.* On *Aen. 4, 242* we have '*fetiales a foedere*'.

cf. Paul. p. 59, foedus appellatum ab eo, quod in paciscendo foede hostia necaretur. Virgilius: 'et caesa jungebant foedera porca'. Vel quia in foedere interponatur fides.

FORMICA, A. 4, 402. sane 'formica' dicta est ab eo, quod *ore* micas ferat.

FORMOSUS, A. 1. 359, formosus a forma, ut a specie speciosus etc.

A. 8. 453, nam forvum est calidum: unde et formosos dicimus quibus calor sanguinis ex rubore pulchritudinem creat.

cf. Paul. p. 59. Forma significat modo faciem cuiusque rei, modo calidam, ut, quum exta, quae dantur, deforma appellantur. Et Cato ait de quodam aedificio aestate frigido, hieme formido.

FORES, A. 1. 449. fores proprie dicuntur quae foras aperiuntur, sicut apud veteres fuit.

FORTUITUS, A. 6. 179. sane 'fortuitus' ab eundo est et a fortuna compositum.

FRAGRANTIA, *vide* FLAGRANTIA.

FRATRIA, A. 7, 286, Argos dipsion . . . , apud quos erat magna societas inter eos qui uno puteo utebantur: unde et fratrias dixerunt ἀπὸ τοῦ φρέατος.

FRENOS, A. 8, 230, nam et frendere significat dentibus frangere . . . . et Varro frenos hinc putat dictos.

FUNDUS, G. 2, 468, fundus dicitur ab eo, quod sit rerum omnium fundamentum.

FUNUS, A. 1, 727, funera dicuntur, quod funes incensos mortuis praeferebant. So on A. 6, 224, and *A. 11, 143*, where another explanation is offered: *alii a fungendo, quod eo supremo in eo qui decessit, officio fungimur, vel quod hi qui mortui sunt 'vita functi' dicuntur.*

cf. Donat. ad Ter. Andr. 1, 1, 88: 'in funus': in ipsum officium aut in pompam exsequiarum: quod a funalibus dictum est et uncis et cuneis candelabrorum, quibus delibuti funes cerei fomites infiguntur.

FUR, FURTUM, *A. 2, 18, nam et furtum ideo dicitur, quod magis per tenebras admittatur; unde fures qui quasi per furvum tempus, hoc est nigrum, aliquid subripiunt.* So A. 9, 348.

G. 3, 407, fur autem a furvo dictus est. . . . . aut certe a Graeco venit; nam fur φὼρ vocatur.

The derivation of 'fur' from 'furvum' is attributed to Varro (in XIV rerum divinarum libro) by Gellius, 1, 18, 4; to Varro (rerum humanarum lib. XIV.) by Nonius, p. 50, 9 M. Gellius himself explains 'fur' as the Latin representative of φώρ.

FURCILLAE, G. 2, 389, nam 'cillere' est movere, unde et furcillae dictae sunt, quibus frumenta cillentur.

HARENA, A. 1. 172, quaeritur, habeat necne nomen hoc adspirationem. Et Varro sic definit si ab ariditate dicitur non habet, si ab haerendo, ut in fabricis videmus, habet. melior tamen est superior etymologia.

HEROAS, B. 4, 35, quidam a terra dictos volunt, quod terra ἔρα dicta sit, unde initio nati creduntur homines, qui nomen a matre traxerunt.

INCLITA, A. 6, 781, inclita Graecum est: nam κλυτον gloriosum dicunt.

cf. Paul. p. 39. clutum Graeci κλυτόν dicunt. Unde accepta praepositione fit inclitus.

INDIGENAE, A. 8. 314; 8. 328. id est inde geniti, αὐτοχθονες

INDIGETES, A. 12. 794. indigetes dii duplici ratione dicuntur: vel secundum Lucretium, quod nullius rei egeant, qui ait (II. 650 sc.) 'nihil indiga curae' . . . vel certe indigetes sunt dii ex hominibus facti, et dicti indigetes quasi in diis agentes. The fuller version adds: *vel quod nos deorum indigeamus . . . , alii patrios deos indigetes dici debere tradunt, alii ab invocatione indigetes dictos volunt, quod 'indigeto' est precor et invoco.*

G. 1. 498, indigetes proprie sunt dii ex hominibus facti, quasi in diis agentes, abusive omnes generaliter, quasi nullius rei egentes.

An incorrect form *Indigens* appears in a Pompeian inscription, *C. I. L. Vol. 1. p. 283*, upon which the editor remarks: '*Indigens* pro *indiges* hoc solo loco reperitur, ni fallor ex fabrili errato'.

INFERIAE, A. 10. 519; 11. 81, inferiae sunt sacra mortuorum, quod inferis solvuntur.

INSERTAS, A. 3. 152, aut clatratas: aut non seratas, ut sit quasi insertas id est non clausas, et dictum quomodo . . . 'conpostus' pro 'conpositus' . . . *vel 'insertas fenestras' quas lumine suo luna inseruerat, ab inserendo, quod se per rimas insereret.*

INSTAURATA, A. 2, 15, 'instar' autem est ad similitudinem: unde non restaurata sed instaurata dicuntur aedificia ad antiquam similitudinem facta.

cf. Paul. p. 79, instaurari ab instar dictum, cum aliquid ad pristinam similitudinem reficitur.

Macrob. Sat. I 11, 5, . . . isque instauratitius dictus est, . . . a redintegratione, ut Varroni placet, qui instaurare ait esse instar novare.

'Instaurare' may be cognate with 'instar', but is hardly derived from it.

INVOLARE, A. 3, 233; G. 2, 88, involare dicimus intra volam tenere. See VOLANDO.

IRRITUM, A. 7, 421, in cassum id est in irritum: incassum autem tractum est a cassibus, id est a retibus.

JUBAR, A. 4, 130, proprie 'jubar' lucifer dicitur, quod jubas lucis effundit: est autem lucifer interdum Iovis: *nam et antiqui 'jubar' quasi 'juvar' dicebant.*

JUNIPER, B. 7, 53, Verrius Flaccus juniperum juvenem pirum ait.

ΛΑΟΙ, G. 1, 63, nam et Graece populi *λαοὶ* dicuntur a lapidibus.

The resemblance between *λαός* people and *λᾶας* stone is implied in Hom. Il. XXIV 611, *λαοὺς δὲ λίθους ποίησε Κρονίων*, and Pindar explains the word from the legend of Deucalion, O. 9, 66, *κτισσάσθαν λίθινον γόνον· Λαοὶ δ' ὀνύμασθεν.*

LATEX, A. 1, 686, proprie aqua est ab eo quod intra terrae venas lateat.

This derivation is accepted by Stowasser. *Dunkle Woerter*, *p.* 5, 'Richtig schon Isidor XIII 20, latex proprie liquor fontis, quod in terra lateat'. It is surely better to regard latex as the Latin representative of *λάταξ*, with which it is sound for sound identical.

LATRONES, A. 12, 7, est Graecum; nam *λατρεύειν* dicunt obsequi et servire mercede, unde latrones vocantur conducti milites. Varro tamen dicit, hoc nomen posse habere etiam Latinam etymologiam, ut latrones dicti sint quasi laterones, quod circa latera regum sunt, quos nunc satellites vocant. *latrones, ab latendo.*

The same three derivations are mentioned by Varro, L. L. VII 52.

cf. Paul. p. 85, Latrones antiqui eos dicebant, qui conducti militabant, *ἀπὸ τῆς λατρείας*. At nunc viarum obsessores dicuntur, quod a latere adoriuntur, vel quod latenter insidiantur.

LEGUMEN. G. 1. 74. dicitur quod manu legatur nec sectionem requirat.

G. 1. 199. 'manu legeret': hinc quidam volunt dictum legumen.

LITUS. A. 2. 557. quod autem Donatus dicit, 'litus' locum esse ante aras, a litando dictum: *vel quod lituo illud spatium designatur*, ratione caret: nam a litando 'li' brevis est, et stare non potest versus.

A. 5, 163, 'litus' est omne quod aqua adluitur.

cf. Sueton. reliqu. p. 244, 5 Reiffers., litus, quidquid aqua adluitur.

LUCUS. A. 1, 22, lucus a non lucendo.

A. 1. 441, 'lucus' autem dicitur quod non luceat, non quod sint ibi lumina causa religionis, ut quidam volunt.

cf. Quintil. 1. 6. 34 etiamne a contrariis aliqua sinemus trahi, ut 'lucus', quia umbra opacus parum luceat, et 'ludus', quia sit longissime a lusu, etc.

Charis. p. 276, 15 K., antiphrasis . . . ut bellum . . . et lucus, quod minime luceat.

Diomed. p. 462, 15 K., antiphrasis . . . ut bellum . . . et lucus, quod minime luceat.

LUGENTES (campi). A. 6. 441. quasi 'lucis egentes'.

LUSTRUM, A. 1. 607. aut 'lustrabant' inumbrabant, unde lustra et ferarum cubilia et lupanaria per contrarium dicimus, quia parum inlustrantur.

MACTARE. A. 4, 57, 'mactant' verbum sacrorum, κατ' εὐφημισμόν dictum, ut adolere, nam 'mactare' proprie est 'magis augere'.

cf. Paul. p. 90, mactus magis auctus.

Sueton. reliqu. p. 275 R. mactatum autem quasi magis auctum.

Non. p. 341. 16. mactare est magis augere.

MAGMENTUM. *A. 4, 57. quasi maius augmentum.*

cf. Paul. p. 91, magmentatum*. magis augmentatum

MALUS, A. 5, 487, dictus est, vel quia habet instar mali in summitate, vel quia quasi quibusdam malis ligneis cingitur, quorum volubilitate vela facilius elevantur.

MANES, A. 1, 139, 'manum' enim antiqui bonum dicebant. . . unde . . . per antiphrasin 'manes' inferi, quia non sint boni.

On Aen. 3, 63, the explanation κατὰ ἀντίφρασιν is repeated, and another mentioned: alii manes a manando dictos intellegunt: nam animabus plena sunt loca inter lunarem et terrenum circulum, unde et defluunt.

cf. Paul. p. 87, inferi di manes, ut subpliciter appellati bono essent, et in carmine Saliari Cerus manus intellegitur creator bonus. cf. Fest. Qu. VIII 38; Paul. p. 131.

Paul. p. 149, Manes di ab auguribus vocabantur, quod eos per omnia manare credebant, eosque deos superos atque inferos dicebant.

MEDIUS FIDIUS, A. 8, 275, 'communem deum' inter deos atque homines: unde medius fidius dictus.

A. 4. 204, ut Sallustius 'quam medius fidius veram licet mecum recognoscas': *id est sis dictis medius: fidius id est* Διὸς υἱός, *Iovis filius, id est Hercules, medium dixit testem.*

cf. Paul. p. 131. Medius fidius conpositum videtur et significare Iovis filius, id est Hercules, quod Iovem Graece Δία et nos Iovem, ac fidium pro filio, quod saepe antea pro L littera D utebantur. Quidam existimant jusjurandum esse per divi fidem: quidam per diurni temporis, id est diei fidem.

MUSTELA, A. 2, 468; 9, 744, 'telum' dictum a longitudine: unde et mustela dicitur, quasi mus longus.

OPPETERE, A. 1, 96, ore terram petere, id est mori.

ORICHALCUM, A. 12, 87, quod et splendorem auri et aeris duritiam possideret.

cf. Paul. p. 7. aurichalcum vel orichalcum quidam putant conpositum ex aere et auro, sive quod colorem habeat

aureum. Orichalcum sane dicitur, quia in montuosis locis invenitur. Mons etenim Graece ὄρος appellatur.

OSCILLA, G. 2. 389, dicta sunt ab eo, quod in his cillerentur, id est moverentur ora: nam 'cillere' est movere. *oscilla autem dicta, sive quoniam capita et ora hostium in summis perticis figebantur, sive quia hunc lusum Osci dicuntur frequenter exercuisse et rem per Italiam sparsisse.*

OSTIUM, A. 6, 43, nam Vitruvius qui de architectonica scripsit, ostium dicit per quod ab aliquo arcemur ingressu ab obstando dictum, aditum ab adeundo, per quem ingredimur.

"Falso Servius 'ostium . . . ingredimur' Vitruvio tribuit": Thilo.

PAGANUS, G. 2. 382, 'pagi' ἀπὸ τῶν πηγῶν, i. e. a fontibus, circa quos villae consueverant condi: unde et pagani dicti sunt, quasi ex uno fonte potantes.

cf. Paul. p. 121, pagani a pagis dicti. pagi dicti a fontibus quod eadem aqua uterentur. Aquae enim lingua Dorica παγαί appellantur.

PALAESTRA, G. 2. 531, vel ἀπὸ τῆς πάλης, id est a luctatione, vel ἀπὸ τοῦ πάλλειν, hoc est a motu urnae, nam ducti sorte luctantur.

On *Aen. 8. 138*, the fuller version tells the story of the Arcadian princess Palaestra, who disclosed to Mercury the nature of her brothers' athletic contests, for which reason all wrestling was known by her name.

PASSUM, G. 2. 93, dicitur a patiendo; nam decoquitur mustum et inde fit passum.

cf. Non. p. 551, 22, Varro de vita populi Romani lib. 1: passam nominabant, si in vindemia uvam diutius coctam legerent eamque passi essent in sole aduri.

PAUSIA, G. 2. 86, a paviendo dicta, id est tundendo: aliter enim ex se oleum non facit.

PECUS, *A. 1, 435, a pascendo.*

PERNIX, G. 3, 230, a pernitendo tractum est.

It seems much better, with Vanicek and Wharton, to derive pernix from perna, 'strong in the ham'. For term.

cf. fel-ix. In the old etymology it is hard to see what becomes of the 't' of nitor.

POLLINCTORES, *A. 9, 485, qui mortuis os polline oblinebant.*

PORRICIAM. *A. 5, 238, id est porro iaciam.*

cf. Paul. p. 273, poriciam porro iaciam.

POSTUMUS, A. 6, 763, postumus est post humationem parentis creatus.

cf. Varr. L. L. 9, 60, postumus . . post patris mortem natus.

Fest. Qu. XII 8, p. 306, postumus cognominatur post patris mortem natus.

Caesellius Vindex ap. Gell. II 16, 5, 'postuma proles' non eum significat, qui patre mortuo, sed qui postremo loco natus est sicuti Silvius, qui Aenea iam sene tardo seroque partu est editus.

PRECIAE, G. 2, 95, quasi praecoquae, quod ante alias coquantur.

PROCUL, A. 3, 13, 'procul' est quasi porro ab oculis.

A. 6, 10, procul enim et quod prae oculis est, et quod porro ab oculis.

PRUNA, A. 11, 788, a perurendo dicta est.

cf. Paul. p. 283, pruina dicta, quod fruges ac virgulta perurat.

RECINUS, A. 1, 282, recinus autem dicitur ab eo, quod post tergum reicitur, quod vulgo maforte dicunt.

cf. Varr, L. L. V 132, ab reiciendo ricinium dictum.

RURA, A. 1, 430, Graece ἄρουρα dicuntur. aphaeresis ergo sermonem fecit Latinum.

SANCIRE, A. 12, 200, 'Sancire' autem proprie est sanctum aliquid, id est consecratum facere fuso sanguine hostiae: et dictum sanctum, quasi sanguine consecratum.

SCOPULUS, A. 1, 180, id est specula.

A. 1, 45, aut a speculando dictus, aut a tegimento navium ἀπὸ τοῦ σκεπάζειν.

SCUTRA, *G. 1, 110, . . . scalebris bullitionibus. unde vulgo vasa, ubi calida solet fieri scutrae appellantur.*

SECURIS, A. 1, 292, quasi semicuris.

SEGNIS, A. 1, 423; *2, 374.* id est sine igni. [Cf. the explanation of CATUS quoted above.]

This etymology is doubtful at best, and, unless Servius means only 'se igni', it is phonetically impossible. Stowasser, *Dunkle Woerter, p. 10*, quotes Isid. or. X 19 (*segnis* aus *se igne*) in support of his explanation of 'prospere'.

SENATORES, A. 5, 758, senatores autem alii a senecta aetate, alii a sinendo dictos accipiunt.

A. 1, 426, legitur apud quosdam. Brutum eos qui se in eiciendis regibus iuvissent legisse in consilium, eumque ordinem senatum appellatum, quod una sensissent. . . . . alii senatum a senectute hominum. †quibi allecti erant, dictum volunt, qui apud Graecos γερουσία appellatur.

SEPULTUS, A. 3, 41, 'sepulto' modo *mortuo vel* jacenti significat . . : nam sepultus est quasi sine pulsu. non enim hunc sepultum possumus dicere, cum sepultura non sit in hoc rite facta, sed fortuita sit obrutus terra.

cf. Donat. ad Ter. Andr. 1. 1. 101. Sepulcrum κατ' ἀντίφρασιν, ut diximus, quod sine re pulcra sit: an quod ibi sine pulsu sint, id est, mortui? . . . . . sepulcrum enim a sepeliendo dictum.

SINISTRUM, A. 2, 693, a sinendo dictum.

cf. Fest. Qu. XV 13. p. 502. Sinistrae aves sinistrumque est †sinistimum auspicium, id quod sinat fieri.

SOLIUM, A. 1, 506, dictum quasi solidum.

A. 7, 169. secundum aliquos a soliditate dictum, secundum Asprum per antistoechon, quasi sodium a sedendo.

SPARUS, A. 11, 682, Varro ait sparum telum missile, a piscibus ducta similitudine, qui spari vocantur. alii 'sparus' a spargendo dici putant.

cf. Paul. p. 489. spara parvissimi generis iacula a spargendo dicta.

STELLA, A. 5, 42. poetice dixit: nam si stella a stando dictae sunt, non fugantur; semper enim fixae sunt praeter planetas.

SUDUM. A. 8. 529, est quasi sub udum. serenum post pluvias. ut ver sudum. *alii 'sudum' semiudum volunt dici, cum per nubes ad nos perveniat solis ictus non integer.*

TELUM, A. 2, 468, telum enim dicitur secundum Graecam etymologiam ἀπὸ τοῦ τηλόθεν, quidquid longe iaci potest. So on Aen. 8, 249; 9, 507; 9, 744.

cf. Fest. Qu. XVI 8, p. 556, Tela proprie dici videntur ea, quae missilia sunt, ex Graeco videlicet translato eorum nomine. quoniam illi τηλόθεν missa dicunt quae nos ennius†.

TERRITORIUM, A. 5, 755. dictum quasi terriborium tritum bubus et aratro.

cf. Varr. L. L. V 21. Terra dicta ab eo, ut Aelius scribit, quod teritur . . . . . Territorium quod maxime teritur.

TORUS, Aen. 5, 388, 'torus' a tortis dictus est herbis. So on Aen. 2, 2; *1, 708.*

cf. Non. 11, 11, Tororum et toralium designator est Varro de vita pop. Rom. lib. I: quod fronde lecticae struebantur, ex eo herba torta torum appellatum.

[cf., however, Varr. L. L. V 167 Torus a torvo, quod is in promptu.]

TUMULTUS, A. 2, 486; 8, 1. dictus quasi timor multus.

cf. Cic. Phil. VIII 1, 3, quid est enim aliud tumultus nisi perturbatio tanta. ut maior timor oriatur? unde etiam nomen ductum est tumultus.

TUS, G. 1, 57. sane 'tus' modo sine aspiratione dicimus; nam antiqui 'thus' dicebant ἀπὸ τοῦ θείου: quod displicuit; tura enim a tundendo dicta esse voluerunt, a glebis tunsis, cum quibus dicitur fluens de arboribus coalescere.

cf. Charis. p. 75, 13 K., tus a tundendo sine adspiratione dicitur, quamvis Iulius Modestus ἀπὸ τοῦ θύειν tractum dicat.

URBS, A. 1, 12. urbs dicta ab orbe, quod antiquae civitates in orbem fiebant; vel ab urvo, parte aratri, quo muri designabantur.

cf. Varr. L.L. V 143, quare et oppida, quae prius erant circumducta aratro, ab orbe et urvo urbes.

URI, G. 2, 374, dicti 'uri' ἀπὸ τῶν ὀρέων, id est a montibus.

VATES, A. 3, 443, vates a vi mentis appellatos, Varro auctor est.

cf. Varr. L.L. VII 36, antiqui poetas Vates appellabant a versibus viendis.

VENENUM, A. 1, 688, venenum dictum quod per venas eat.

A. 4, 2, quia per venas amor currit . . . sicut venenum.

VERBENAE, *A. 12, 120, quidam sane veris proximi herbas verbenas dicunt.*

Buc. 8, 65, a viriditate verbenae appellantur.

VESTIBULUM, A. 2, 469, vel quod ianuam vestiat. . . . vel quoniam Vestae consecratum est.

A. 6, 273, vestibulum ut Varro dicit, etymologiae non habet proprietatem, sed fit pro captu ingenii: nam vestibulum, ut supra diximus, dictum ab eo, quod ianuam vestiat. alii dicunt a Vesta dictum per inminutionem: nam Vestae limen est consecratum. alii dicunt ab eo, quod nullus illic stet; in limine enim solus est transitus: quomodo vesanus dicitur non sanus, sic vestibulum quasi non stabulum.

Sulpicius Apollinaris, quoted with approval by Gellius XVI 5, derives vestibulum from ve + stabulum, but gives a different force to the prefix. Nonius, p. 53, prefers this explanation to that from Vesta.

VICTIMA, Aen. 1, 334, victimae . . . sacrificia quae post victoriam fiunt.

cf. Fest. Qu. XVI 25, p. 562, victimam Aelius Stilo ait esse vitulum ob eius vigorem. alii aut quae vincta adducatur ad altare aut quae ob hostis victos immoletur.

VIPERA, G. 3, 416, quae vi parit.

VIRGA, A. 4, 242, dicta quod vi regat.

VIRGO, Buc. 3, 30; 6, 47, a viridiore aetate.

VIROSA. G. 1. 58, dicta ab eo, quod est virus; alii fortia accipiunt a viribus.

VITULA, Buc. 3, 30. a viridiore aetate dicta.

A. 1, 533. Graeci boves ἰταλοὺς, nos vitulos dicimus.

VOLANDO, A. 6. 198, alii 'volando' ambulando dicunt: vola enim dicitur media pars pedis sive manus.

cf. Non. Marc. p. 32. 31 M., involare est inruere, insilire, aut a volatu aut a vola, id est media manu, dictum.

PROPER NAMES.

No part of a language allows freer play to popular etymologizing than its proper names, for no part of a language is more difficult to explain. Servius himself appreciated in some measure the difficulty of this part of his task. On Aen. 7, 678, he says that it is not surprising that many conflicting explanations are offered for ancient Italian names, 'nam antiquitas ipsa creavit errorem'. On Aen. 1, 273, Daniel's Servius quotes a great many explanations of the name 'Roma'. These are prefaced by the remark: *sed de origine et conditore urbis diversa a diversis traduntur*. Accordingly, many such words as Argiletum (Aen. 8. 345); Aventinus (Aen. 7, 657); Palatinus (Aen. 8, 51) are provided with several etymologies each.

No attempt is here made to pronounce upon all Servius' explanations of proper names, but the following brief list will show most of the phases of popular etymologizing which are to be expected in an ancient commentary upon words of this class. "False etymologies are of themselves the fruitful causes of myths" (Sayce, *Science of Language, Vol. II, p. 259*). "The eponymous heroes from whom tribes and nations have been supposed to derive their names owe their existence to the same popular etymologizing etc." [*Ibid. p. 247.*] In our commentary myths are recounted *ad nauseam*, and the hero eponymous appears on almost every page. Sometimes these ancient tales are rejected as fabulous, and rational explanations inserted to replace them. Thus Servius explains

away the stories of the origin of the Myrmidones (Aen. 2, 7), the strange birth of Orion (Aen. 1, 535), and of Minotaurus (Aen. 6, 14), the she-wolf that suckled Romulus and Remus (Aen. 1, 273), etc.

In any language foreign words are especially liable to corruption. cf. Paul. p. 13, 'Alumento pro Laumedonte a veteribus Romanis necdum adsuetis Graecae linguae dictum est. sic Melo pro Nilo, Catamitus pro Ganymede, Alphius pro Alpheo dicebatur'. See also Paul. p. 6 (v. Alcedo); p. 31 (v. Catamitum); p. 89 (v. Melo). If a language could represent *Γανυμήδης* by Catamitus, *Σεμέλη* by Stimula, it is not surprising to find its ancient commentators offering numerous popular etymologies for its proper names—and especially for names of foreign origin—as they manfully tried to assign meanings to these unintelligible words by deriving them from words of similar sound. Many of these explanations are obviously suggested by well known peculiarities of climate, product, or character.

Finally may be mentioned one or two examples of a 'perverse ingenuity', which discovers a certain appropriateness in the names of some of the actors in the Aeneid. Not content with the statement on Aen. 12, 391, "Iapix, aptum nomen medico; nam *ἰᾶσθαι Graeci* dicunt curare," Servius has two curious comments on Achates' name, on Aen. 1, 174 and 1, 312. For these see the first word in the following list.

ACHATES, Aen. 1, 312, diximus quaeri, cur Achates Aeneae sit comes. Varia quidem dicuntur, melius tamen hoc fingitur, ut tractum nomen sit a Graeca etymologia. *ἄχος* enim dicitur sollicitudo, quae regum semper est comes.

Aen. 1, 174, adlusit ad nomen. nam achates lapidis species est: bene ergo ipsum dicit ignem excussisse. unde etiam Achatem eius comitem dixit.

ACHERON, Aen. 6, 107, quasi sine gaudio.

ACIDALIA, Aen. 1, 720, Acidalia Venus dicitur vel quia inicit curas, quas Graeci *ἄκιδας* dicunt, vel certe a fonte Acidalio qui est in Orchomeno . . . . .

AFRICA, Aen. 6, 312, . . . apricis, quasi ἄτερ φρίκης, id est sine frigore; unde non nulli et Africam dictam volunt.

Aen. 5, 128, 'apricum' autem quasi ἄνευ φρίκης, sine frigore: *inde et Africa, quod est calidior.*

cf. Paul. p. 2, apricum locum a sole apertum a Graeco vocabulo φρίκη appellatum, quasi ἀφρικής, id est sine horrore, videlicet frigoris, unde etiam putatur et Africa appellari.

AMAZON, Aen. 1, 490, dictae vel quod simul vivant sine viris, quasi ἅμα ζῶσαι, vel quod unam mammam exustam habeant, quasi ἄνευ μαζοῦ.

Aen. 11, 651, nam hoc est Amazon, quasi ἄνευ μαζοῦ, *sine mamma.*

ARCTURUS, Aen. 1, 744; G. 1, 67, quasi ἄρκτου οὐρά.

CHARON, Aen. 6, 299, κατὰ ἀντίφρασιν, quasi ἀχαίρων.

The resemblance between the words χαίρων and Χάρων was sufficient for a Greek pun (Ar. Ran. 184, χαῖρ' ὦ Χάρων), but Servius' rule for agreement of quantity should have prevented him from connecting them etymologically.

CUMAE, Aen. 3, 441; 6, 2, vel ἀπὸ τῶν κυμάτων, vel a gravidae mulieris augurio, quae Graece ἔγκυος dicitur.

CYBELE, Aen. 3, 111, 'Cybeli' *id est* montis Phrygiae, a quo et Cybele dicta est. alii . . . . a Cybelo sacerdote, ... alii ἀπὸ τοῦ κυβιστᾶν τὴν κεφαλήν, id est a capitis rotatione.

ETRURIA, *Aen. 10, 164, ab Etrusco principe.*

Aen. 11, 598, Etruria dicta est, quod eius fines tendebantur usque ad ripam Tiberis, quasi ἑτερούρια; nam ἕτερον est alterum, ὅρος finis vocatur.

HECATE, *Aen. 4, 511, quidam Hecaten dictam esse tradunt, quod eadem et Diana sit et Proserpina,* ἀπὸ τῶν ἑκατέρων: *vel quod Apollinis soror sit, qui est* ἑκατήβολος.

HYADES, Aen. 1, 744, hyades stellae sunt in fronte tauri, quae quotiens nascuntur pluvias creant: unde et Graece ὑάδες dictae sunt ἀπὸ τοῦ ὕειν, Latine suculae a suco . . . alii

dicunt hyadas dictas vel ab Y littera vel *ἀπὸ τοῦ ὑός*. id est sue, in cuius formatae sunt faciem. The same three explanations are given on G. 1. 138. Daniel's Servius adds on Aen. 1, 744. *quidam hyadas ab Hyante fratre etc.*

cf. Cic. N. D. II 43, § 111; Tullius Tiro ap. Gell. XIII 9. 4; Plin. N. H. XVIII 26, § 247.

IRIS. Aen. 5, 606; 9, 2; Iris dicta quasi *ἔρις*.

IANUS, *Aen. 7, 610, quidam Ianum Eanum dicunt ab eundo.*

cf. Cic. N. D. II 27, 67, Ianum . . . . quod ab eundo nomen est ductum.

IOVEM, Aen. 1, 47. Iovem autem a iuvando dixerunt. See IUPPITER.

IUNO. Aen. 1. 4. cum a iuvando dicta sit Iuno etc.

cf. Varr. L. L. V 67, dicta quod una cum Iove iuvat. Iuno.

Cic. N. D. II 26, 66, Sed Iunonem a iuvando credo nominatam.

IUPPITER, Aen. 4, 638, 'Iuppiter' iuvans pater.

cf. Cic. N. D. II 25. 64, sed ipse Iuppiter id est 'iuvans pater', quem conversis casibus appellamus a iuvando Iovem.

Gell. V 12, 4, Iovem Latini veteres a iuvando appellavere, eundemque alio vocabulo iuncto 'patrem' dixerunt. Nam quod est elisis aut inmutatis quibusdam litteris 'Iupiter', id plenum atque integrum est 'Iovispater'.

ΚΡΟΝΟΣ, Aen. 3, 104. quasi *Χρόνος*.

cf. Cic. N. D. II 25, 64, *Κρόνος* enim dicitur, qui est idem *χρόνος*, id est spatium temporis.

LENAEUS, Aen. 4, 207; G. 2. 4. *ἀπὸ τοῦ ληνοῦ*. In each passage Servius rejects Donatus' explanation, mentioning him by name in the second: nam quod Donatus dicit ab eo, quod mentem deleniat, non procedit: nec enim potest Graecum nomen Latinam etymologiam recipere.

LIBER, Aen. 4, 638. a libertate.

G. 1, 166, ab eo, quod liberet, dictus.

G. 1, 7, quamvis Sabini Cererem Pandam appellent, Liberum Loebasium, dictum autem, quia graece λοιβή dicitur res divina.

LIBYA, Aen. 1, 22, dicta autem Libya vel quod inde libs flat, hoc est africus, vel, ut Varro ait, quasi *ΛΙΠΠΥΙΑ*, id est egens pluviae.

LUPERCAL, Aen. 8, 343, sub monte Palatino est quaedam spelunca, in qua de capro luebatur, id est sacrificabatur: unde et lupercal non nulli dictum putant. alii quod illic lupa Remum et Romulum nutrierit: alii, quod et Vergilius, . . . ergo ideo et Evander deo gentis suae sacravit locum et nominavit lupercal, quod praesidio ipsius numinis lupi a pecudibus arcerentur.

MAENALA, G. 1, 17, mons Arcadiae, dictus ἀπὸ τῶν μήλων, id est ab ovibus, quibus plenus est.

MERCURIUS, Aen. 4, 638, quod mercibus praeest.

Aen. 8, 138, alii Mercurium quasi Medicurrium a Latinis dictum volunt, quod inter caelum et inferos semper intercurrat. hic etiam mercimonii deus est.

cf. Arnob. III 32, Mercurius etiam quasi quidam Medicurrius dictus est.

For a curious explanation of this word see Wilmanns, *De Terent. Varr. Libris, p. 175:* 'Mium et commircium per I antiquis relinquamus, apud quos aeque et Mircurius per I dicebatur, quod mirandarum esset rerum inventor, ut Varro dicit'.

NILUS, Aen. 9, 30; G. 4, 291, dictus quasi νέαν ἰλύν, hoc est novum limum trahens.

PAN, Buc. 2, 31, deus rusticus, in naturae similitudinem formatus, unde et Pan dictus est, id est omne etc.

cf. Hom. Hymn. XVIII 47, Πᾶνα δέ μιν καλέεσκον, ὅτι φρένα πᾶσιν ἔτερψε.

POENINAE, Aen. 10, 13, loca quae rupit (Hannibal sc.) Poeninae Alpes vocantur. *quamvis legatur a Poenina dea etc.*

PRAENESTE, Aen. 7, 678, locus dictus ἀπὸ τῶν πρίνων, id est ab ilicibus.

Aen. 7, 682, Cato dicit quia is locus montibus praestet.

PROMETHEUS. Buc. 6, 42, ἀπὸ τῆς προμηθείας.

SABAEI, G. 1, 57, ἀπὸ τοῦ σέβεσθαι, quod apud eos tus nascitur, quo deos placamus.

SUCULAE, Aen. 1, 744, Graece ὑάδες dictae sunt ἀπὸ τοῦ ὕειν, Latine suculae a suco.

Daniel's Servius on *Aen. 1, 744*, and *G. 1, 138*, gives the other explanation, '*a suibus*'.

cf. Gellius, XIII 9. 15, quod ab illis 'ὑάδες', a nobis primo 'syades', deinde suculae appellantur.

TUSCI, Aen. 2, 781; 8, 479: *10, 164*, a frequentia sacrificii dicti, hoc est ἀπὸ τοῦ θύειν.

cf. Paul. p. 537, Tusci a Tusco rege, filio Herculis, sunt dicti, vel a sacrificando studiose, ex Graeco velut θυσκόοι.

TARTARUS, Aen. 6, 577, ἀπὸ τῆς ταραχῆς, aut, quod est melius, ἀπὸ τοῦ ταρταρίζειν id est a tremore frigoris.

VESTA, Aen. 1, 292, ἀπὸ τῆς ἑστίας . . ., vel quod variis vestita sit rebus.

Aen. 2, 296, . . . quod vi sua stet.

cf. Ovid. Fast. VI 299, stat vi terra sua: vi stando Vesta vocatur.

ΖΕΥΣ, Aen. 1, 388, Iuppiter, quo constant omnia, Ζεύς vocatur ἀπὸ τῆς ζωῆς, id est vita.

cf. Plat. Cratyl. 396 B, Ζῆνα . . . δι' ὃν ζῆν ἀεὶ πᾶσι τοῖς ζῶσιν ὑπάρχει.

NOTES ON JEFFERSON'S DRAFT OF THE ORDINANCE OF 1784.

By W. M. HALL.

THE "SEVENTEEN STATES" PLAN.

McMaster speaks of the plan for seventeen States as if it were wholly thrown aside by Jefferson's committee. "One plan was to divide the ceded and purchased lands into seventeen States. Eight of these were to lie between the banks of the Mississippi and a north and south line through the falls of the Ohio. Eight more were to be marked out between this line and a second one parallel to it, and passing through the western bank of the mouth of the Great Kanawha. What remained was to form the seventeenth State. But few supporters were found for the measure. and a committee, over which Jefferson presided, was ordered to place before Congress a new scheme of division. Chase and Howe assisted him, and the three devised a plan whereby the prairie lands were to be parted out among ten new States." *

These "ten new States" lay north of the Ohio; except, as will presently be seen, parts of two of them. Nevertheless it seems clear that the Ordinance was meant to provide government for all the Western country that the States might cede, down to the southern line of the United States at 31°. The surrender of some State claims south of the Ohio was expected, and North Carolina did pass a short-lived act of cession a few weeks after. Jefferson was laboring to persuade the Southern States to cede all their territory west of the Great Kanawha meridian. The Ordinance reported by his committee begins by saying that "the territory ceded or to be ceded by indi-

* Volume I., page 165. But since the following comments were read before the Society, Volume III. of Mr. McMaster's History has been published, containing (at page 101) a seemingly full recognition of the substantial identity of the two plans.

vidual States to the United States . . . . shall be formed into additional States, bounded . . . . as nearly as such cessions will admit . . . . northwardly and southwardly by parallels of latitude, so that each State shall comprehend, from south to north, two degrees of latitude, beginning to count from the completion of thirty-one degrees from the equator; but any territory northwardly of the forty-seventh degree shall make part of the State next below." (Then are described the north-to-south lines, viz., the Mississippi and the meridians of the Ohio falls and the mouth of the Great Kanawha.) If we were to understand "the territory ceded or to be ceded by individual States" as an unguarded phrase meaning merely such territory north of the Ohio (a strange oversight when cessions south of the river were under discussion), the expected cessions being those of Massachusetts and Connecticut only, it would be hard to account for the irrelevant mention of the thirty-first degree line, which is about six degrees south of the southernmost point of Illinois; and it would be equally hard to see why, in the detailed statement of boundaries further on, a State ("Pelisipia") was made of two small blocks of land between the thirty-ninth degree line and the Ohio east and west of Cincinnati, which should naturally have been attached to the adjoining square State, Saratoga, as the points of the two Michigan peninsulas had been attached to other States.

The difficulties vanish when we regard the reported Ordinance as dealing with the whole Western country, from 31° up, the first paragraph laying an outline of States by drawing east-and-west lines every two degrees up to 45°, and a meridian line through the falls of the Ohio (Louisville)* dividing each of these strips into two States, with another meridian line through the mouth of the Kanawha cutting off what is now Eastern Ohio into a State by itself. Here we have two rows of eight States each, and an odd State further east; the

* Apparently the meridian of Louisville was believed to enter Lake Michigan near the southern end. For the Illinois river, *i. e.*, its Kankakee branch, is mentioned in the later paragraph of details as one of the rivers whose sources lie in Metropotamia, east of that meridian. The same is suggested, though less clearly, by the description of Michigania as wholly west of the lake, without remark upon this as a deviation from the meridian system.

"seventeen States" still undiminished. This is the general scheme; and a later paragraph applies it in detail for the region north of the Ohio. which was already ceded except the two strips claimed by Massachusetts and Connecticut; the northeast corner State of the general scheme, consisting chiefly of lake surface, is extinguished and the points of the Michigan peninsulas are given to the contiguous States. Two of the ten States here bounded and named lie partly in Kentucky, Pelisipia being thus no mere remnant north of the Ohio. Six States of the general scheme are not here named nor particularly bounded; doubtless because they lie in the still unceded region south of the Ohio, and the eastern boundaries of the expected cessions are not certainly known. The "ten States" plan is thus a mere definition of part of the "seventeen States" plan, and Jefferson's draft represents both.

RIDICULE OF THE DRAFT.

The proposed list of State names (rejected by Congress) was a cause of great hilarity in its day, and the pleasure is shared by some recent historians. It can scarcely be that their amusement comes from any other feature of the plan, for the comparatively small size of the States was due to an express requirement in Virginia's deed of cession,* and the attempt to lay down State lines was not untimely, for the lines drawn by the more famous Ordinance of 1787 are, with little change. those that separate Ohio, Indiana, and Illinois. We must look to the names alone for the supposed absurdity, and there a mere glance seems to prove it—Sylvania, Michigania, Chersonesus, Assenisipia, Metropotamia, Illinoia, Saratoga, Washington, Polypotamia, Pelisipia.

At a second glance the list is less formidable. Michigania, Illinoia, Washington, are names that have since won the preference of States; and Sylvania is a manifest improvement upon Pennsylvania. Saratoga is a blameless commemorative name. The absurdity is reduced to the other five. As to those, no one will pretend that better names, for three at least, might not have been found easily. But there is room for

*The larger States of the Ordinance of 1787 were made in view of an expected modification of Virginia's deed, which duly followed.

question whether they are worse, or much worse, than the names that States have deliberately taken to themselves or received from their European sponsors. Two come legitimately enough from the Indian names, Assenisipi and Pelisipi, of the Rock and Ohio rivers: and it is not easy to frame a canon of euphony that will reject these and accept Mississippi, Massachusetts, Connecticut and Minnesota. The Grecian or would-be-Grecian aspect of Metropotamia, Polypotamia and Chersonesus need not have offended the sensibilities of a Congress that was accustomed to hold sessions in Annapolis or Philadelphia. Indeed, in the short list of our cities of a hundred thousand inhabitants appear three names, either Greek or of a lamentable Greek admixture—Philadelphia, Minneapolis and Indianapolis, besides other curiosities like Cincinnati, Chicago, Omaha, Milwaukee, Allegheny, Providence and Buffalo. And if the comparison be kept strictly to names of States, the error of Jefferson's list towards Greek is perhaps offset by its superiority in other points; it has no name out of a story-book like California; none so commonplace as New Hampshire, New Jersey and New York; none so angular as Wisconsin, Arkansas, Massachusetts and Connecticut; none so mongrel as Pennsylvania; none so cumbrous as Rhode-Island-and-Providence-Plantations.

It may accordingly be urged that the amusement over Jefferson's names comes not from any unusual oddity in them, but from the fact that others have been taken instead. If Assenisipia and Polypotamia had been adopted, they would doubtless pass unnoticed now, like Mississippi and Indianapolis.

SOME NOTES ON BLAYDES' NUBES.

By AUGUSTUS T. MURRAY.

It is is not the purpose of this paper to give a detailed review of Blaydes' work. His editorial methods are well known and his thorough acquaintance with comic diction is deservedly praised. It is in this respect that his editions are most useful, giving as they do a wealth of illustration invaluable to every student of Aristophanes. Of course this must be offset by his contempt of manuscript tradition, shown in this very play by the introduction or suggestion of many hundred variations from the vulgata.

Added to this is the thoroughly unscientific character of the commentary in which views are maintained and retracted, and utterly divergent ones substituted, apparently according to the whim of the moment. This very volume might have been reduced in bulk surely one third, if a thorough revision had been carried out. Many passages are now absurdly inconsistent (see *e. g.* text, critical note and addenda, vs. 811), and while in Blaydes' eyes it may be a small sin to adopt one reading in the text and another in the commentary, yet to all believers in painstaking accuracy this fault is a grievous one. The same carelessness shows itself in matters of fact: *e. g.* on vs. 151 we are told that Dindorf and Teuffel give the reading *φυγείσῃ*, whereas both have the earlier Attic form *φυχείσῃ*. Again the addendum on vs. 1365 is said to be drawn from "Teuff. II" while it really comes from Kock. (The same error in addendum to vs. 1192.) In the commentary on vs. 225 [Plat.] Axiochus 372 B. is referred to as Aeschin. Axioch. 22. But surely the identification is ungrounded; cf. Teuff. Uebersicht p. 35. After reading in the text *ἀείδειν*, (vs. 298) it is absurd to read in the critical note "Dedi *ἀοιδῶν*," although it is true we find half a dozen lines further down "Vel potius *ἀείδειν*."

Such examples might easily be multiplied, and the editor himself is not unaware of their existence; see introd. p. xxxvii. It is, however, foreign to my purpose to criticise these details. I wish rather to discuss certain of his readings.

It will be interesting to start with his own exposition of his views in regard to Aristophanic criticism. Regretting the fact that for the Nubes the Ravennas and Venetus have not as yet been as thoroughly collated as we could wish, he continues: "Fatendum tamen est non valde magnum auxilium vel ab iis expectandum esse, quum mendorum quibus nostri textus maculatur non pauca inveterata sint et ejusmodi ut sine conjecturae ope nunquam tolli queant." This is the ground for the adoption or suggestion of many hundred variations from the vulgata, as if he were correcting a schoolboy's exercise! What shall we say to the array of "hariolationes" in the critical note to vs. 995? or to the samples of his critical acumen given in the introd. p. xxxv f.?

I propose to discuss certain passages in detail.

Vs. 24.—Strepsiades is cursing himself for having bought the ἵππον κοππατίαν and exclaims—

εἶθ' ἐξεκόπην πρότερον τὸν ὀφθαλμὸν λίθῳ.

There is not the slightest ground for rejecting the reading of the MSS. and Scholia. Close parallels are given by Teuffel, especially Plaut. Men. 152 (158) oculum ecfodito mihi si ullum verbum faxo.

Kuster however suggested ἐξεκόπη and has been followed by Hermann, Meineke and Dindorf, the last of whom gravely remarks, "Equo si elisus fuisset oculus, non emendus fuisset Strepsiadi." Blaydes not only adopts this reading but suggests further τὸν ἕτερον ὀφθ. "quum non μονόφθαλμοι sint equi." This is triviality itself. Of course the dual and plural often occur, but there are a number of cases, some cited by Blaydes himself, where the singular is found.

See *e. g.* Aves, 1611 ff:

ὅταν ὀμνύῃ τις τὸν κόρακα καὶ τὸν Δία,
ὁ κόραξ παρελθὼν τοὐπιορκοῦντος λάθρᾳ
προσπτόμενος ἐκκόψει τὸν ὀφθαλμὸν θένων.

Ach. 91 ff.—*Καὶ νῦν ἄγοντες ἥκομεν Ψευδαρτάβαν*
τὸν βασιλέως ὀφθαλμόν. Dic. *ἐκκόψειε γε*
κόραξ πατάξας τόν γε σὸν τοῦ πρεσβέως.

Dem. 18. 67, of Philip.—*τὸν ὀφθαλμὸν ἐκκεκομμένον.*

Lucian Tox. 24.—*τὸν ὀφθαλμὸν ἐκκεκομμένη.*

Men as well as horses are not *μονόφθαλμοι.*

Vs. 151.—*κᾆτα ψυχείσῃ περιέφυσαν Περσικαί.*

So Dind. and Teuff. The MSS. differ only in giving the later form *ψυγείσῃ*. Kock and Meineke give the easy reading *ψυχέντος* (Herwerden *ψυγέντος*) of which Teuff. well says: "Die Aenderung *ψυχέντος* liegt zu nahe als dass sie wahrscheinlich waere." Why in the world Aristophanes should have preferred to say "when the flea became cold," rather than "when the wax cooled," is a question that has not been answered and the reading is suspicious. Certain, however, it is that *ψυχείσῃ* did not come from the easy *ψυχέντος*. Nothing is gained by Blaydes' adoption of *ψυχεῖσι* (*τοῖς ποσί*) and he will find no followers.

Kaehler's *ψυχεῖσαι* (proleptic) is worth noting.

It is characteristic that while Blaydes gives *ψυχεῖσι* in his text, we read in the commentary "*ψυχείσῃ* sc. *τῇ ψύλλῃ*. Sed corrigendum videtur *ψυχέντος* (*τοῦ κηροῦ*)."

Vs. 225 f.—Soc. *ἀεροβατῶ καὶ περιφρονῶ τὸν ἥλιον.*

Streps. *ἔπειτ' ἀπὸ ταρροῦ τοὺς θεοὺς ὑπερφρονεῖς*
ἀλλ' οὐκ ἀπὸ τῆς γῆς, εἴπερ;

Blaydes changes *ὑπερφρονεῖς* to *σὺ περιφρονεῖς*. (an exceedingly slight change and a very clever one), on the ground that *ὑπερφρονεῖς* should govern the genitive and because the sense demands *περιφρονεῖς*.

We would at first glance expect the same verb, but the reading is supported by all the MSS. and the scholiast, and the verb on closer inspection seems purposely varied. Socrates means, "I am speculating about the sun," (for this sense of *περιφρονεῖν* cf. 741. 1503), but the words *can* be taken as. "I am making no account of Helios," and it is in this sense that Strepsiades takes them. This meaning of *περιφρονεῖν* does

occur, although Thuc. 1, 25 seems to be the only classic instance. Others can, however, be adduced from later writers; see Krüger's note on Thucyd. loc. cit.

Strepsiades therefore uses the common word for contemning, ὑπερφρονεῖν.

As to the acc. with this last vb. cf. Thucyd. 6. 68. 2. 3. 39. 5. Aesch. Pers. 825. Eur. Fr. 545 N².

For Strepsiades' taking words in a different sense from the speaker, see 236, although there it is from inability to understand them. Perhaps 248 is to be explained in the same way. (Teuff.²)

In my opinion the passage is perfectly sound, and Blaydes' suggestion, while very ingenious, is to be discarded.

Vss. 369 ff.—Soc. has denied the existence of Zeus. Strepsiades then asks, amazed, ἀλλὰ τίς ὕει; to which Socrates replies:

> αὗται δήπου. μεγάλοις δέ σ' ἐγὼ σημείοις αὐτὸ διδάξω.
> φέρε, ποῦ γὰρ πώποτ' ἄνευ Νεφελῶν ὕοντ' ἤδη τεθέασαι;
> καίτοι χρῆν αἰθρίας ὕειν αὐτόν, ταύτας δ' ἀποδημεῖν.

In vs. 369 Reisig reads τοῦτο from inferior MSS., so Blaydes, a reading which is easier but unnecessary. In 371 Blaydes takes exception to αὐτόν and casts it out, accepting οὔσης before ὕειν, which is also found in some MSS. That is, however, unquestionably a gloss to explain αἰθρίας and can readily be paralleled, while the use of the genitive alone is very common.

Blaydes' grounds for ejecting αὐτόν are that it itself is apparently a gloss. (ὕοντα is used absolutely in 370), and that there is nothing for it to refer to. "nisi illud additum hic statuas majoris emphasis causa propter sequens ταύτας." But that is exactly the case, and the reference is clear from what precedes. There is no difficulty whatever and the αὐτόν adds force.

Vs. 493.—δέδοικά σ', ὦ πρεσβῦτα, μὴ πληγῶν δέει.

Blaydes with Dindorf, but against the MSS., reads δέῃ, noting that the subj. is necessary. In this he seems clearly wrong, despite Elmsley's authority. See Goodwin G. M. T.²

§ 369 and Krüger 54. 8. 12. Some of Blaydes' passages are not to the point and in others the verbs are indicative.

Vs. 575. ὦ σοφώτατοι θεαταί, δεῦρο τὸν νοῦν προσέχετε.

So the MSS. and Teuff. (Kaehler). Kock and Mein. follow Bentley in reading πρόσχετε. Dindorf and Blaydes πρόσσχετε, in order to avoid the tribrach in the fourth dipody.

Blaydes brings forward a number of passages to show that the verb is not exclusively in the present tense. These are worth examining; what he does not tell us is, that in every case cited from Aristoph. the best MSS. have the present.

Of the sixteen passages he cites, Nub. 635, Eq. 1014, Pax 174, Thesm. 25, 381, Plut. 113, Pherecrat. II 340, Cratin. II 189, Antiph. III 29, Alex. III 503, may be dismissed off hand as giving simply πρόσεχε τὸν νοῦν, generally in trimeter, and without a variant.

In Nub. 1122 (Trochaic tetram.), Eq. 503 (anapaestic tetram.), Vesp. 1015 (anapaestic tetram.), Av. 688 (anapaestic tetram.) and in the present passage, the MSS. agree in giving προσέχετε. (In Vesp. 1015 *BΔ* have πρόσχετε, but RV show προσέχετε). Bentley changed throughout to πρόσχετε, and is followed by most editors. Dindorf prefers πρόσσχετε throughout, while Bergk. Teuff. and Kaehler retain the present. Blaydes is hopelessly inconsistent both in text and commentary, vacillating from one reading to another. In his latest note, however, (addenda vs. 575 Nub.), he inclines to Dindorf's view. In view of this, Kaehler's note, (Anhang p. 197), is to the point: "Die Herausgeber haben vielfach in πρόσχετε πρόσσχετε geaendert, ohne Grund, wie es sich nach dieser auffaelligen Uebereinstimmung zeigt, nur einem metrischem Gesichtspunkte zu liebe."

There remain two instances of πρόσισχε (one is plural), Eupolis II 438=K. 37, and II 575=K. 386, (the first is not in Kaehler or in Jacobi's index s. v. νοῦς), and finally Blaydes omits the only instance of the aorist in comedy, Pherecrates II 283=79 K. πρόσχετε τὸν νοῦν. In so short a fragment metrical grounds cannot be conclusive, but this aorist may be so accounted for.

Therefore, while it must be admitted that the aorist improves the metre in tetrameters, Blaydes' rule, (Aves. 688 addenda), "Scribendum ut in libris *προσέχετε*," or Nub. 575 not. crit.), "Quare non nimis festinanter corrigendum *πρόσσχετε*" is a safe one. He has not, however, followed it himself.

Vs. 592. *εἶτα φιμώσητε τούτου τῷ ξύλῳ τὸν αὐχένα—.*

Few will follow Blaydes here in accepting from Lys. 680 the verb *ἐγκαθαρμόζειν* and rewriting the present passage thus:

ἐγκαθαρμόσητε τούτου 'ς τὸ ξύλον τὸν αὐχένα,

although the phrase *φιμοῦν τὸν αὐχένα τῷ ξύλῳ* does not occur elsewhere.

φιμοῦν from its very unusualness must be the true reading, and the question is only whether we should read *τῷ ξύλῳ* or *'ν τῷ ξύλῳ.*

In view of Eq. 394 *ἐν ξύλῳ δήσας*, Eq. 705 *ἐν τῷ ξύλῳ δήσω*, and similar passages, it would seem that we must follow Elmsley in inserting the preposition in that phrase even where the MSS. do not have it, *e. g.* Eq. 367 *οἷον σὲ δήσω τῷ ξύλῳ.* Eq. 1049 is interesting; there the MSS., (R. excepted, which has *ἐκέλευσε*), have,

τουτονί

δῆσαι σ' ἐκέλευε πεντεσυρίγγῳ ξύλῳ.

The preposition is found in the quotation in the Etym. Mag. p. 346, 18. Evidently the scribes mistook *ἐκέλευ' ἐν* for *ἐκέλευεν*, and then the *ν* was lost before the consonant.

From this analogy *ἐν* may be expected in our phrase; so Sauppe, Dind., Kock, Mein. and Elmsley on Ach. 343.

I am inclined, however, with Teuff. and Kaehler, to regard the dative as causal.

Vss. 743 ff.—

ἔχ' ἀτρέμα κἂν ἀπορῇς τι τῶν νοημάτων,
ἀφεὶς ἄπελθε κᾆτα τὴν γνώμην πάλιν
κίνησον αὖθις αὐτὸ καὶ ζυγώθρισον.

The variants are *ἀτρέμας*, many MSS., and *πάλαι* for *πάλιν* R.S.V. R. has also *κατα* for *κᾆτα.*

The passage can hardly be sound as it stands, although it is kept by Bergk, Dind., Mein., Teuff. Bergk suggested *καὶ*

κατὰ for κᾆτα, but that does not satisfy, for the phrase κινεῖν τὸ νόημα κατὰ τὴν γνώμην is strange enough. Blaydes quotes the suggestion with approval.

Meineke proposes τῇ γνώμῃ or αὖ σύ, (for αὐτό), of which the latter is certainly not to be accepted, as there is no stress upon the pronoun. The former, τῇ γνώμῃ, had already been suggested by Reiske and is, I think rightly, accepted by Kock and Kaehler. νόημα τῇ γνώμῃ κινεῖν is not tautological. Blaydes objects, "quum κινεῖν τὴν γνώμην bene dicatur, non item κινεῖν νόημα," after which it is strange to read in the commentary "τὴν γνώμην κίνησον—nisi corrigendum τῇ γνώμῃ—κίνησον, mente id (τὸ νόημα) agita"!

Blaydes further adopts ἢν δ' for κἂν to avoid a difficulty which is only apparent, as ἄπελθε refers to a different activity and does not contradict the ἔχ' ἀτρέμα.

Again in vs. 745 he reads αὐτό τε, assigning to κίνησον and ζυγώθρισον different objects. The sense is good but the change is needless, and the τε solitarium should not needlessly be foisted into trimeters. Not satisfied with this, he offers the choice between αὐτό τε, καὶ τάχα, καὶ πάλιν, καὐτίκα, αὐτίκα τε, εἶτα δὲ.

Accepting Reiske's τῇ γνώμῃ, I think the passage is sound without further change.

Vs. 823.—καί σοι φράσω τι πρᾶγμ' ὃ μαθὼν ἀνὴρ ἔσει.

So R. V. and others while many MSS. give ὃ σύ and omit τι. For once Blaydes gives us no critical note, although he follows Dindorf in rejecting the reading of R. V. Why the other reading should be preferred it is hard to say, and to give it to one's readers without at least noting the alternative is inexcusable in an edition which claims to be critical.

Vs. 1046.—ὅτιὴ κάκιστόν ἐστι καὶ δειλὸν ποιεῖ τὸν ἄνδρα.

So the MSS. save that R. has δειλότατον; which, however, is easily explained by the preceding κάκιστον (Dind). All editors, Blaydes excepted, accept the reading and there is no serious objection to it. Blaydes comments: "Versus valde frigidus et nostro indignus. Displicet inter alia τὸν ἄνδρα." He reads, therefore, ὅτιὴ κάκιστον αὐτὰ καὶ δειλότατον ἄνδρα ποιεῖ,

as Reiske had suggested, but as if to show his independence, suggests alternatives which are characteristic enough to be quoted in full. "Vel *ὅτι ἡ κακὸν μάχην τε . . . . .* Vel *ὅτι ἡ κακὸν τὸ λῆμα καὶ δειλόν τιν' ἄνδρα ποιεῖ.* (vel *δειλότατον ἄνδρα ποιεῖ*). Vel *ὅτι ἡ κακόν τ' ἐς τοὺς πόνους καὶ δειλὸν ἄνδρα ποιεῖ.* Vel *ὅτι ἡ μαλακὸν ἐς. . . . .* (cf. 1048. 1049.), Vel *ὅτι ἡ κακὸν τὸν χρώμενον,* (vel *λούμενον*) *καὶ δειλὸν ἄνδρα ποιεῖ.* Vel *ὅτι ἡ γυναικώδη τε καὶ κακὸν* (*δειλὸν*) . . . ." And yet another; but one's patience gives out. Who can call this criticism?

Vs. 1415.—*κλάουσι παῖδες πατέρα δ' οὐ κλάειν δοκεῖς.*

Blaydes brackets this line as spurious, wrongly as it seems. Metrical difficulty there is none. The additions in various MSS. are attempts to complete the tetrameter, but the poet purposely inserts the trimeter to make the parody more apparent; and in view of the metrical scolium there can be no doubt that the trimeter is intended.

More reasonable is the objection to *δοκεῖς* in this sense (hence Cobet *σὺ χρῆναι*, Herwerden *δοκεῖς προσήκειν*), but in my opinion the close resemblance to Euripides' verse excuses it, and any addition makes the resemblance less close.

To cast out the verse is surely wrong, but it is in line with Blaydes' treatment of another parodic passage, vs. 30, where on the traditional *τί χρέος ἔβα με*, he remarks "An *χρέως* (debitum)?" a suggestion which robs the passage of half its force.

ON EURIP. IPH. TAUR. Vss. 351 ff.

By AUGUSTUS T. MURRAY.

καὶ τοῦτ' ἄρ' ἦν ἀληθές, ᾐσθόμην, φίλαι·
οἱ δυστυχεῖς γὰρ τοῖσιν εὐτυχεστέροις
αὐτοὶ κακῶς πράξαντες οὐ φρονοῦσιν εὖ·

So the MSS. save for Dindorf's certain correction of ᾐσθόμην for ἠχθόμην.

Seidler changed the κακῶς of vs. 353 to καλῶς, and paraphrases thus: "Infelices, qui ipsi aliquando felices fuerunt, felicioribus se non bene volunt." He is followed by Dindorf. Nauck, Paley, and Jerram, (of the editions I have at hand). Schoene and Flagg keep the κακῶς. which Schoene explains as causal,—"weil sie unglücklich geworden sind." That seems. however. impossible. Nor will many follow Wecklein in reading τοῖσι δυστυχεστέροις and κακῶς, and interpreting, "Die Unglücklichen meinen es nicht gut mit den noch Unglücklicheren wenn sie selber Leid erfahren haben."

The objection to Seidler's rendering is, to my mind, twofold. In the first place, we need a ποτέ or πάλαι; so that Dindorf's πάλαι καλῶς is an improvement; or. better. Rauchenstein's αὐτοί ποτ' εὖ πράξαντες; and secondly, in what respect are Orestes and Pylades more fortunate than Iphigenia? They *had been*. it is true, but now that they are brought face to face with death, their lot is not an enviable one. If in Iphigenia's eyes death was preferable to her state. it was an alternative she could at any time have chosen.

The reference must be to those who have been more fortunate but who have fallen into adversity, (as have Orestes and Pylades). The envy and bitterness caused by their former good fortune steel the hearts of the δυστυχεῖς against them.

Hence Kirchoff, (followed by England), would read αὐτοῖς κακῶς πράξασιν. But why the αὐτοῖς? αὐτοί referring to the δυστυχεῖς has its proper force. but αὐτοῖς is more than superfluous with τοῖσιν εὐτυχεστέροις.

I feel certain, therefore. that we are nearer the truth in reading:

οἱ δυστυχεῖς γὰρ τοῖσιν εὐτυχεστέροις
ὅταν κακῶς πράξωσιν οὐ φρονοῦσιν εὖ.

The αὐτοί may have been added by some scribe who mistook the subject of πράξωσιν and wished to make the reference clearer. When once αὐτοί had crowded out ὅταν the subj. would naturally be changed to the participle.

DRAPER'S BAROGRAPH.

By FLORIAN CAJORI.

The description of Draper's Barograph in the *Scientific American Supplement*. No. 209. 1880, contains the following passage: "Heat has a slight effect on them (the springs), causing them to lengthen about $\frac{1}{16}$ of an inch for 90 degrees Fahr.; . . . . *Otherwise this instrument gives the correction for temperature (or reduction to 32 degrees) from the fact that it weighs the mercury instead of measuring its length, which is affected by heat.*" This statement appears to be considered correct. for we are told that in the use of Draper's Barograph no corrections are made for variations in temperature of the mercury.

It is the object of this paper to point out that the statement quoted in italics is erroneous. The instrument in question consists of a tube 36 inches long, the upper portion being of larger diameter than the lower. The tube is held firmly in a fixed position and filled with mercury. Its lower end dips into a movable cistern which is suspended on two spiral springs. When the atmospheric pressure diminishes, part of the mercury flows from the tube into the cistern: this becoming heavier, stretches the steel springs. causing the ink pencil fastened to them to mark downwards. If the pressure increases, the opposite movement takes place.

The error referred to can be established by the following *reductio ad absurdum:* All the mercury in the instrument is supported by the movable cistern except that which is in the fixed tube above the surface of the mercury in the cistern. Now, if we suppose the atmospheric pressure to be constant. but the temperature of the mercury to vary (the temperature of the springs remaining the same), then, if the quoted statement is correct, the cistern should neither rise nor fall, else

the instrument would wrongly indicate a change of atmospheric pressure. If the cistern does not move, it supports the same weight as it did before the change of temperature. Hence the mercury in the fixed barometric tube extending from the surface of the mercury in the cistern upwards, must have the same weight as it had before. This weight is in part balanced by the atmospheric pressure and in part rests upon the fixed tube at the place near its upper end where it becomes suddenly broader. The atmospheric pressure, being assumed constant, supports the same weight of mercury as before; hence the fixed tube likewise supports the same weight as before. But the tube supports always a definite and constant percentage of the entire mass of mercury in the broad part. Hence mercury cannot pass in either direction between the broad and narrow parts of the tube, for such a passage would alter the total amount of mercury in the broad end and also the portion upheld by the tube. If these deductions are true, then mercury in the narrow part of the tube cannot expand when the temperature rises, since it is impossible for the mercury to pass either into the wider part of the tube above or into the cistern below. The statement quoted at the beginning must therefore be wrong, for it leads to an absurdity. A change of temperature in the mercury, the pressure of the atmosphere remaining constant, *does* affect the reading of the instrument.

What actually takes place during a rise of temperature, at constant pressure, may be briefly indicated as follows: The mercury in the narrow tube begins to expand downwards as well as upwards, causing a small amount to escape into the cistern. The increase in the mass of the liquid in the cistern, together with the lengthening of the steel springs, due to the rise in temperature, will thereupon cause the cistern to fall. By the several readjustments the mercury column supported by the atmospheric pressure is lengthened to such an extent as to make the weight of the new column supported by the atmospheric pressure equal to the weight of the old one, which was shorter on account of the greater density of the mercury at a lower temperature. There is now less mercury than be-

fore in the wide part of the fixed tube and more of it in the cistern. Hence, a rise in temperature of the mercury tends to make the instrument read too low. With the aid of somewhat elaborate equations the writer has calculated that for a rise of 45 degrees Fahr. in the mercury (at constant atmospheric pressure) the cistern will fall about .174 inches. According to the figures in the S. A. S. No. 209, the springs will stretch *about* .03 inches, making an aggregate depression of .204 inches for 45 degrees, and thereby causing the atmospheric pressure to appear .048 inches lower than it actually is. An experiment with the barograph at Colorado College caused the cistern to fall .211 inches for a rise of 45 degrees Fahr. The experimental and the calculated figures are in fairly close agreement.

The conclusion then is that Draper's Barograph reads too low for temperatures above the standard temperature and too high for temperatures below the standard. It is not sufficient to make corrections for the stretching of the springs, for the errors arising from variations of temperature in the mercury are over five times greater.

It has been the practice to compare the barograph, from time to time, with a standard barometer, but the results are not satisfactory. Suppose comparisons are made at 12 M. It is easy to adjust the barograph so that it will indicate very nearly the same atmospheric pressure as does the barometer at noon, except on days when the temperature varies widely from the average noon temperature. But what about the barograph readings in the morning before sunrise, when the temperature is much lower? Will the barograph then not indicate a pressure altogether too high? Suppose the temperature in the early morning be 10 degrees less in the barograph case than it is at noon, then if the barograph reads correctly at noon, it will read nearly .02 of an inch too high in the morning. This is a serious error. If tables be constructed for temperature corrections in Draper's Barograph, it will not be difficult to use the barograph independently of a standard barometer and to secure results of much greater accuracy than those obtained at present.

THE CONDITIONAL IN GERMAN.

By SYLVESTER PRIMER.

Two opinions obtain in regard to the origin of the two so-called conditional modes in German, ich wuerde loben, ich wuerde gelobt haben, the one advanced by Jacob Grimm in his Deutsche Grammatik, 4. 183 ff., the other first proposed by Vernalaken, Deutsche Syntax, II. 283 ff., which soon found favor with others. However, these two opinions are not so opposed that they cannot be considered complements to each other.

In O.H.G. the Latin Imperfect, Perfect and Pluperfect Subjunctives were expressed by the Imperfect Subjunctive (wari, etc.); the Latin Perfect Subjunctive was also expressed by the Present Subjunctive. The M.H.G. added a Perfect form (Compound of the Present), but the Imperfect was not limited to the expression of the Latin Imperfect Indicative or Subjunctive, for it still retained its old Perfect force. Thus the Imperfect finally expressed the Latin Imperfect proper and the Aorist Perfect, while the Compound of the Present (bin gewesen) expressed absolute past time. The result was a loss in the differentiation of the Imperfect and Aorist, but a gain in that of the Aorist and Perfect. Right here the Romance Languages excel the German and Latin, for they have three tenses (j'étais, je fus, j'ai été, etc.). If, then, for the Indicative the differentiation between the Aorist and Perfect was more important and essential than that of the Imperfect and Perfect, the exact opposite was true with regard to the Subjunctive, where there is little to narrate, but a clearer differentiation of incomplete and complete past time is essential. Grimm therefore argues that the Imperfect Subjunctive could not suffice for the Imperfect tense, since there were unavoidable encroachments of the simple Preterite (Aorist) upon the Present on account of the wearing away of the form of

the Present Subjunctive. The result of this was a new periphrasis, by means of which all modern German Languages have enlarged their Subjunctive by one more tense than the Indicative. This is called the *Conditional*, in imitation of the Romance Grammarians. The auxiliaries forming the future also serve in the German Languages for this Periphrastic Conditional, and its signification is just the same as in the Romance Languages. It cannot, according to Grimm, have arisen earlier than the Periphrastic Future.

This conditional idea, says Grimm further, was at first expressed by *sollen*, and still is in dialects where *sollen* is used for the future (cf. Netherl. *zaude*, Engl. *should* and *would*, Swed. *skulle*, Dan. *skulde*). Mid.H.G. also uses *solde* and *wolde* in the same sense. A Mid.H.G. *wuerde* with the Infinitive is just as unheard of in the poets of the 13th cent. as the present tense-form *wirde* as periphrastic future. In the 14th and 15th cent. isolated examples are found: *wuerdent schaetzen* (*aestimarent*). Ls. 1, 15, and in the 16th cent. *wuerde sagen* is as common in the language as *werde sagen*. The Subjunctive forms *sollte* and *wollte*, which had become less clear because they lacked the umlaut, is easily recognizable in *wuerde;* it must therefore be considered a Subjunctive tense and not Indicative.

Furthermore this periphrastic form coincides with the simple Preterite Subjunctive in its signification. The Mid.H.G. *disiu zuht gienge billicher ueber mich*, Iw. 1678, *vor im genaese niemen*, Ben. 380, corresponds to a Mod.H.G. *wuerde ergehen*, *wuerde genesen*, though the simple tense may still be used, as in Mid.H.G. might have been said, *solde gan*. Both tenses, the simple and periphrastic, thus compete according to circumstances, as in the Indicative *gieng* and *ist gegangen*. But *gienge* has a broader field and in many cases cannot be replaced by *wuerde gehen*, while *gienge* can almost always replace *wuerde gehen*. *Wuerde gehen*, *wuerde lieben* never have optative force, and are thus differentiated from the passive periphrastic form of the Preterite Subjunctive, which is not to be placed on the same footing with it and was present much earlier in the language. *Wuerde uf getan* (aperire-

tur), Iw. 1264, and similar expressions are found everywhere in the 13th cent. and earlier, never *wuerde uf tuon* (aperiret), and the origin of the periphrasis of the two is different. *Wuerde aufgethan* fills quite the role of the Preterite Subjunctive and has optative force. A passive conditional acquires further periphrasis by *wuerde aufgethan werden*, and this parallels the active *wuerde aufthun*. From this we see that *wuerde aufthun* cannot be a mere Subjunctive form of *ward aufthun*, since these Indicative periphrastic forms have not been maintained in the language (Grimm, loc. cit. 185 f.).

The second opinion is thus set forth by Vernalaken (loc. cit. II, 283 f.). It is well known that the future *er wird fragen* had an imperfect form *ward* or *wurde fragen*. *Ward*, in the beginning joined to the preterite participle, then to the infinitive, umlauted and *wuerde* was used consequently as conditional along with *sollte* and *wollte*. All four absolute forms of the Subjunctive, *kaeme*, *waere gekommen*, *wuerde* (*sollte*) *kommen*, *wuerde* (*sollte*) *gekommen sein*, are used as conditionals. Since the subjunctive form is less readily recognized in *sollte*, *wollte* than in *wuerde*, the subjunctive form of the *futurum* and *futurum exactum* has become more frequent in late years. However, it must not be overlooked that all four absolute subjunctive forms can be used both independently (that is, in independent clauses), and dependently (that is, in dependent clauses). Moreover it belongs to the nature of the conditional sentence to employ the absolute future Subjunctive, viz., the conditional present and past, both indicatively and subjunctively, as in French, where the forms are just the same, *e. g.*, in the subordinate clause: Il jurait qu' il ne se rendrait (conditional instead of future) jamais coupable d' un tel crime. The same can be used indicatively: Je ne me rendrait jamais coupable, etc. As *wuerde gehen* is nothing else than an absolute future conditional, so the French conditional "Je dormerais" denotes no definite time, no peculiar mode, but only such a subjunctive as is used conditionally and optatively. While the Indicative of the Imperfect is lost in the dialects of Upper Germany and the periphrastic Preterite (Perfect) is used for it, the Subjunctive of the Imper-

fect is retained with the conditional force. One can hear in the Western Alps (Switzerland) the periphrasis with "i wurdi, wurd (wur)," in the Eastern, more often the periphrasis with "i thaet", but also the weak and strong forms of the Imperfect Subjunctive.

Vernalaken explains the origin of the future signification of *werden* as follows: *Sein* and *werden* with the present participle are analogous to *sein* and *werden* with the infinitive, though this construction with *sein* is considerably older than with *werden* (cf. Grimm, Gr., 4, 125; for an example of *werden* with the present participle compare *die werdent got schende*). With both auxiliaries the infinitive soon replaced the participle; the present of *werden* joined to the infinitive was quite suitable to denote the present as initial point of the future (that is the inception of the action); for the transition was very near. We find this already prepared in the 13th and 14th centuries, when the periphrasis with *sollen* was gradually disappearing. The passive periphrasis with *werden* may properly be connected with this (cf. Grimm, Gr., 4, 182). The M.H.G. furnishes examples like ich waene ir *werdent* mirs *jehen*, daz ir iuch Koufes nit begant, Flore 3115: Der *wirt* iuch wol *enthalten*, ibid. 3580. In the 15th century the future signification of the present of *werden* became more decided, *e. g.*, so *wird* in Jhesus *wisen;* Wann er umb sich hin und her *sehen wuert;* Wann dann der sudenwind ungestuemlichen *ween wuerd* (spiret), so godt dann die gezierd in doernen hinweg (anfaengt zu wehen). Toward the end of the 15th century the vacillation between the participle and infinitive was greatest, but finally the infinitive carried the day (cf. sie *werdent* wisheyt *sehen* an und den lon, der drumb ist bereit und *werend wurt* jn ewikeyt).

If we now turn to the past tense, we shall find the same idea of inceptive action expressed by the past tense of *werden*, at first joined with the participle and then with the infinitive, *e. g.* do sie got an sehende *wurden* (cf. above, die *werdent* got *sehende*); Sie *wurden* lihte *spotten* min; Und da bi *wart* man sie *erkennen* (" this led to their recognition"): So wanne si dat kint an sach, so *wart* si *weinen;* again, gar haisz sy *wainen*

ward ("she began to weep bitterly"); even in the 16th century it retained this signification, *e. g.*, und als er schneiden solt und die scher nit enfand, *ward* er neber sich *sehen* und uff die bueni *klopffen* mit den henden ob er die scher nit hoerte; do er sich weret und *schrygen ward*, schluogen sie, etc. Vernalaken, I, 29 ff., has any number of examples all of which show that this verb then denoted the beginning of an action, as may be seen from those already given.

For a complete understanding of these periphrastic forms we must carefully consider the predication, or predicate, where it will be necessary to distinguish beginning, middle, end, duration and repetition of the action and for all three tenses. The inception or beginning of an action always has the accessory idea of *will*, of *intention*, and is frequently denoted in language by periphrasis (cf. Grimm, Gr., 4, 182), especially by *sollen*, *wollen* and *werden*. Compare for instance the Latin *hortaturus sum*, *eram*, *ero*, French *aller* with the infinitive, English *going* to denote the future, etc. Naturally these periphrases acquire certain other accessory ideas which are more or less independently developed in each language. Other periphrases of this nature will readily occur to all. Let us now examine more closely the exact shade of thought that these auxiliaries were and still are, to a certain degree, intended to express. For besides *will* and *intention* they also expressed *duty*, and language is constantly enriching itself with expressions that convey to the mind those various shades of thought relating to the future. What peculiar shade of meaning is there, then, in these auxiliaries that led to their use for expressing future time?

The Gothic, Isidor (8th cent.), Otfrid (9th cent.), and Notker (10th cent.) did not employ *werden* as future auxiliary. They in common use the present as future, or when they wish to distinguish the future they use the auxiliary *sollen* (Ulf. Mc. 8th, 31: jah dugann lais jan ins thatei skal sunus mans filu winnan; hva skuli thata barn wairthan? Seimon, skal thus hva qithan, ith is qath: laisari, qith. Otf. I, 23: Thu scalt beran einan, etc.; 17, 4: scal ih iz mit uuillen, etc.). The A.S. also uses the present as future, but when the addi-

tional idea of duty and intention is to be expressed it employs *sceal* and *wille* (cf. Fr. il *doit* arriver, er wird ankommen). Notice furthermore the Gothic sa auk habaida ina galewjan ains wisands thize twalibe (is eum proditurus erat, Jno., 6, 71; also 12, 26: tharuh sa andbahts meins wisan habaith).

We cannot, however, consider the absolute temporal relations of the present, the past, and the future without at the same time discussing those of inception, completion, duration and repetition of the action, though we are here only concerned with the inceptive action and the modal relations, to which we shall confine ourselves. In A.S. gann (began) has an inceptive force, *e. g.*, ic gann drinkan (cf. I am going to write, je vais boire, mod. Eng. begin, Lat. coepi, etc.). In German the future formed by *sollen* and *wollen* corresponds to this French je vais vous dire, j'allais partir and the English I was about, was going to do it, rather than to the Latin periphrastic future. In Gothic we find only one form with *sollen*, saei skulda qiman (Matt. 11, 14, qui venturus erat), which seems to correspond more to the periphrastic future of the Latin. Though *sollen* and *wollen* per se express modal relations, they, however, have an inceptive signification in such expressions as, Es will regnen, Die Wunde will heilen, Es sollte eben getanzt werden, als, etc., Der Tisch soll gerade gedeckt werden, just as much as the French and English forms cited above. A relative temporal relation to a preceding event is inceptively denoted by the Imperfect, *e. g.*, Die Wunde wollte eben heilen, als sie wieder aufgerissen wurde, Der Verbrecher sollte eben hingerichtet werden, als die Nachricht von seiner Begnadigung ankam. Before proceeding farther we shall have to discuss the modal signification of these auxiliaries in order to determine why and when they should be used and how their modal and inceptive signification led to their future use until differentiation settled modern use.

A real necessity, that is, a necessity imposed upon matter by nature, is expressed by *muessen* (vacare; cf. Musse, muessig). A moral necessity imposed, not by the will of the individual but by a general law, is not different from a real necessity, and is therefore likewise expressed by *muessen*,

e. g., Alle Menschen muessen sterben; Man muss der Obrigkeit gehorchen. A moral necessity is expressed by *sollen* and *wollen*. *Sollen* (debere), which in Gothic and O. German very frequently denotes the temporal relation of the future, has in Gothic the above mentioned signification of *muessen*, *e. g.*, Tharuh is qath du im thatei jah thaim antharaim baurgim wailamerjan ik skal bi thiudangardja guths, unte duthe mik insandida. Luc. 4. 43; unte thatei skuldedum taujan gatawidedum. Luc. 17. 10; Qathuth than jah gajukon im du thammei sinteino skulun bidjan. Luc. 18. 1; ik skal waurkjan waurstwa this sandjandins mik. Jno. 9, 4; jah bi thamma witoda unsaramma skal gaswiltan. Jno. 19, 7; in presnt use it denotes the moral necessity only in so far as it is imposed by the will of another, by command. Every *sollen* corresponds to a *wollen*, *e. g.*, Du sollst schweigen (Ich will es). Ich soll kommen (Er will es). *Wollen* denotes the moral necessity in as far as it is imposed by the will of the predicated agent. The action denoted as necessary is either the action of the predicated (willing [participle]) agent, *e. g.*, er will sprechen, or the action of another agent, *e. g.*, Er will, dass ich spreche. As the English uses these auxiliaries for the temporal relation of the future it must use other auxiliaries (I wish, I mean, I intend, etc.,) to express these relations.

These same three auxiliaries, *muessen*, *sollen*, *wollen*, likewise represent the logical necessity with various shades of meaning. Like *koennen*, *duerfen*, *moegen*, they are also employed in the present in the sense of a logical possibility, even when the declaration of the predicate is past; or in the imperfect when the declaration of the predicated agent is placed in the past, *e. g.*, Er will ihn gesehen haben, Er wollte ihn gesehen haben. *Muessen* denotes the pure logical necessity as one imposed by the judgment of the speaker himself, *e. g.*, Er muss krank sein. *Sollen* denotes the logical necessity imposed by the judgment of another and corresponds to the Latin *decor*, *e. g.*, Er soll krank sein. Ich soll das gesagt haben. *Wollen* denotes the logical necessity imposed by the judgment of the predicated agent, *e. g.*, Er will dich kennen, er will dich oft gesehen haben (asserts). Moreover it often

denotes that the judgment of the predicated agent and the logical necessity thereby imposed is only pretended. *e. g.*, Er will unschuldig sein (he pretends to be innocent), Er will kuenftige Dinge wissen (he pretends to know future things).

It is well known that language often represents the relation of necessity as a temporal relation, though more often it employs the temporal forms of the future. On the other hand the temporal relations of the future are often expressed by those auxiliaries which denote necessity, especially by *sollen* and *wollen*. It must be remembered that the fine distinction now observed in the use of *shall* and *will* did not exist in A.S. where the employment of these verbs corresponded more to that of the German verbs. In German the future is very often denoted by *wollen* and in restricted (not free) actions by *sollen*. *e. g.*, Ich will dich morgen besuchen; Ist es wahr, dass mir ein Tag zwei Soehne rauben soll? By means of *wollen* and *sollen* partly inceptive forms and partly temporal forms are created, the former of which alone concerns us here, as they form a strong basis for the future element of these verbs (cf. above).

Thus we see that however poor the German is in forms for future time, it is rich in expressions of *will*, *intention*, *duty*, and *necessity*. *Sollen* is least of all employed to denote mere future time in modern German, though it is on the other hand used imperatively, which always has reference to future action: ihr sollt kommen! ihr sollt mir willkommen sein! *Wollen* denotes rather the free decision and belongs preferably to the first person, *sollen* to the second, to which the imperative refers, while *werden* denotes the pure, abstract future and belongs to the third person.

To return, after this digression, to the two prevailing opinions in regard to the origin of the conditional forms in German we have: 1st, that of Grimm, that the present of the Conditional (kame) in the verbs of the new conjugation (redete) in German is not different from the imperfect Indicative and this caused the introduction of the future form (wurde) to supply the place of the failing subjunctive form; secondly, the inceptive preterite (ward weinen) is employed later as

conditional Subjunctive because it expresses just what the Conditional Mode should express. By Conditional Mode I do not mean *Conditional form*, for we have several forms to express the same idea, but *the peculiar mode of thought* which these various forms all more or less distinctively convey to the mind of the hearer or reader. What is the truth in regard to the matter? In other words, what do we mean by *Conditional Mode?* These are questions that cannot be answered until we consider the various modes of thought and their form of expression. Then we can resume our consideration of the origin of the two conditional forms *wurde lieben, wurde geliebt haben* and show the gradual differentiation in the meaning of *sollte, wollte* and *wurde*, giving the reason for present usage. Let us then briefly consider the different modes in their relations to, and differences from, one another.

The Indicative expresses the simple thought of the speaker which corresponds exactly to the reality, while the Conditional expresses a thought which corresponds to a reality only assumed for the time being by the speaker. The Subjunctive also expresses a thought, but only as considered by the speaker. Generally speaking, the Indicative and Conditional are the modal forms for judgments of the speaker, and hence the modal forms of independent clauses, whereas the Subjunctive is the modal form of thoughts which represent, not the judgment of the speaker, but a thought considered by him; it is therefore the modal form of dependent clauses.

Following the example of the Latin Grammar the German Grammar did not distinguish the Conditional as a special mode in earlier times, but included it, inasmuch as it is formed from the past tense-forms, among the tense-forms of the past Subjunctive. If we examine the signification of these forms, we must distinguish the Conditional, if we have any decided idea of modal relations at all, as that form of a conceptive thought which assumes something contradictory (antithetical) in its very nature, from the Subjunctive, as that form of thought which is only contemplated. Several older Latin and German grammarians, following the example of the Greeks, have really distinguished the Conditional as a special

mode from the Subjunctive. The simple forms of the Conditional, *e. g.*, spraeche. haette gesprochen. are formed, like the forms of the Subjunctive, from the tense-forms of the Indicative; but they by no means correspond in meaning to these latter forms: in time spraeche does not correspond to sprach, but to the present spreche; and haette gesprochen not to the pluperfect hatte gesprochen, but to the perfect habe gesprochen. This is also the case with the Latin conditional forms, *e. g.*, *dicerem, amarem. dixissem, amarissem,* yet with the difference that *dicerem, amarem,* etc., are not formed from the preterite of the Indicative. The modal relation of the Conditional—the assumed reality of a predicate which is not real in itself for the speaker—is a special relation decidedly different from the modal relation of the Subjunctive—a relation merely contemplated by the speaker, and therefore for him only a logically possible thought; in all languages, therefore, it is distinguished in form from the relations of the Subjunctive. All languages have not, indeed. like the Indic. Greek and German languages. special flexional forms for this modal relation. The Slavic, as well as the Semitic languages, have no special form for the Conditional, just as they have none for the Subjunctive: they none the less express both modal relations and clearly distinguish them by denoting the relation of the Subjunctive by the tense-forms of the future, but the Conditional by the tense-forms of the preterite. Thus the logical possibility (Subjunctive) is represented as something which can become real, and the reality which is only assumed (Conditional) as something that has been and therefore is not real. The Conditional is often represented both in Greek and German in the same way, *e. g.*, εἴ τι εἶχεν, ἐδίδου ἄν, wenn er etwas haette, so wuerde er es geben; Maria Stuart war noch heute frei. wenn ich es nicht hindert; Wenn dieser starke Arm Euch nicht hereingefuehrt, Ihr sahet nie den Rauch von einem fraenkischen Kamin steigen. It is especially the negatived reality which language represents by using a preterite tense instead of the Conditional; the latter (negatived reality) is often denoted by a mere preterite, *e. g.*, Jene hat gelebt. wenn ich dies Blatt aus meinen Haenden gebe. Notice the following in Greek where the particle ἄν

marks an action as contingent on an unfulfilled supposition, and therefore contrary to fact (hypothetical Indicative; Hadley Sec. 895): ἔδωκεν ἄν, εἴ τι εἶχεν, "he would have given, if he had had anything". Sometimes in the conclusion the aorist refers to the present and is used of the inception or bringing about of the action, *e.g.*, εἰ ἐγώ σε ἐτύγχανον ἀνερωτῶν, τί ἄν μοι ἀπεκρίνω, "if I happened to be asking *you, what would you* (proceed to) *answer?*"

On the other hand every thought of cognition — a judgment or a question — is either a logically real or a logically possible thought. We call it logically real when it is an intuitive thought of the speaker; we call it logically possible when it is a thought intuitively conceived by the speaker and is assumed as a grammatical member in an intuitive conception. The intuitive thought of the speaker is expressed in an independent clause, *e.g.*, Er hat immer Wort gehalten. From those intuitive thoughts which are really in the relation of logical reality we must distinguish those intuitive thoughts in which the predicate stands in an antithesis to the reality merely assumed by the speaker, *e.g.*, Jedermann wuerde ihm vertrauen, wenn er immer Wort gehalten haette. The intuitive thought of the speaker is expressed, when it really stands in the relation of logical reality, by the Indicative; when the predicate of the thought stands in an antithesis to the reality assumed by the speaker, it is then it is expressed by the Conditional. The one common point which the Conditional has with the Indicative is that it expresses a real judgment — an intuitive thought — of the speaker, and hence, like the Indicative, stands in independent clauses; it is distinguished from the Indicative by always representing the predicate in the relation of a reality, though only assumed by the speaker. The sentence Waere er ein guter Haushaelter gewesen, so waere er der reichste Mann in der Stadt expresses, just as Wenn er ein guter Haushaelter ist, wird er ein reicher Mann, or Er ist der reichste Mann im Lande, a real judgment of the speaker, though the predicate is not, as in the last sentence, in the relation of unconditional reality, but the reality of the predicate is conditioned, and, indeed, so conditioned

that it is not thought of, as in the second sentence, in the relation of possibility, but with condition at the same time in the relation of negative reality; the reality of the conditioned conception, like that of the condition, is only assumed by the speaker. Since the condition expressed in the dependent clause likewise stands in the relation of a negative reality. it is also expressed by the Conditional. We must take that signification which the Conditional has in the independent clause as the real fundamental signification of this modal form. For, though the Conditional is used in dependent clauses, aside from the case of a merely assumed condition just mentioned, it assumes the signification of a Subjunctive by representing the relation of a logically possible thought in the form of an assumed reality. We must, however, distinguish this derivative signification of the Conditional from its fundamental signification. The Conditional stands much nearer the Indicative in its fundamental signification than it does to the Subjunctive, as is seen from the fact that it frequently exchanges with the preterite of the Indicative in the ancient and modern languages without varying the signification in the least, *e. g.*, εἴ τι εἶχον, ἐδίδων ἄν. Moreover in many other relations, and especially in hypothetical sentences, the Conditional is also just like the Indicative. In hypothetical sentences, where the relation of a necessary condition is to be expressed, either with the Conditional or the Indicative, the Greek uses εἰ and not ἐάν as with the Subjunctive, *e. g.*, Εἰ θελοίμεν σκοπεῖν τὰς φύσεις τὰς τῶν ἀνθρώπων, εὑρήσομεν. Cf. with this the German: Haett' er mein Auge, oder stuend' ich oben, das Kleinste nicht entginge meinem Blick; O waerst Du wahr gewesen und gerade, nie kam es dahin, Alles stuende anders.

The meaning of the Conditional explains why its forms are derived from the forms of the preterite and why languages which lack the special modal forms of the Conditional use a preterite of the Indicative instead. If we must assume that language did not originally positively distinguish the modal relations from the relations of time and represented the reality as present in time and the other modal relations as relations of time not present, it is then natural that language

should represent the relation of a negatived reality by that relation of time which denied most decidedly the reality in time (present): viz., by the past. Grammar especially should consider speech-forms, not according to the relation of the outer form, but according to their signification; it should therefore present the forms of the Conditional, not as the Latin Grammar, and following this usage the German Grammar earlier, as tense-forms of the Subjunctive, but as special modal forms. In the dependent clause the use of the Conditional is like a preterite of the independent clause, but it has here, as above stated, lost its fundamental meaning. The fundamental idea of the Conditional excludes time altogether, or at least distinguishes time less than any other mode. The same tense-forms of the Conditional, though derived from a preterite tense-form, denote, with more freedom than the present of the Indicative, present and future time, *e. g.*, Waere er (jetzt) krank, so arbeitete er nicht (or, so wuerde er nicht arbeiten); Wenn er morgen kaeme, waere er mehr willkommen sein, als heute (or wuerde er mehr willkommen sein); Utinam cras veniret! It is remarkable in this respect that the O.H.G. had only one form of the Conditional to express not only the present and future but also the past, *e. g.*, Ni wari tho thiu giburt, tho wurti worolt firwurt; sia satanas ginami, ob er tho ni quami; Oba er (Adam) iz firliazi, odo iz got biliazi, ouh worolt ni gigiangi in thesa goringi (Otfrid). Even in dependent clauses where the Conditional assumes the signification of the Subjunctive, the present also expresses the past, *e. g.*, Ouh n' ist, ther er gihorti so fronisg arunti; n' ist wib, thaz io gigiangi in merun goringi; Thaz thu unsih nu gidua wis, oba thu gotes sun sis, zi kriste er thih ginanti, ioh hera in worolt santi (Otfrid). The periphrastic Conditional (ich wuerde sprechen) cannot therefore be found in O.H.G. because in O.H.G. the future of the Indicative is not at this time formed by *werden*. Thus in dependent clauses where the Conditional has the signification of the Subjunctive and where we in general express the future by a future form we always find the present in O.H.G., *e.g.*, Ther gotes geist, ther mo ana was, ther gihiaz imo thaz, thaz krist er

druagi in henti er sines dages enti, er todes io ni koreti, er er den drost habeti (Otfrid).

In Mod.H.G., however, besides the present ich spraeche, and the preterite ich haette gesprochen, the Conditional also has a future ich wuerde sprechen and a future perfect ich wuerde gesprochen haben, as we now call these forms. But these futures, if they are used in the real signification of the Conditional, do not have the temporal (tense, time) signification corresponding to the form of the simple future; for the future is not distinguished from the present, and the future perfect from the preterite, *e. g.*, Waere er abgereiset, so wuesste ich es, or, so wuerde ich es wissen: waere er eingeladen worden, so waere er gekommen, or, so wuerde er gekommen sein. Only when the futures stand in a dependent clause and have the signification of the Subjunctive do they denote the temporal relation corresponding to their form, *e. g.*, Ich hoffte, er wuerde kommen; Er sagte, er wuerde kommen, instead of er werde kommen; Er glaubte, er wuerde die Arbeit schon vollendet haben, ehe du abreisest. The use of the future perfect, as in the Indicative, is here also very limited. In the real signification of the Conditional the futures are used only in the independent clause, *e. g.*, Waere er nicht krank, so wuerde ich ihn besuchen; haette er den Brief geschrieben, so wuerde ich ihn erhalten haben. The Romance and English languages, which in other respects express the relation of the assumed reality by the preterite of the Indicative, generally use in the conditional sentences a form of the future, *e. g.*, s' il avait faim, il mangerait; s' il le savait, il vous l' aurait dit; If I were thisty, I should drink.

The use of the forms of the future here (in the cases we have just explained) has its justification in the special relation of the conditional sentence as a causal relation. For in truth the conditioning thought is conceived as a possible reason and the conditional thought as that caused by the reason; thus here the condition is represented as something following in time the conditioning thought and as something future in reference to the conditioning thought. We can furthermore add to this also that the relation of possibility expressed in

the conditional sentence is very frequently represented by the temporal relation of the future. This relation of reason is really at the same time a temporal relation, if it is a real one. We therefore make a special use of these forms of the future, whenever the possible reason (the condition) is to be represented as a real one (not as a logical one). Thus we say, *e.g.*, Wenn er Wein getrunken haette, wuerde er berauscht sein; wenn es warm waere, wuerde das Eis schmelzen, but never Wenn er berauscht waere, wuerde er Wein getrunken haben; Wenn das Eis schmoelze, wuerde es warm sein. The real signification of the Conditional is the fact that it serves, like every abrogative antithesis, to emphasize the thought presented. (Cf. Dr. K. F. Becker, Ausf. d. Gramm., II, p. 58 ff., from whom most of this discussion is taken.)

If we now compare the various significations of these modes with those which obtained in the older languages, we shall see that the germ of even the nicer shades of distinction existed at that early day. The future forms also existed and are described by Whitney, Skr. Gramm. Sec. 940 as follows: "From the future-stem is made an augment-preterit, by prefixing the augment and adding the secondary endings, in precisely the same manner as an imperfect from a present-stem in *a*. This preterite is called the conditional.

"It stands related to the future, in form and meaning, as the French conditional *aurais* to the future *aurai*, or as the English *would have* to *will have*—nearly as the German *wuerde haben* to *werde haben*."

It is, however, the rarest of all forms of the Sanskrit verb, the R.V. containing but one example, *abharisyat*, "was going to carry off," and the later literature not many more. Whitney defines the Conditional thus: "The conditional would seem to be most originally and properly used to signify that something 'was going to' be done. And this value it has in its only Vedic occurrence, and occasionally elsewhere. But usually it has the sense ordinarily called 'conditional'; and in the great majority of its occurrences it is found (like the subjunctive and optative, when used in the same value) in both clauses of a conditional sentence. Thus, yó vṛtráya sínam

átrá 'bharişyat prá tám jánitri vidúşa uvaca (R.V.), 'him, who was going here to carry off Vritra's wealth, his mother proclaimed to the knowing one'; catayum gam akarişyam (A.B.), 'I was going to make (should have made) the cow live a hundred years.'" (Sec. 950.)

Of the other three modes, Subjunctive, Optative and Imperative, but little need be said. The Subjunctive proper early became nearly extinct. "Its fundamental meaning," says Whitney, Sec. 574, "is perhaps that of requisition, less peremptory than the Imperative, more so than the Optative." The Optative soon replaced it. Command, requisition, wish, with no sharp line of division between, would characterize the shades of thought expressed by these three modes. Whitney says they may all be specialized uses of forms originally equivalent—having, for instance, a general future meaning. "This, however, (in Skr. as in other languages) is by no means always of the same force; the command shades off into a demand, an exhortation, an entreaty, an expression of an earnest desire. The Imperative also sometimes signifies an assumption or concession; and occasionally by pregnant construction, it becomes the expression of something conditional or contingent; but it does not acquire any regular use in dependent clause-making." (Sec. 572.)

As our desires naturally find utterance in the form of a request or an entreaty, the Optative often becomes a mild Imperative. It can also express what is generally desirable or proper, or what should or ought to be and thus become the mode of prescription. It may also weakly signify what may or can be, what is likely or usual, and thus become a softened statement of what actually is. "In dependent clauses, with relative pronouns and conjunctions, it becomes a regular means of expression of the conditional and contingent, in a wide and increasing variety of uses. Thus, in A.V., we have in imp.: catám jiva carádah, 'do thou live a hundred autumns'; ubhaú taú jivatam jarádasti, 'let them both live to attain old age';—in subj., adyá jivani, 'let me live this day'; catam jivati carádah, 'he shall live a hundred autumns';—in opt., jívema carádam catáni, 'may we live hundreds of autumns';

sárvam áyur jivyasam (pre.), 'I would fain live out my whole term of life'. Here the modes would be interchangeable with a hardly perceptible change of meaning." (Sec. 573 ff.) In many of its uses the Optative takes the place of our Conditional and it has one other characteristic like it, viz., its use in conditional clauses contrary to fact.

If we consider the use of old prose in Greek and compare the usage of the Greek aorist, we cannot doubt that the usage of the Indic aorist extends back to a time of the inceptive action. Bartholomae asserts that the Iranian aorist does not essentially differ from the Greek aorist, only that the Iranian still denotes by the imperfect one part of what the Greeks express by the aorist. It is therefore probable that the Greek aorist in the main corresponds to the Indo-Germanic aorist, though varying here and there in its final development. But the Indic aorist expresses the inceptive action in nearly every case and this use is found in Greek oftener than one would suppose from the statements of the grammarians. Naturally the "now" does not inhere in the aorist, but is expressed by a particle like *νῦν*, *e. g.*,

> *Ζεύς με μέγα Κρονίδης ἄτῃ ἐνέδησε βαρείῃ,*
> *σχέτλιος, ὃς πρὶν μέν μοι ὑπέσχετο καὶ κατένευσεν*
> *Ἴλιον ἐκπέρσαντ᾽ εὐτείχεον ἀπονέεσθαι,*
> *νῦν δὲ κακὴν ἀπάτην βουλεύσατο.—B 114.*
>
> *μή με, γύναι, χαλεποῖσιν ὀνείδεσι θυμὸν ἔνιπτε.*
> *νῦν μὲν γὰρ Μενέλαος ἐνίκησεν σὺν Ἀθήνῃ,*
> *κεῖνον δ᾽ αὖτις ἐγώ.—Γ 439.*

"In the conclusion, the aorist sometimes refers to *present* time, being used of the inception or bringing to pass of the action, *e. g.*, *εἰ ἐγώ σε ἐτύγχανον ἀνερωτῶν, τί ἄν μοι ἀπεκρίνω*, "if I happened to be asking you, what would you (proceed to) answer?" Had. Sec. 895. In Indo-Germanic, then, we may conclude that the aorist-stem marks the inceptive action, which the Indicative transfers to the past; the distance of the time from the speaker was not indicated. Cf. Delbruck, Griech. Synt., p. 107. The aorists are frequently translated by the present in colloquial style, *e. g.*, *νῦν δέ σευ ὠνοσάμην*

πάγχυ φρένας (*P. 173*); *ἥσθην, ἐπῄνεσα, ἐγέλασα*, etc., are so translated.

The Subjunctive and Optative aorists are modes of the inceptive action, nothing else; they do not there contain a single sign of temporal gradation, nor do they have either in Sanskrit or Zend a temporal sense, and just as little in the independent clauses of the Greek. In the developed sentence of the Greek, however, their surroundings are such that the sense of the past time enters, or seems to enter into them, which is specially true in the following cases. In prior relative and conjunctive sentences the Subjunctive aorist seems to have the sense of past time, *e. g.*, *ὃς μέν κε βάλῃ τρήρωνα πέλειαν πάντας ἀειράμενος πελέκεας οἶκόνδε φερέσθω. Ψ 855.* All the sentence means is "whoever shall hit the dove". The hearer will understand the hitting precedes the prize. Pedantically we might translate "whoever shall have hit", though the future perfect does not inhere in the aorist, but is transferred there by the hearer from the context of the aorist. In such surroundings the aorist is nearly always chosen because the inception of the action is emphatic, not its course. The Skr. does not distinguish action so accurately as the Greek and uses the present in similar cases, and this was very probably done in Greek also in colloquial style. When the Optative is in the dependent question, it does contain the sense of the past, *e. g.*, *ἐπειρώτα, τίνα δεύτερον μετ' ἐκεῖνον ἴδοι*; meaning "whomever he might have found." However this sense does not belong to *ἴδοι* as such, but only in so far as it is the representative of an *εἶδες*. From the sentence *εἴ τινα εἶδες* we have by means of the transmutation of persons and modes *εἴ τινα ἴδοι* and in the transmutation the temporal sense of the original *εἶδες* is transferred to *ἴδοι*. This transmutation finds moreover no analogy in the Indic group, but is a special development of the Greek. (Cf. Delbruck, Griech. Synt., p. 107 ff.)

The results of our investigation so far are then that the Conditional is the form of a conceptive thought which assumes something contradictory (or antithetical) in its very nature. Its modal relation—the assumed reality of the predicate, which is not real in itself for the speaker—is a special

relation essentially different from that of the Subjunctive proper. It represents the assumed reality as something that has been and therefore not real, that is, a logically possible thought intuitively conceived by the speaker and forming a grammatical member in an intuitive conception. Its real significance is that it serves, like every abrogative antithesis, to emphasize the thought presented, especially when the Indicative form is used. Its most original signification would seem to be "was going to", or "is going to" be done. We have furthermore seen that the Indo-Germanic aorist expresses the inceptive action in nearly every case and that this use is also found in Greek. "In the conclusion the aorist sometimes refers to present time, being used of the inception, or bringing to pass of the action." The inceptive action leads gradually to the future idea, and, if Whitney is right in his conjecture that the Imperative, Subjunctive and Optative "may all be uses of forms originally equivalent—having, for instance, a general future meaning"; if, moreover, our explanation of the use of the future forms in the special relation of the conditional sentence as a causal relation (see above, p. 68,) is true, then it is easy to understand the use of the different forms, both simple and periphrastic, to express the modal relation of the Conditional. For the inception of the action always has the accessory idea of *will*, of *intention*, and is denoted by periphrasis, especially by *sollen*, *wollen*, *werden*. Cf. the Latin *hortaturus sum*, *eram*, *ero*, etc. In order to express this modal and inceptive shade of thought (just given in summary) which we have concluded to call "conditional" par excellence, the German language has used various forms, simple and periphrastic. In O.H.G., as we have seen, the preterite form (the so-called imperfect Subjunctive) was always used. Later in the development of the language, as the endings wore away and the different forms were no longer distinguishable, *sollen*, *wollen*, and *werden* were used. Let us turn to these auxiliaries and consider the appropriateness of each for expressing this conditional thought. Their use to express future time has already been stated. To this we can add that *werden* is surer, the speaker accepts the news as his own, *e. g.*, Der liebe Gott *wird* uns helfen; er theilte ihr mit.

er wolle Unterkommen suchen, sie alsdann abholen, und er hoffe, sie *werde* ihm ihre Hand nicht versagen. The same confidence lies in *sollen*, *e. g.*, Sie riefen hoehnisch aus: die sollen mir nicht entwischen; more emphatically, Der Brief *soll* Morgenabend mit der reitenden Post abgehen; Es *soll* das naechste Mal geschehen, etc. *Sollen* is besides used to express the conditional idea, *e. g.*, Sollte ich mich irren? Wenn er kommen sollte. However, this use of the modal auxiliaries to denote the modal relation of the predication in German is far less frequent than in languages which have no complete modal flexions, as the English, which always expresses the Conditional in independent clauses by auxiliaries, *e. g.*, "I should have sent you the book, if I had known that you would be at leisure to read it".

In an earlier stage of the German language *sein* with the present participle paraphrases the incomplete tense of the active and *werden* was also soon used in the same manner. The former combination denotes the duration of the action, while the latter mostly denotes the inception of the action. Soon, however, the present participle became the infinitive both with *sein* and *werden*. In Mod.H.G. only a few examples are left with *sein*, but it is the only construction with *werden*. This inceptive force of *werden*, that is, the *Werden des Zustandes*, was quite common in Mid.H.G., *e. g.*, Wan si noch hinaht swanger wirt und einen sun wirt tragende. etc. The future idea comes very near this, so that the employment of *werden* with the present participle to express future time is in Mid.H.G. the usual form, though *sollen* and *wollen* were also used. It is the continuation of the inceptive force of this combination and was replaced by *werden* and the infinitive when the infinitive crowded out the present participle. As we have seen above the preterite of *werden* and the infinitive (earlier present participle) retained still later than the present its inceptive force. In the 13th cent. it is common, *e. g.*, daz wart in tragen sit; daz Morice na der art wahsen an dem kinde wart; der kunic wart si vaste klagen, etc.

We have now arrived at that point of our investigation where we can roughly state the cause of the differentiation of

the different auxiliaries. Naturally certain accessory ideas join themselves to the periphrases which are developed in every language for increasing the number of ways of expressing the different shades of thought. In *sollen* we have the accessory idea of *necessity*, of *must;* in *wollen*, the *inclination*, the *determination* of the agent. Cf. the English *shall*, *will*. In *werden* we have at first the idea of the inception of the action, later the pure future idea. This differentiation had already begun when the language, on account of the loss of the endings, was seeking a modal form to replace the fast disappearing conditional form, especially in the weak verbs (redete) where Indicative and Conditional were indistinguishable. *Sollte* and *wollte*, both of which had a general future meaning, perhaps a common quality of the Imperative, Subjunctive, Optative and Conditional and logically based on reason, as we have seen above, were often used to supply this modal form of thought. In the strong verbs. and in other verbs also where the form was distinguishable, the real conditional form was retained. *Sollte* and *wollte* soon fell into the same category as *redete*, which could not be distinguished from the Indicative as a conditional form; but the peculiar appropriateness of *werden* to express the true modal relation of a Conditional was recognized and replaced nearly all other periphrastic forms. *Sollen* is still used, as we have already stated, though with a slightly different shade of meaning.

Present usage presents two phases of the Conditional (kaeme, gaebe, wuerde kommen, wuerde geben and their compounds); it is used in the independent and in the dependent clause. It stands in the purely hypothetical period (protasis and apodosis) and both forms (simple and periphrastic) may be used and are often interchangeable, though the simple form is much more frequent than the periphrastic. But it must be remembered that these two forms are not absolutely identical in sense. The periphrastic form may sometimes be preferred where the idea of futurity is prominent, *e. g.*, lebtest du noch, so wuerde ich dich von dieser Zeit an lieben, "wert thou still alive, I should love thee henceforth." Again for formal reasons where the simple form would not be distin-

guishable from the indicative form (as in redete) the periphrastic form is preferred.

In the dependent clause present usage is unsettled: the real Subjunctive is often used where the Conditional (both forms) ought to be used. But whenever the reference to the future is plain, then the periphrastic conditional form is preferred, *e. g.*, er wusste, dass diese Anerbietungen den Kreuzzug nicht aufhalten wuerden (for werden, which would not be distinguishable from an Indicative). "he knew that these offers would not detain the crusade."

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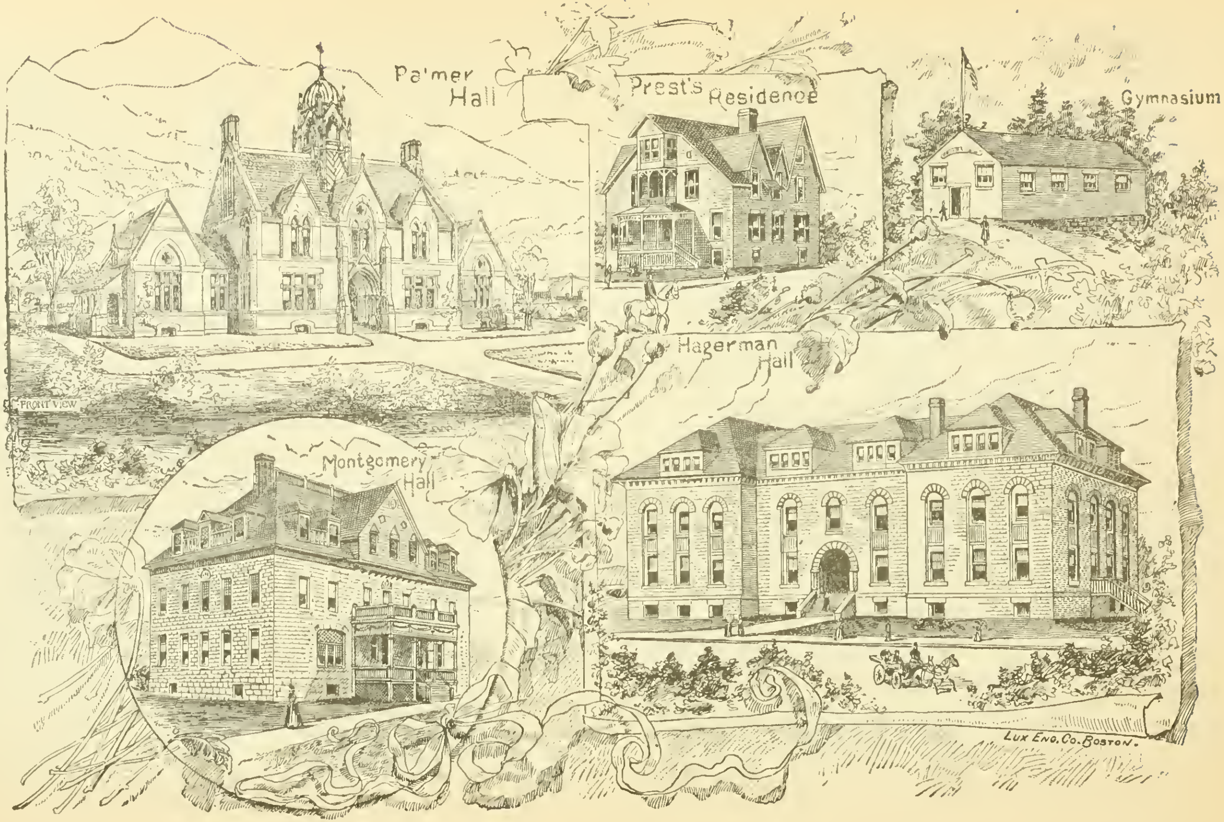
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ANNOUNCEMENT.

The following is a complete list of the papers read at the monthly meetings of the *Colorado College Scientific Society* during the past year. Three of the papers are printed in full in this pamphlet, while others will be published elsewhere.

October 18, 1892:

Folk Etymology in Latin, - - - W. P. Mustard.

November 18, 1892:

A Construction for the Imaginary Points and Branches of Plane Curves, - - F. H. Loud.

December 16, 1892:

State Bank Notes, - - - - - W. M. Hall.

January 27, 1893:

Friction Tests in Water-pipes and Fire-hose, - - - - - - - W. Strieby.

February 24, 1893:

Prayer in a Universe of Law, - - - E. S. Parsons.

March 24, 1893:

Acidimetry, - - - - - - D. J. Carnegie.

On the Eight Lines Usually Prefixed to Horat. Serm. I. 10, - - - - W. P. Mustard.

April 28, 1893:

Kant's Theory of Space and Time, - - Marion McG. Noyes.

On the Multiplication of Semi-convergent Series, - - - - - - Florian Cajori.

May 19, 1893:

The Essential Element of Religion, - - F. R. Hastings.

Multipolar Dynamos, - - - - Florian Cajori.

The Circular Locus, - - - - F. H. Loud.

THE CIRCULAR LOCUS

Geometrically Constructed to Show Imaginary Values of the Variables.

By FRANK H. LOUD.

In the paper on "A Geometrical Construction for the Imaginary Points and Branches of Plane Curves," which I read before this society on November 18, 1892, the general method of construction therein outlined was illustrated by the simple example of the equation $X^2+Y^2=r^2$; and it was shown by a diagram how the full significance of this equation, for imaginary as well as real values of the variables, may be geometrically interpreted by the aid of additional curves, accompanying the circle described around the origin. A more full and methodical treatment of the theory of this plan of construction, in its general application, has since been contributed to the *Annals of Mathematics*, but at the date of the present reading has not appeared in print. It is the object of the present paper to continue somewhat further the treatment of the circular locus just mentioned, as a special case of the general theory, borrowing, from the two papers above named, as much as may be necessary to render this intelligible to a reader who has seen neither of the others.

The principle of construction employed is of course based upon the well-known geometric interpretation of imaginaries, in which $a+i\alpha$ is represented by a line drawn from the origin to a point a units to the right of a vertical axis and α units above a horizontal axis, called the imaginary and the real axis respectively.

The apparatus postulated in this usual representation of imaginaries—a pair of rectangular axes in a plane—coincides precisely with that of the still more familiar Cartesian method of translating algebraic equations into

geometric forms, upon which is reared the edifice of analytical geometry. In the customary treatment of the latter, while imaginary values of the variables appear frequently in the algebraic work, and are recognized as of high importance to the theory of curves (witness the "circular points at infinity"), they are excluded from geometric interpretation. This is not necessary. Imaginaries may be admitted, with "real" quantities, into the constructions as well as the arguments of analytical geometry, if we are content to lay aside that habit of associating each axis with a single variable, which has resulted in the names "axis of X" and "axis of Y" respectively. Let us regard these lines rather as an axis of reals and an axis of imaginaries, and then express the usual Cartesian method of locating a point by its coördinates, by saying that the point (x, y) means the point at the extremity of the vector $x+iy$. When x and y are real, this will locate the point just as Descartes does, but if either coördinate be imaginary, the point has still a definite position in the plane. Thus if $x=5-2i$ and $y=4+3i$ we find $x+iy=5-2i+4i-3=2+2i$, and the point is situated 2 units to the right of one axis, and 2 above the other, just as if its coördinates were 2, 2.

Thus, when imaginaries are admitted, any point of a plane serves for the geometric representation of an infinite number of sets of coördinates, all distinct from one another, and we can no longer infer from the position alone of a point what are its coördinates. If we are informed by some other means what is the imaginary part of each coördinate, then this knowledge, combined with that of the position of the point, suffices to determine the real part, and hence the coördinates in full. Thus in the instance given, if we know that $x+iy=2+2i$, and that the imaginary part of x is $-2i$, we find by subtraction that the imaginary part of iy is $4i$, hence that the real part of y is 4. In like manner may be found the real part of x.

An equation between the variables, such as $x^2+y^2=r^2$, serves to restrict the number of sets of coördinates belonging to any one point; never depriving any point, however,

of its capacity of representing at least one such set. To render visible this effect of the equation, it is necessary to indicate, for points in different parts of the plane, by what coördinates they may, consistently with the equation, be represented; and this is effected, as already mentioned, by indicating the imaginary part of each coördinate. If a line is drawn through all these points of the plane for which the imaginary part of X has a certain constant value, say $1i$, another through all those points for which its value is $2i$, etc., then this system of lines will indicate to the eye the value of the imaginary part of X for all points of the plane. Another series of lines will indicate in the same way the manner in which the imaginary part of Y varies over the plane; and when these two things are known for any point, the real parts of the coördinates become also known, and the geometric significance of the equation is fully set forth. It is true that unless a point be situated exactly on one of the lines of each series, its coördinates are only approximately indicated; but, as will be seen, it will always be possible to draw a line of each set through any assigned point of the plane, and ascertain the value of the coefficient of i which remains constant upon that line: thus the determination of the coördinates may be made precise.

To these lines I have ventured to give the name of *comitants*, designating the two series as x-comitants and y-comitants respectively, and affixing to either name the number or symbol denoting the constant value of the coefficient of i in the expression for the coördinate of any point upon it. Thus the x-comitant μ is defined to mean a line (or curve) drawn through every point for which the imaginary part of the value of X has the value $i\mu$.

I will also adopt in the remainder of this paper, the rule of denoting by small capitals A, B, etc., quantities which are entirely unrestricted in value, and hence are, in general, complex; while for their real parts I use italic letters, and Greek letters for the coefficients of the imaginary

unit. Thus $\text{A}=a+i\alpha$, $\text{B}=b+i\beta$, etc. The Greek letter answering to C is taken to be γ, and that answering to Y to be η.

To return to the comitants, it is plain that one of each series is of chief importance, namely, the comitant-zero. This may be compared to the equator on a map, while the comitant $+1$ and the comitant -1 are on opposite sides like parallels of latitude. The x-comitant 0 passes through all the points for which X is real, and the y-comitant 0 through all those for which Y is real, so that all "real points" of the locus—that is, those having both coördinates real—must lie on both these lines. For all real curves, therefore, the comitants zero will have a common arc, and this arc will be—or, at least, will include—the curve itself, as known to the ordinary constructions of analytical geometry. Thus for the equation of our present example, $\text{X}^2+\text{Y}^2=r^2$, the x-comitant 0 consists of the circle of radius r described about the origin, and in addition of a line coinciding with the real axis. The y-comitant 0 consists of the same circle with the addition of the axis of imaginaries.* The other comitants, of either series, are cubic curves situated on opposite sides of the axis named, and having oval branches within the circle, as will shortly be seen.

A rule for constructing the comitants by points may be derived by inspection of the equation. For the x-comitants, we may put the latter in the form $-\text{Y}^2=\text{X}^2-r^2$, when we have, by extraction of the square root, and the addition of X to each member,

$$\text{X}+i\text{Y}=\text{X}\pm\sqrt{(\text{X}+r)\ (\text{X}-r)}.$$

The *mean proportional* between $\text{X}+r$ and $\text{X}-r$, which, as the radical indicates, is to be found, is to be understood of course not merely as having the length implied, in the Euclidean use of the word, but as bisecting the angle at

*Beside having the circle in common, these two comitants zero intersect also at the origin, for this point of the locus may be regarded as having *either* of its co-ordinates real, but since the latter cannot both be real at once, the origin is not included in the real part of the locus.

the origin formed by the two vectors x+r and x−r. Similarly, the principle of vector-addition is to be borne in mind in uniting the result with the term x. The ordinary geometric constructions of a bisectrix and a mean proportional may be combined with a fair degree of convenience as follows: Let the two axes, OJ and OI,

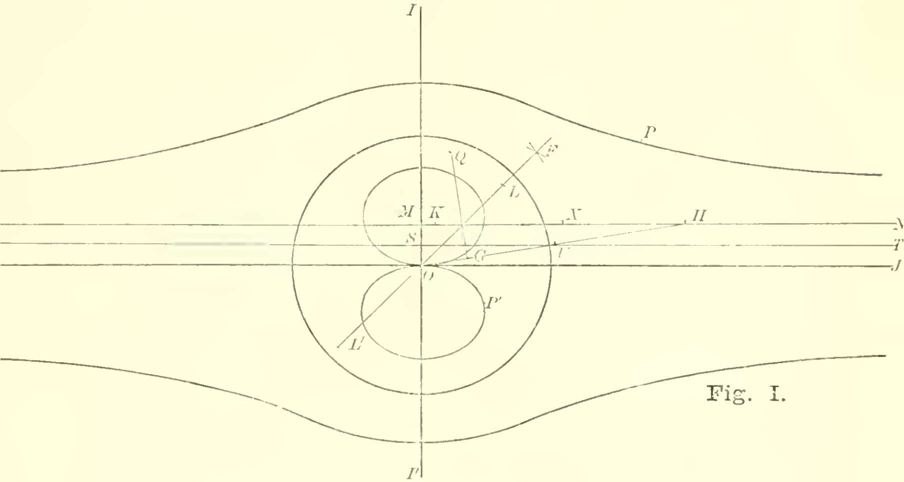

Fig. I.

and the circle of radius r about the origin be first drawn, in ink, or so as to remain permanently through the construction. If the x-comitant μ is to be drawn, where μ is a given quantity, let a point M be taken on the axis of imaginaries, at a distance μ above the origin; also a point S midway between O and M; and let lines MN and ST be drawn parallel to the real axis to an indefinite length. These two lines will remain during the construction of a single comitant, but all that follow must be erased and drawn anew for each point constructed. On MN take any point X, and on the same line two points, H and K, at distances equal to r, on opposite sides of X. Of these, let H be on the side of X opposite M. Draw OH. With center O and radius OK describe an arc intersecting OH at G, and at G erect a perpendicular to OH. From center U (the point where OH meets ST) with radius UH, describe an arc cutting this perpendicular at Q. From K and G, with equal

radii of any convenient length, describe arcs meeting at F; draw OF, and on it take OL and OL′ in opposite directions, each equal to OQ.* On the side of LOL′ toward X, describe arcs from centers L and L′, with radius OX; and intersect each of them by an arc drawn from center X with radius OL. Then P and P′, the points of intersection, will be points on the comitant curve, one on the infinite branch, the other on the oval. Two more points on the same comitant are situated symmetrically to these, on the opposite side of the imaginary axis. Also, the x-comitant $-\mu$ is equal to the x-comitant μ, and the real axis is an axis of symmetry to the two curves. Further, if either of them be rotated through a right angle, a y-comitant is produced; but it must be noted that while, for all positive values of μ, the infinite branch of the x-comitant μ is situated above the real axis, that of the y-comitant μ is to the left of the axis of imaginaries. Thus a single construction virtually determines sixteen points—four on each of two x-comitants and as many more on two y-comitants.

Each comitant, excepting the comitants zero, proves when drawn to consist of a conchoidal and an oval branch, the former approaching at infinity a line drawn parallel to an axis (the real axis in the case of x-comitants) and at a distance therefrom equal to double the coefficient (μ) of i, which distinguishes the particular comitant; while the oval lies on the opposite side of this axis, touching the latter at the origin. The breadth of each branch, in a direction perpendicular to the above-named axis, is the same, and is easily found to be $\sqrt{r^2+\mu^2}-\mu$; while $\sqrt{r^2+\mu^2}+\mu$ is the greatest ordinate of the curve, belonging to the point where the conchoidal branch cuts the other axis, which divides it symmetrically. Regarding together all the comitants of one series, it is apparent that all have a common point at

*Another method of finding a geometric mean between two lines which extend from a common point is as follows: First, bisect the angle of the lines and also its adjacent supplementary angle. On the latter bisectrix lay off half the longer line, and from its extremity, in both directions, half the shorter, thus obtaining the half sum and half difference of the lines. From the extremity of the half-difference, with radius equal to the half sum, cut off on the bisectrix of the angle a segment, which will be the required mean proportional.

infinity, also all the ovals are mutually tangent at the origin. Beginning with a very small value of μ, the corresponding comitant of course lies near the comitant zero, which consists of a circle and a line; and it is seen that the conchoidal part of the comitant answers to one semicircumference and to that part of the line which is outside the circle, and lies just outside these parts of the comitant zero. The oval, on the other hand, answers to the other half the circumference and to its diameter, and lies inside the semicircle close against the boundary. Accordingly, its curvature is rapid on the side remote from the origin, while near the latter it is almost straight. The succeeding comitants have their conchoidal parts successively further and further from the axis, each encompassing its predecessor, and more nearly straight than it, while the ovals lie each one within the preceding, and are rounder as well as smaller. Hence for $\mu=\infty$, the comitant should consist of a point (the origin), and a line at an infinite distance.

We are thus able to construct and describe the systems of comitants from the equation $X^2+Y^2=r^2$ alone; but if we wish to determine the order of these curves, so as to study them in the light of related forms, it becomes desirable to pass to new equations. In the article already mentioned, contributed to the *Annals of Mathematics*, it is shown that from the equation of any locus we may derive that of another, whose *real part* shall coincide with any specified comitant curve of the given locus. In the present instance we shall obtain, for the general x-comitant—the x-comitant ξ—the following result:

$$x^2y+y^3-r^2y-2\xi\,(x^2+y^2)=0;$$

and for the y-comitant η,

$$x^3+xy^2-r^2x+2\eta\,(x^2+y^2)=0.$$

These may be called the "equations of the comitants," for the sake of brevity of expression, if it be borne in mind that from our present point of view the only equation which truly represents the comitants *and nothing more*, is

that which at the same time represents them all, viz., the equation $X^2+Y^2=r^2$.

The new equations show that all the comitants are cubics; and the existence, as previously ascertained, of an oval branch in each, at once refers them to that one of the genera of Cayley and Salmon which consists of the projections of the "*parabola cum ovali.*" If a more minute identification is desired, a transformation of coördinates, easily effected, throws either equation into the standard form discussed by Sir Isaac Newton in his "*Enumeratio Linearum Tertii Ordinis.*" The curves thus prove to be of the kind described by him as "defective hyperbolas having a diameter," and to belong to the species numbered 40, whose distinguishing mark is the presence of the oval on the concave side of the conchoidal branch. To write either equation in the form adopted by Plücker as a standard, no change of axes is necessary, but the equation of the x-comitants, for instance, becomes by a purely algebraic modification,

$$(y-2\xi)\left[x^2+\left(y-\frac{r}{2}\right)^2\right]-\left(2r\xi+\frac{r^2}{4}\right)y+\frac{1}{2}r^2\xi=0.$$

In this expression, the existence of the asymptote $(y-2\xi=0)$ and of the "asymptotic point" $\left(0,\frac{r}{2}\right)$ becomes manifest, as also the position of the "satellite line" whose equation is $\left(2r\xi+\frac{r^2}{4}\right)y=\frac{1}{2}r^2\xi$. Among the diametral defective hyperbolas—or, to use language better corresponding to the vocabulary of Plücker, among cubics whose single asymptote is osculating—the group numbered XXXIV is distinguished by the fact that that line passes through the asymptotic point, while the next preceding group, XXXIII, differs from all others in which the asymptote and asymptotic point are separate, by the fact that the satellite line does not cut the curve. In the special case in which ξ has the value $\frac{r}{4}$, the comitant belongs to XXXIV, otherwise to XXXIII. In either case, the existence and position of the

oval determine the species, in the classification of Plücker as in that of Newton; so that if $\xi = \frac{r}{4}$, the curve is of species 152, but, for other values of ξ, of species 148.

The equations of the comitants also suggest a process of geometrical construction entirely different from that deduced from the original equation of the locus. (See Fig.

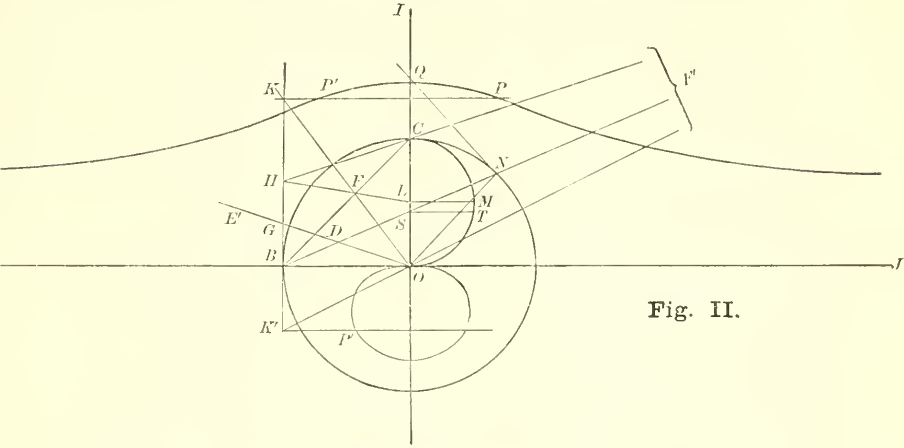

Fig. II.

II.) If, as before, it is desired to construct the x-comitant μ, let the axes and the circle of radius r and center O be again drawn, and, in addition, a semicircle having for its diameter that radius, OC, of the former circle which extends along the imaginary axis in the direction indicated by the sign of μ. Draw also, through B, one of the points in which the circle of radius r meets the real axis, a tangent to this circle. All the foregoing may remain unerased during the entire construction. On the tangent last drawn, take points G and H, so that $BG = \mu$ and $BH = 2\mu$. Draw OG, and produce it sufficiently, so that a distance equal to GB may be laid off from G on the produced part, to E, while the same distance from G toward O, fixes the point D. Then OE will be the maximum distance of the conchoidal branch and OD that of the oval branch, from the axis of reals, and the extremities of these maximum ordinates are to be located on the other axis, in opposite directions from the

origin. In finding other points, the construction must proceed somewhat differently for the two branches. For the infinite branch, take on the line BG a distance BK greater than BH and less than OE. Draw BC and OK, and call their point of intersection F. Draw HF, intersecting the imaginary axis at L, and then draw LM parallel to the real axis and meeting the semi-circumference, $\left(\text{of radius } \frac{1}{2} r\right)$ at M. Draw OM, and produce it to meet the circle of radius r at N. Draw NQ, parallel to MC, meeting the imaginary axis at Q. Finally, describe about O, with radius OQ, an arc meeting at P and P′ a parallel to the real axis drawn through K. Then P and P′ are points of the conchoidal branch.

For the oval, extend the tangent GB on the opposite side of the axis of reals, and take BK′ in that direction, of any length not exceeding OD. Draw OK′ and CH, to intersect at F′, and then F′B, meeting the imaginary axis at S. Through S, a parallel to the real axis is to be drawn, meeting the semi-circumference at T. Then an arc, with radius OT, described about O, will meet the parallel to the real axis drawn through K′, in points belonging to the oval.

If a parallel ruler is used in drawing, this construction may prove more convenient than the preceding, since the compasses will have to be adjusted only once in locating each pair of new points. It should be mentioned that after the distances OD and OE have been determined, we may, if convenience require, replace the points B, G, H, K, K′, on a tangent to the circle, by points at equal heights on any other parallel to the imaginary axis.

In each of the foregoing constructions for a comitant μ it has been assumed that the value of μ is directly given. An important modification of the problem may be stated as follows: Through any given point of the plane to draw the comitants of a circular locus of given radius, whose center is at the origin. Here it is necessary to determine at the outset the quantity (coefficient of i) which

characterizes each of the required comitants, and for this purpose the comitant equations may be employed. Thus, in the case of an x-comitant, the equation may be written $2\xi=\frac{y(x^2+y^2-r^2)}{x^2+y^2}$, a formula which directly suggests the following geometric process: Draw by the usual methods the polar of the given point with respect to the circle of radius r, and from the point in which this polar meets the line joining the given point to the origin draw a parallel to the real axis, then the perpendicular distance from this line to the given point is equal in absolute magnitude to double the quantity ξ, answering to the μ of the preceding constructions. Having found this quantity, the comitant is constructed as before.

It is to be observed that when ξ and η are found by this process, the coördinates of the given point are fully known, hence this construction solves also the problem: To determine geometrically the coördinates of a given point of a plane, when regarded as a point of a circular locus of given radius, having the origin as center.

Having now sufficiently considered the form and position of the comitant curves which make up the circular locus, the next step will be to examine the most important properties of the locus as they are discussed in the elementary analytical geometry, and observe what new light is shed by the present construction upon the familiar processes and results.

"A right line," it is commonly said, intersects the circle in two points, which may be real and discrete, coincident, or imaginary. Just as the "circle," here used to mean the geometric equivalent of the equation, is in the present interpretation replaced by a double system of curves, together forming the *circular locus*, so the right line gives place to a double system of lines. These intersect the comitants of the circular locus in every part of the plane, but since these indefinitely numerous intersections of the separate comitants are not liable to be confused with the two special points mentioned in the theorem, the word "intersect"

may be retained, and the proposition will read. "A circular and a linear locus intersect at two points." By a point of intersection of two loci must of course be meant one whose coördinates, when it is regarded as a point of the one locus, are the same as when it is taken to belong to the other. If such a point, then, lies on the x-comitant ξ of one locus, it is on the x-comitant ξ of the other (the ξ having the same value for the two), and a similar statement is true of the other system of comitants. Thus a point of intersection of two loci may be defined as *a point in which two comitants (viz., one of each series) belonging to one locus are met by the corresponding comitants of the other locus.*

The theorem of analytical geometry, above quoted, specifies not only the number of intersections of the two loci, but their kinds. This, however, is done under the tacit assumption that both loci are real. The full statement intended, therefore, is as follows: "A circular and a linear locus, each of which has a real branch, intersect at two points, which may be real and discrete, real and coincident, or conjugate imaginary." The locus of a linear equation with real coefficients (called, for brevity, a real linear locus) has the two comitants zero coinciding in a single right line, while each of the series of comitants consists of lines parallel to this. Real intersections, whether in discrete or consecutive points, are formed by the real parts of the two loci, just as in the ordinary constructions of analytical geometry. The situation of the imaginary intersections may be easily studied by combining the two equations $X^2+Y^2=r^2$ and $X \cos\varphi+Y \sin\varphi=p$, while assuming $p>r$. Elimination gives $X=p \cos\varphi\pm i \sin\varphi\sqrt{p^2-r^2}$, and $Y=p \sin\varphi\mp i \cos\varphi\sqrt{p^2-r^2}$, whence

$$X+iY=(p\pm\sqrt{p^2-r^2})(\cos\varphi+i \sin\varphi).$$

It is at once apparent that the two intersections lie on the perpendicular drawn from the origin to the real part of the linear locus, at equal distances on opposite sides of the latter, but always on the same side of the origin. The intersection nearest the origin is within the real part of the circular locus, since $p-\sqrt{p^2-r^2}<r$,

hence the branches of its comitants which are here met by the corresponding linear comitants are both ovals, while at the other intersection of the loci, the conchoidal branches alone are present. If p is made indefinitely great, the latter intersection recedes to infinity, while the former approaches the origin as its limiting position. In this way is indicated the geometric representation of the "circular points at infinity," only one of which, it appears, is situated at an infinite distance on the plane, and this one must be regarded as in an indeterminate direction from the finite points of the locus. But the further examination of these important points may be deferred a little until the subject of the asymptotes is reached.

The geometric construction of the finite imaginary points in which the circular locus may be intersected by a real linear locus needs little further statement. The real parts of the loci—the circle and non-secant line—are supposed given. A perpendicular is to be drawn to the line from the center of the circle (the origin), and from its foot a tangent to the circumference. The length of this tangent, laid off on the above-mentioned perpendicular in both directions from its intersection with the given line, marks out the points required.

By the aid of the circle may also be solved a problem, which may often be required, viz: To draw other comitants of a real linear locus, when the comitants zero (*i. e.*, the real line) are given. The distance between two comitants of one system, say the x-comitant μ and the x-comitant ν, for a given difference between μ and ν, depends on the direction of the real line, and is independent of the latter's distance from the origin. Hence if the given line happens to be a secant, a parallel may be drawn to it which will lie outside the circumference, and then the points of intersection of the loci found as above. Selecting one of these points—for instance, the exterior one—its distance from the given line proves to be to the coefficients of i in its coordinates X and Y, as unity to $\sin\varphi$ and $-\cos\varphi$ respectively. The corresponding distance of the two comitants μ will be to μ in the same ratio, hence these distances are $\mu \operatorname{cosec}\varphi$ and $-\mu\sec\varphi$.

The rule that imaginary intersections of a circular and a linear locus must be conjugate, and hence discrete, ceases of course to apply if the linear locus is imaginary, when two imaginary intersections may be coincident. In this case, at the point of contact of the two loci an x-comitant of one must be touched by the corresponding x-comitant of the other, and also a y-comitant of the first by its corresponding y-comitant of the second, the two curves having in general different directions at the point, since in an imaginary linear locus the two series of comitants need not be parallel to each other. Two such linear loci, having a real point Q in common, may touch the circular locus. The usual form of this statement, that "two imaginary tangents to a circle may be drawn through any point within the latter," must not be allowed to give the impression that the comitants of the linear loci, which are the actual tangents, will pass through the point Q. This would of course be impossible, since Q, as the sole real point of each linear locus, is at the intersection of its comitants zero. In fact, since the polar of Q with its comitants constitutes a real linear locus, intersecting the circular locus in the points of contact, the latter points must, as already shown, be on the perpendicular from the origin on this polar: that is, on the line OQ.

The complete geometric construction, exhibiting the "tangents to a circle from a point within," may be made as follows: (See figure, page 19).

The circle drawn around the origin as center, and the point Q (which, to avoid undue simplification, may be assumed to be on neither axis) are the only data needed. The polar of Q is first to be drawn, and the intersections made by the locus to which it belongs are to be found in the manner just stated. These are the points of contact. By a previous construction the coördinates of T, one of these points, may be determined geometrically, and it will be well (in order to render apparent the significance of the subsequent construction) to sketch at the same time the circular comitants which pass through T. Now if the algebraic expressions for the coördinates of T be

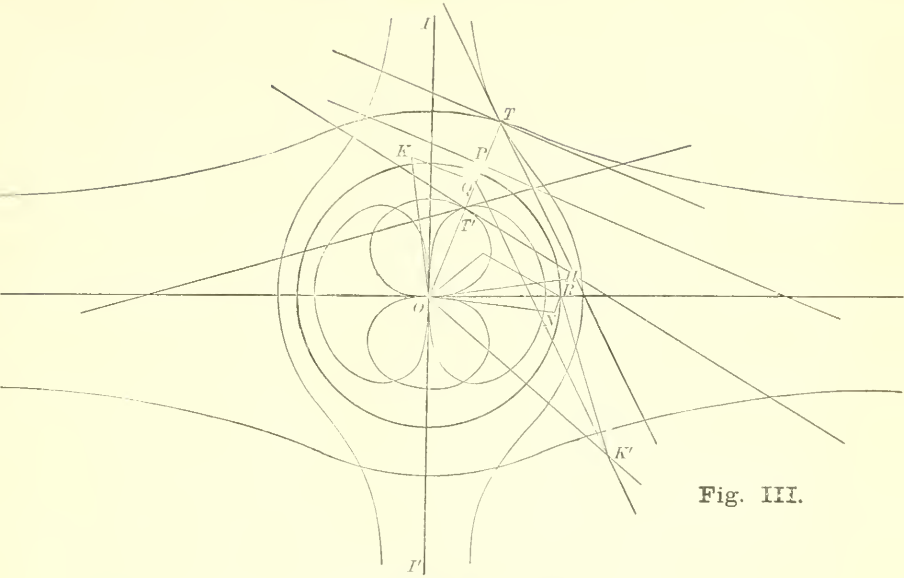

Fig. III.

M and N, the equation of the tangent is $MX+NY=r^2$; whence it is seen that. for the point of the tangent at which $X=0$, Y will be $\frac{r^2}{N}$. Accordingly, that point may be located by finding (in direction as well as in magnitude) a third proportional to the quantities N and r. For this purpose, if the vector ON expresses the quantity N, while OR is that radius of the circle which coincides with the real axis, an angle ROH must be made equal to NOR. and an angle ORH equal to ONR, when the distance OH. cut off by the bounding line of one of these angles on that of the other, will represent the value of Y. If OK make a positive angle of 90° with OH, and be made equal to OH in length, it will represent iY and hence $X+iY$; so that K is the point of the tangent locus for which $X=0$. But this point must be on the x-comitant zero. and that line is accordingly QK. A parallel to QK through T is the linear x-comitant at that point, and will be found to touch the x-comitant of the circular locus. Similarly, by first finding the point of the tangent locus for which $Y=0$, the tangent y-comitant may be constructed; and in like manner both the comi-

tants of the other tangent locus may be drawn through T′, the second point of contact.

By a slight modification of this construction, a tangent may be drawn at a given imaginary point of the locus. For if T be given, the points of the tangent locus for which X and Y respectively vanish may be found as before, but Q must be determined in order to obtain the directions of the comitants zero. From the relations already stated it is easily shown algebraically that if OT be a given distance d, then OQ is found from the proportion

$$d^2+r^2 : r^2=2d : \mathrm{OQ};$$

which admits of ready geometric construction.*

Again, the construction of the polar of an imaginary point whose coördinates are given is obtained in a similar manner. For the equation of the polar of a point M, N is in the same form $\mathrm{MX}+\mathrm{NY}=r^2$ already used for the tangent, hence the points of the required locus for which X and Y are severally zero are found as before, but the real point must be separately determined. For this purpose we may put for M and N respectively their expanded values $m+i\mu$, $n+i\nu$ (where m, n, μ and ν are given real quantities), and then if we assume that X and Y are the real quantities x and y, we can separate the equation $(m+i\mu)\,x+(n+i\nu)y-r^2=0$ into two (since the sums of the real and imaginary terms must separately vanish), which will be $mx+ny-r^2=0$ and $\mu x+\nu y=0$. The real point of the locus is then at the intersection of two lines, whereof the former is the polar of a known point m, n; and the latter is a line through the origin, the tangent of whose inclination to the real axis is $-\frac{\mu}{\nu}$. With the determination of this point, two points of each of the comitants zero become known, and hence these lines may be drawn. To fully construct the locus, however, it is needful to have the coördinates of one imaginary point, that the distance between successive comitants

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*In this is included the solution of a problem relating to a comitant curve considered as a variety of the cubic, and independently of its connection with the other comitants forming the locus; viz: To draw a tangent at a given point of the curve.

of each series may be known. Following the analogy of the previous part of the problem, we may inquire for the point, both of whose coördinates are pure imaginaries. Writing $i\xi$ for X and $i\eta$ for Y, the equation becomes $(m+i\mu)\,i\xi+(n+i\nu)\,i\eta-r^2=0$ and is resolved into $-\mu\xi-\nu\eta-r^2=0$ and $m\xi+n\eta=0$. Now the point whose coördinates are $i\xi$ and $i\eta$ is the same as that whose real coördinates are $x=-\eta$, $y=\xi$; hence the foregoing resolved equation may be interpreted as directing to a point at the intersection of two lines, one of which is the polar of the known point ν, $-\mu$, and the other is drawn through the origin at an inclination to the real axis whose tangent is $\frac{n}{m}$. The perpendicular distance of the point thus found from the real axis is in the same ratio to its distance from the x-comitant 0 as the unit of linear measure is to the distance between the x-comitants 0 and 1; and a similar proportion gives the interval between successive y-comitants.

Since every linear equation (unless its absolute term is zero) may be thrown into the form $\text{M}\text{X}+\text{N}\text{Y}=r^2$, by multiplying it through by the (real or imaginary) ratio of r^2 to this absolute term, we have in the foregoing a direction for constructing all the comitants of any such linear locus given by its equation. The coefficients M and N, found in this way, are the coördinates of the pole of the given locus.

The problem of finding the points of contact of the two linear tangent loci which have a given imaginary point in common is equivalent to that of finding the points of intersection of the circular locus by the polar of the given point; and this may be done as follows, without the labor of constructing the polar locus. The method is based on an inspection of the result of a solution of the two equations $\text{X}^2+\text{Y}^2=r^2$ and $\text{M}\text{X}+\text{N}\text{Y}=r^2$. It is found algebraically that $\text{X}+i\text{Y}=\frac{r}{\text{M}-i\text{N}}(r\pm\sqrt{r^2-\text{M}^2-\text{N}^2})$. To express this result geometrically, we have first to find $\sqrt{\text{M}^2+\text{N}^2}$, a mean proportional between $\text{M}+i\text{N}$ and $\text{M}-i\text{N}$; then $\sqrt{r^2-\text{M}^2-\text{N}^2}$,

a mean proportional between $r+\sqrt{\text{M}^2+\text{N}^2}$ and $r-\sqrt{\text{M}^2+\text{N}^2}$, and finally a fourth proportional to $\text{M}-i\text{N}$, r, and $r\pm\sqrt{r^2-\text{M}^2-\text{N}^2}$.

If P is the given point, whose coördinates are M, N, and if M is the point M, 0, then MP is equal to iN, and by producing PM to Q, making MQ=PM, the point M, −N, is found. Hence the first step is accomplished when a distance equal in magnitude to a mean proportional between OP and OQ is laid off both ways from O on the line bisecting the angle POQ. From each of the points thus fixed a distance r is then to be measured to the right, parallel to the real axis, so fixing the points H, K. Now the angle HOK is to be treated as was POQ; *i. e.*, on its bisector is to be laid off in each direction from O a length equal to a mean proportional between its sides, and from each of the points so found a distance r measured to the right, to the points E and F respectively. If R be the right-hand extremity of that diameter of the circle which lies on the real axis, we have next to make a triangle OET similar to OQR, on the base OE homologous to OQ (and with the angle TOE equal in sense as well as in magnitude to ROQ), and T will be one of the required points of contact; the other having the same relation to OF that T has to OE. (Figure omitted.)

The remark has been already made that any linear locus which does not contain the point 0, 0, may be regarded as represented by the equation $\text{MX}+\text{NY}=r^2$; hence the foregoing construction of the intersections of such loci needs only to be supplemented by discussing those of the locus $\text{CY}=\text{SX}$, to complete the treatment of linear intersections. Let the points C and S be the extremities of vectors extending from O and having the values C and S respectively, and let P be the point whose (imaginary) coördinates are C, S. Produce PC to Q making CQ=PC, then the vectors OP, OQ are $\text{C}+i\text{S}$ and $\text{C}-i\text{S}$ respectively; so that OL and OL′ (denoted by $\pm\text{L}$) will represent $\pm\sqrt{\text{C}^2+\text{S}^2}$, if L and L′ be taken on the bisector of the angle POQ and at distances each way from O equal to the mean proportional between the lengths OP, OQ. The cöordinate X of

the point of intersection is now to be found as a third proportional to $\pm$L, C and r (that is, the triangle ROX is to be made similar to LOC), and similarly Y is a third proportional to L, S and R. (Figure omitted.) The quantity X$+i$Y might, of course, be found at once, as in previous cases; but the construction of the separate coördinates may be more useful.*

It has now been shown how to find the intersection of the circular locus by any linear locus whatever; a brief mention should be made, however, of another mode of attacking this problem, which may be best presented by considering first the simpler one, to find the point of intersection of two given linear loci. Suppose that, in each, the two comitants zero are given in position, also in each, the x-comitant ξ (the ξ having the same value in one as in the other), and in each, the y-comitant η. Join the intersection of the x-comitants 0 to that of the x-comitants ξ. Now, since in either locus the distances from the x-comitant 0 to any two x-comitants, ξ and ξ', are as the numbers ξ and ξ', it follows that the line just drawn will pass through the intersections of *any* two like x-comitants of the two loci. So also the line joining the intersection of the two y-comitants 0 with that of the y-comitants η will pass through the intersections of all pairs of like y-comitants. The point in which these two lines meet is obviously the required point of intersection of the given linear loci.

The points of intersection of any two algebraic loci whatever may be determined by an application of the same principle; that is, the curves which pass respectively through the intersections of like x-comitants, and through those of like y-comitants are always algebraic curves, whose equations result from the elimination of ξ or of η from a pair of comitant equations; and their real intersections must always correspond in position with the (real or imaginary) intersections of the two given loci. But the application of this method to determine the inter-

*The discussion of the intersections made with a circular locus by imaginary radii leads directly to the theory of the trigonometric functions of an imaginary variable, but this subject is much too extensive to be here entered upon.

sections of the circular and linear locus is not practically appropriate, since it invokes the aid of higher curves to treat a problem which is in fact amenable to the ruler and compasses. A similar objection must hold in other cases.

The problems thus far considered embrace the construction of linear loci as secants, tangents, or polars, when imaginary quantities enter the problem in any way whatever; and are hence adaptable to the treatment of any specific elementary question which involves only one circle. But there is one particular case of tangents which, on account of its importance as well as of the special peculiarities it exhibits, requires a separate investigation. This case is that of the asymptotes.

The equations of the asymptotes of the locus $X^2+Y^2=r^2$ are $X+iY=0$, or $\frac{Y}{X}=i$, and $X-iY=0$, or $\frac{Y}{X}=-i$. It is at once apparent that the equation $X+iY=0$ for any (finite) values of X and Y can be satisfied only at the origin; though from the form $\frac{Y}{X}=i$, it appears that infinite values of X and Y might differ numerically by any finite amount. Hence the finite comitants of the locus all pass through the origin, while any line whatever of the plane may be taken as a comitant infinity. Now a real "line at infinity" is characterized by opposite properties, *i. e.*, its comitants μ, when μ is any finite quantity, lie altogether in the infinitely distant region of the plane, unless the comitant zero be regarded as parallel to an axis, when any other parallel to the same, though at a finite distance therefrom, becomes a finite comitant. But by assuming μ infinite we may identify its comitants with any lines in the finite region, irrespective of direction. Hence the first "circular point at infinity" regarded as the intersection of these two loci, is at any finite point; and the statement that all circles pass through this *one* "circular point," is only in this sense true, that one analytical expression will serve to designate for all circles the infinite coördinates which will satisfy their equations; while, as the geometric equivalent of this analytical expression is indeterminate, the point in question is in fact geometrically different for different circles. It is, actually,

in each circle, situated at the center. Accordingly, in examining the intersection of $X \cos \varphi + Y \sin \varphi = p$ with the locus, no indeterminateness was found to attach to the position of the interior point of intersection, when p was increased indefinitely; on the contrary, the limiting position of the intersection was found to be the center of the circle. Through this point every comitant passes, and each is there touched by the corresponding comitant of the asymptote; while this point of contact, in the case of a given comitant—say the x-comitant μ—has for its coördinates $X = \infty + i\mu$, $Y = iX$. It is to be observed that although in this way each single comitant exhibits the position of the point at infinity, the entire series of comitants can illustrate only one form of infinite value which might be assigned to the variable X. For beside the value $\infty + i\mu$, where μ is finite, X might have the infinite value $\infty(x + i\xi)$ with a finite ratio between x and ξ, or indeed it might be $x + i\infty$, where x, the real part, is finite. But in these cases the point of contact would fall on a comitant infinity.

This remark applies with still greater significance to the consideration of the second asymptote, $X - iY = 0$. This locus has nothing of the unique quality which makes the former asymptote a correlative to the line infinity. Its x-comitant 0 lies on the real axis and its y-comitant 0 on the imaginary axis, while any comitant μ is at a distance 2μ from the comitant zero. On the x-comitant μ the point whose coördinate X has the value $\infty + i\mu$ is at an infinite distance from the origin; and similarly for the y-comitants. So long, then, as μ is finite, the point of intersection with the line infinity is represented by the *two* points infinitely distant on the two axes. Accordingly, every (finite) comitant of the circular locus has for its individual asymptote the corresponding comitant of the locus $X - iY = 0$. But the second circular point at infinity must not on this ground be asserted to be represented *only* by the infinitely distant points of the axes, for if we take into account the comitants infinity, the direction of the point in which the asymptote locus meets the line infinity becomes indeterminate; and this agrees with the fact that the exterior inter-

section of the line $X \cos \varphi + Y \sin \varphi = p$, when p was made infinite, was found to be infinitely distant on the radius bounding the angle φ.

Thus the circular points at infinity are not definitely localized, but it is to be noted that their indeterminateness results, not from their imaginary quality alone, nor yet wholly from the infinity of their coördinates, but from the combination of both characters, and the absence of any means of imposing a definite value upon the ratio of the real and imaginary parts.

The problems which have been discussed in the present paper are mostly of the class which relate to properties of the locus considered as a whole, rather than to those of individual comitants. As such problems when analyzed will, as a rule, resolve themselves into investigations of the intersections of the locus by other loci, they will generally be distinguished by the mark that the comitants of the two series enter similarly into the geometrical constructions; thus at the intersection of two loci the like x-comitants and also the like y-comitants meet in the same point. Of the other class of investigations mentioned, viz., that in which the properties of individual comitants are the subject of inquiry, an instance occurs in obtaining the "comitant equations." In such problems it is frequently necessary to employ separate symbols for the real and imaginary parts of constants or of variables; and if in the subsequent course of the investigation, a symbol which has thus been taken to represent a real quantity occurs as the representative of the unknown quantity in an equation, imaginary roots of such an equation must of necessity be treated as impossible, since they contradict the hypothesis of reality. This is just as if m, having been put to represent the integral part of a mixed number, in some algebraic problem, should afterward be determined by a quadratic equation, one of whose roots should prove to be integral, the other fractional. The latter root would of course be disregarded, without the imputation of any unreal quality as belonging to fractions.

To illustrate by an example this class of inquiries, suppose that we desire to determine at what point a line, tangent to one of the comitants, cuts the same comitant again. Let the x-comitant μ be the curve in question, then we know that its tangent at the point M, N (*i. e.*, the point $m+i\mu$, $n+i\nu$, when these quantities satisfy the equation $\text{M}^2+\text{N}^2=r^2$), is the x-comitant μ of the locus $\text{M}x+\text{N}y=r^2$. But the point at which these two lines again intersect will not have the same coördinates, regarded as a point of the linear locus, which it has as a point of the circular locus. It is easily shown that a coincidence in position of points on two like x-comitants belonging to different loci, *without a concurrent intersection of the* y-*comitants*, implies only that the real parts of the coördinates x for the coinciding points differ by the same amount as the coefficients of i in the imaginary parts of their coördinates y. If we represent this unknown difference by d, we have the three equations

$$(x+i\mu)^2+(y+i\eta)^2=r^2, \qquad (m+i\mu)^2+(n+i\nu)^2=r^2$$

and

$$(x+i\mu+d)(m+i\mu)+(y+i\eta+id)(n+i\nu)=r^2$$

from which to find x, y, and η, eliminating d. Since each of the equations can be resolved into two, by separating the real and imaginary terms, we can eliminate also two of the constants, as m and ν, and find y in terms of r, μ, and n. Two values of y will of course be found equal to n, and represent the coincident intersections at the point of contact, but the third, which is the subject of the inquiry, is determined by the linear equation

$$[(\mu^2-n^2)^2+r^2\mu^2]y+\mu[(\mu^2-n^2)^2+r^2(\mu^2-2n^2)]=0.$$

From this value of y, that of x and η can be found, and the tangential point is completely determined. As the equation is linear, no trouble arises from imaginaries.

But let it be now required to find on the x-comitant μ a point of inflection, using for the purpose the property that at such a point the above-determined tangential point must coincide with the point of contact. Then n is to be substituted for y in the above equation, and the whole

solved for n. One root, $n=\mu$, can be immediately found, and this will be seen to indicate the existence of a point of inflection at an infinite distance. After dividing out the factor $n-\mu$, there remains the biquadratic

$$n^4+2\mu n^3-2\mu(\mu^2+r^2)n-\mu^2(\mu^2+r^2)=0.$$

Two of the roots of this equation are imaginary, and with them a third must be rejected on the ground that it cannot consist with real values of m and ν. The remaining root shows the position of the two real, finite, points of inflection. The three rejected roots are just sufficient in number to represent the six imaginary inflections of the cubic curve, whose real part coincides with the x-comitant μ; but it seems plain that, consistently with the fundamental notions of the present scheme of interpretation, it can only be said that the latter curve, considered as a comitant of the locus $X^2+Y^2=r^2$, does not possess these six inflections.

As all of the foregoing discussion has been limited to the interpretation of the equation $X^2+Y^2=r^2$, the circular locus which has been treated is not the most general one to which the name applies, but is merely the real circular locus having its center at the origin. It seems proper in a closing paragraph or two, to indicate what modifications of the foregoing constructions would result from their extension to the circular locus in general.

The removal of the center from the origin is effected by a parallel transformation of coördinates, in respect to which it is only necessary to remark that transformation to a real point simply changes the relative position of the axes in respect to the entire system of curves, leaving the latter unaffected, while transformation to an imaginary point disturbs. not the form of the comitant curves, but the numbers by which they are characterized, thus the comitant zero may become the comitant μ, while some other comitant of the same series becomes the new comitant zero. From this statement, the effect of such a change on the foregoing geometrical constructions may be easily estimated.

A more important modification results from admitting imaginary values of the radius. The equation $X^2+Y^2=R^2$ (when $R=r+i\rho$), is at once seen to belong to a singly infinite series of different loci, which vary in form with the magnitude of the ratio $\rho : r$. The case already considered is that in which this ratio is zero; and the most closely analogous case, as may be readily conjectured, is that in which the ratio is infinite. These two cases alone can be represented by equations with real coefficients.

The comitants of the locus $X^2+Y^2+\rho^2=0$, like those of $X^2+Y^2=r^2$, belonging to Newton's "defective hyperbolas having a diameter," being symmetrically divided by one of the axes. The x-comitant 0 is a straight line, coinciding with the real axis, and the successive comitants ξ, when ξ is small, are conchoidal curves, *convex* toward this axis, and all touching it at the origin. Each lies within its predecessor, and exhibits a greater curvature, until the x-comitant ρ is reached, when the curve, whose shape at first might suggest a bowl, and later a purse with a narrowing neck, has at length completely closed together, and has a node on the imaginary axis, at a height of ρ above the origin. The succeeding comitants have ovals, like the comitants of the locus $X^2+Y^2=r^2$, but these lie on the same side of the real axis as the conchoidal branches, and opposite the convexity instead of the concavity of the latter. All the ovals, as in the former case, touch the real axis at the origin. Thus it is seen that in a single series of comitants occur successively representatives of Newton's species 45, 41, and 39—the conchoidal hyperbola *pura*, *nodata*, and *cum ovali*, the last having the oval on the convex side. As ξ increases beyond ρ to infinity, the conchoidal branch grows more straight, and the oval shrinks to a point at the origin.

Between this form and that of the locus $X^2+Y^2=r^2$ there is an unbroken gradation, corresponding to the positive values of the ratio $\rho : r$; and similarly another for the negative values. In the intermediate forms, however, the comitants are no longer symmetrical in respect to either axis.

and must be classed among Newton's "defective hyperbolas *not* having a diameter." The "comitant equations" are

$$x^2y+y^3-2\xi(x^2+y^2)+2r\rho.x+(\rho^2-r^2)y=0,$$
$$x^3+xy^2+2\eta(x^2+y^2)+(\rho^2-r^2)x-2r\rho.y=0;$$

(and from these as general forms the special cases already considered are derived by making ρ or r equal to zero.) All the comitants of a series pass through the origin, and have there a common tangent. This tangent, however, for the x-series, is no longer the real axis, but has a slope of $\frac{2r\rho}{r^2-\rho^2}$, so that in the loci in which $\rho=\pm r$, the common tangent coincides with the imaginary axis. The infinite branches of the comitants are not conchoidal, but serpentine, crossing their asymptote (which is still parallel to the real axis, and at a distance 2ξ therefrom) at a point distant by $\frac{(r^2-\rho^2)\xi}{r\rho}$ from the imaginary axis. In the case of the comitant zero, this crossing is at the origin, and the serpentine curve is symmetrical to that point as a center, thus belonging to the Newtonian species numbered 38. But as ξ increases from the value 0, the successive comitants, no longer possessing any kind of symmetry, belong at first to the species 37, as they have no oval and no singularity. There is, however, the same gradual closing into the form of a narrow-necked purse, already observed in the case $r=0$, and the comitant ρ is again nodate, so belonging to the 34th species. For still greater values of ξ, the curve is of species 33, having an oval, which, as in all cases, shrinks to a point at the origin as ξ becomes infinite, the serpentine branch in the meantime straightening toward coincidence with its asymptote. The node on the x-comitant ρ is at that point of the locus for which $Y=0$; and it may be remarked, as true of all forms of the locus, that the four points characterized by zero values of one or the other coordinate are double points of the comitants of opposite name on which they fall.

Very slight and obvious modifications are alone required to adapt the first method given in the present paper for the construction of comitants, to use with any given values of r and ρ.

ON THE EIGHT LINES USUALLY PREFIXED TO HORAT. SERM. I. 10.[1]

By WILFRED P. MUSTARD.

The eight lines usually prefixed to Horace, Satires, I. 10 are found only in some of the mss. of Keller and Holder's third class. They are unknown to the mss. of classes I and II, and to z and the whole $R\pi$ family of class III. They were apparently unknown to the Scholiasts, who would surely have considered them obscure enough to require some explanation. Mavortius did not know them. In $F\lambda'$ and some other mss. they appear as the beginning of satire 10, while in $\beta\mu np$ they form a continuation of satire 9.

On this external evidence almost all the editors have condemned the lines as an interpolation, and either marked them off by brackets or omitted them altogether.[2] They appear as part of the text in Zarotto's Milan edition, in the first and second Aldine editions, and in the Paris edition by R. Stephanus. But even in the fifteenth century Landino rejected them, and most of the older editors followed his example. Some editors have separated them from the text but prefixed them to the satire, others have printed them separately in their commentaries, while many have omitted them altogether. Thus they do not appear in ten of the Venice editions (for the omission in the first eight Landino was responsible), in Bentley's, Wakefield's and some twenty others. Lambin ascribes them to some 'semidoctus nebulo' who wished to explain the open-

[1] This paper offers no new theory as to the meaning, authorship or date of these obscure lines. It is merely an attempt to collect and arrange the various opinions that have been expressed with regard to them.

[2] I owe the greater part of the facts presented in this and the following paragraph to Kirchner's edition of the first book of the Satires (Leipzig, 1854), p. 142.

ing word 'nempe.' Jacobus Cruquius barely mentions them in his commentary as the words of a 'simius Horatianus.' Bentley omits them without mention.

Others have defended the lines. Gesner restored them. Valart thought they were the work of Horace. Heindorf, followed by Bothe and others, thought that Horace had written them as an introduction to this satire but afterwards threw them aside and commenced in a different tone; or that they were an unfinished introduction to some satire discovered after his death and, with the addition of the expletive words 'ut redeam illuc,' prefixed to Sat. I. 10, on account of the similarity of subject. Jo. Val. Francke proposed to insert them after verse 51 of this satire, Reisig after verse 71. Morgenstern held that Horace had written the lines, but afterwards rejected them. Schmid[3] virtually said that they were the work of Horace. Apitz[4] ascribed them to Horace, but bracketed verse 8. Urlichs[5] said that the old question is really one of subjective feeling as to what is worthy or unworthy of Horace. He thought the lines genuine, though he admitted their obscurity and considered the text corrupt. Döderlein found nothing seriously objectionable in the lines, and was quite certain of their genuineness. He maintained that the fact that they are not found in many mss. does not prove them spurious; this might be the result of chance, or even of a recension by Horace himself. W. Teuffel's[6] verdict was similar to Morgenstern's.

The text of these obscure lines is very corrupt. The mss. of most importance for determining the original reading are F$\lambda'\beta'$. F. the principal representative of the large third class, is the assumed common source of the '*gemelli Parisini*,' φ 7974 and ψ 7971; λ' the archetype of a similar pair, λ *Leidensis* and l *Parisinus;* β' that of β *Bernensis* and f *Franckeranus* (now *Leeuwardensis*).

[3] *Philol.* XI. pp. 54–59.

[4] *Coniectan. in Q. H. F. Satiras* (Berlin, 1856), p. 86.

[5] *Rhein. Mus.* XI. p. 602.

[6] *Rhein. Mus.* XXX. p. 621.

These mss. agree very closely, and establish the text as follows:

> Lucili, quam sis mendosus, teste Catone
> defensore tuo pervincam, qui male factos
> emendare parat versus, hoc lenius ille
> quo melior vir est, longe subtilior illo
> qui multum puer et loris et funibus udis
> exoratus, ut esset opem qui ferre poetis
> antiquis posset contra fastidia nostra,
> grammaticorum equitum doctissimus. ut redeam illuc,

"How full of faults you are, Lucilius, I shall clearly prove from the testimony of Cato, your champion, who is preparing to revise your ill made verses. He will deal more gently with them inasmuch as he is a better man, of far finer tastes, than the scholar who in his boyhood felt the vigorous persuasion of moistened thong and rope, in order that there might be one who could lend a helping hand to the poets of old against the carping criticism of our day, the cleverest of aristocratic grammarians. To return to that point,"

NOTES ON THE TEXT.

Vs. 1. 'quod sis' (*codd. plerique ap. Lamb.*). Some of the abbreviated forms of 'quam' and 'quod' in minuscular writing are very much alike.[7] Unless very carefully written these words might be readily confused, and so 'quod' may have appeared here. When once it had appeared in a ms. it might easily be retained because of its use in late Latin to introduce substantival clauses after 'verba dicendi et sentiendi.'[8]

Vs. 2. 'convincam' (*ed. Landini ex mss.*) for 'pervincam,' which as the more difficult reading should be retained. One ms. (*Kirchneri cod. L in Dresd. III.*) gives 'devincam.' Peerlkamp suggested 'prope vincam.'

Vs. 4. 'quo melior vir est.' This is the reading of the most important mss. The false quantity in 'vir' has

[7] Chassant, *Dictionnaire des abreviations, latines et francaises*, Paris, 1876, p. 77.

[8] Draeger, *Hist. Syntax der latein. Sprache*, Vol. II., p. 229.

given rise to many attempts at improving the line. Thus one ms. has 'quo vir melior est,' another 'quo vir est melior,' a third 'quo est vir melior,' while several read 'est quo vir melior.' The last arrangement of the words gives undue emphasis to 'est.' Lambin conjectured 'quo melior is est,' and the *Martinius* of Cruquius, the only one of his mss. that contained these eight lines, had 'quo melior hic est.' But there are pronouns enough already in 'ille . . . illo.' Several mss. had 'quo melior vir et est longe subtilior.' Meineke defended this hyperbaton for 'quo melior vir est et longe subtilior,' appealing to Sat. I. 3, 63; I. 4, 68; I. 9, 51. This, however, gives the impossible combination 'quo longe subtilior.' Heindorf found 'adest' in *Berol. 5* and accepted it.

Vs. 5. 'puer et.' The obscurity of this line has given rise to several emendations: 'puer est' (Gesner); 'pueros' (Urlichs); 'puerum est' (Reisig); 'nuper' (Rutgers); 'fuerit' (Praedicow, who also read 'quem' and 'exhortatus'); 'pueros' (Nipperdey[9]). W. Teuffel[10] suggested 'me olim' for 'multum' and defended 'olim' by a reference to Sat. I. 4, 105.

Vs. 6. 'exoratus' is confirmed by the number and importance of the mss. in which it is found. The other mss. readings 'exortatus' and 'exhortatus' are only possible with 'puerum' in the preceding line, for there is very little authority for the active form or passive meaning of 'exhortor.' In any case the omission of 'est' is a difficulty, and hence, apparently, Peerlkamp's conjecture 'est hortatus.' The conjectures 'exornatus' (Glareanus) and 'est ornatus' (Valart) are obviously suggested by such expressions as 'adeo exornatum dabo, adeo depexum, ut dum vivat, meminerit mei.'[11] Horkel apparently wanted a good strong word after 'loris et funibus,' and settled upon 'excoriatus,' which Meineke and Schütz approve.

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[9] *Opusc.* 493.

[10] *Rhein. Mus.* XXX. p. 622.

[11] Ter. *Heaut.* 5, 1, 77.

Vss. 4–6. In the *Rheinisches Museum für Philologie*, XLI. pp. 552–556, F. Marx offered the following emendation:

> —hoc lenius ille,
> quo melior *versu* est, longe subtilior illo
> qui multum puerum et loris et funibus *ussit*
> exoratus,—

His explanation and defense of these changes are given below.

COMMENTARY.

In the very first verse there is evidence of the spurious nature of this fragment, for (1) the promise 'quam sis mendosus, *teste Catone*, pervincam' is not fulfilled, and (2) the sentiment is unlike Horace. In the tenth satire he defends the opinion he had pronounced upon Lucilius in Sat. I. 4, but with full recognition of his peculiar merits, and elsewhere he very modestly claims for himself a lower place than for his predecessor.[12] "To Lucilius he pays also the sincerer tribute of frequent imitation. He made him his model, in regard both to form and substance, in his satires: and even in his epistles he still acknowledges the guidance of his earliest master."[13]

'Teste Catone.' The Cato here referred to is the grammarian Valerius Cato, who is mentioned in Suetonius[14] as 'poetam simul grammaticumque notissimum,' 'summum grammaticum optimum poetam.' 'Cato grammaticus, latina Siren.' Another section of Suetonius tells of Cato's interest in the works of Lucilius, 'quas (*sc.* Lucili saturas) legisse se apud Archelaum Pompeius Lenaeus, apud Philocomum Valerius Cato praedicant.'[15]

Those who see in the person here compared with Cato the 'plagosum Orbilium' of Horace, Epp. II. 1. 70, assume that the writer of these lines knew that epistle, which is

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[12] Sat. II. 1. 29. 'me pedibus delectat claudere verba, Lucili ritu, nostrum melioris utroque.' *Ibid.* 74, 'quicquid sum ego, quamvis infra Lucili censum ingeniumque.'

[13] Sellar, *The Roman Poets of the Republic*, 3d ed., 1889, p. 249.

[14] *De Gramm.* 4 and 11.

[15] *De Gramm.* 2.

assigned by Vahlen to B. C. 14. Suetonius, *de gramm.* 11, says of Cato, 'vixit ad extremam senectutem,' so that 'emendare parat' might be literally true if the lines were genuine. Marx claims that the words need mean only 'emendare studet, emendationi operam dat, emendaturus est,' comparing Juv. 8, 130, 'per oppida curvis unguibus ire parat nummos raptura Calaeno.' Moreover, he maintains, the author of these lines pronounces upon the whole recension of Cato, implying that it was already finished, so that they were not necessarily composed in the time of Horace.

Keller objects even to the sentiment of 'teste Catone' that (1) Horace required no one's authority to confirm his opinion of Lucilius, and (2), in view of Epp. I. 19, 39–40, it is not likely that he would have appealed to the authority of any grammarian.[16] This he regards as another evidence of interpolation.

Vs. 3. Some editors punctuate with a period after 'versus,' and another after 'doctissimus,' verse 8. With this punctuation 'hoc' would most naturally be taken as accusative after a finite verb understood. It seems better to point with commas and supply such a participle as 'facturus,' taking 'hoc' as the ablative corresponding to 'quo.'

Vs. 4 is certainly corrupt.

(*a*) It is strange that 'melior' should be given as a reason for 'lenius.' It must have been this difficulty that gave rise to the variant 'lenior.' Cato's moral character is not at all concerned. All that is required of him is ability to correct metrical errors and halting sense in Lucilius' verses, defects which had probably been multiplied even in his day by mistakes of the copyists. Nor does 'subtilior' suit 'lenius,' for Lucilius' verses are 'male facti.'

(*b*) There is a false quantity in 'vir.'[17]

[16] 'non ego, nobilium scriptorum auditor et ultor,
grammaticas ambire tribus et pulpita dignor.'

[17] The Italian dialects show that the 'i' in 'vir' was once long (*veir*): cp. Buecheler, *Lex. Ital.* p. 30.

(*c*) 'Longe subtilior' is irregular. "Cicero and the older writers did not use 'longe' to strengthen the comparative, though it appears in poets of a later age and in the more recent historians."[18] Wölfflin[19] says that Horace kept strictly to the old rule of 'multo' with the comparative, using 'longe' only in one anomalous case. He would therefore not have written 'longe' here instead of its metrical equivalent 'multo,' and its use is one proof of the spurious nature of these eight lines.

(*d*) 'Ille' and 'illo,' ending consecutive lines and referring to different persons, are strange and confusing as to meaning. Suetonius rejected a certain prose epistle which purported to have been written by Horace, 'epistula etiam obscura, quo vitio minime tenebatur'.[20] He would scarcely have found the transparency of genuineness in verses 3–4. To avoid the difficulties in 'lenius' and 'ille . . . illo' Schütz would strike out the two half-lines and read 'emendare parat versus subtilior illo."

Vs. 5. If the genuineness of verse 4 may be questioned on the ground of obscurity, still more objectionable is verse 5. It seems impossible to explain this and the following lines in their best attested form. For example, who is the person compared with Cato?

(*a*) Because Horace says, Epp. II. 1. 70, that he studied the poems of Livius Andronicus in his boyhood under the 'plagosus Orbilius,' many editors have made 'qui puer . . . exoratus' refer to the poet himself. It may be doubted whether Horace would have thus spoken of himself, but a greater difficulty awaits us in verse 8, 'equitum doctissimus.' These words most naturally refer to the same person as 'qui . . . exoratus,' and Horace was not an 'eques.'

(*b*) Reisig, who reads 'puerum . . . exhortatus,' makes 'puerum' refer to Horace, 'qui' to Orbilius. But to this Schütz objects that 'puerum' would be too indefinite without 'istum' or 'illum.'

[18] Hand, *Tursellinus*, III. p. 551.
[19] *Comparation*, p. 40.
[20] *Horatii Poetae Vita.*

Schmid[21] also read 'qui . . . puerum . . . exhortatus,' referring 'qui' to Orbilius.

W. Teuffel[22] refers 'puerum' to Scribonius Aphrodisius, 'qui' to Orbilius. To this also Schütz objects that Scribonius was 'Orbili servus atque discipulus,'[23] and that 'puerum' would not imply all this. He might more reasonably have repeated his objection to Reisig's explanation, that the unmodified 'puerum' is too indefinite.

These three interpretations are obviously based upon the mention of the 'plagosus Orbilius,' Epp. II. 1, 70, and they receive some support from the words 'grammaticorum equitum doctissimus,' in verse 8. These words naturally refer to the same person as the clause 'qui . . . puerum . . . exhortatus,' and Orbilius might, at least ironically, be called a knight.[24] There is, however, no evidence that he revised Lucilius' 'ill made verses,' or that he paid special attention to them.

(*c*) J. Becker[25] thought that either Florus or Titius is meant. Very little is known of these men except from Horace, Epp. I. 3, and II. 2. Horace merely says that Florus has ability enough to win distinction in oratory, in law, or in poetry.[26] Porphyrio says 'hic Florus [scriba] fuit satirarum scriptor, cuius sunt electae ex Ennio, Lucilio, Varrone.' Kiessling hints that the old commentator inferred all this from Epp. I. 3, 21, 'quae circumvolitas agilis thyma?' Whether right or not, Porphyrio apparently means that Florus rewrote some of the poems of these earlier authors, adapting them for the readers of his own day. Even if this be accepted, it is hard to suppose that Horace would refer to Florus in the language of these eight lines, and yet address him fifteen years later as a young man who had not written much.[27] Of Titius still less is known. Horace asks Florus whether he is still

[21] *Philol.* XI. pp. 54–59.
[22] *Rhein. Mus.* XXX. p. 622.
[23] Sueton. *De Gramm.* 19.
[24] Sueton. *De Gramm.* 9, 'deinde in Macedonia corniculo, mox equo meruit.'
[25] *Philol.* IV. p. 490.
[26] Epp. I. 3, 23–25.
[27] Epp. I. 3, 22–25.

writing odes or trying his hand at tragedy, 'Titius Romana brevi venturus in ora.'[28] All that the scholiasts have to say about him may very well have been derived from the text. Thus Becker's theory seems to have very little support, except Porphyrio's statement that Florus was a writer of satires, and the fact that Titius and Florus were both noblemen of a literary turn, and might be called 'equitum doctissimi.' That either of them could be called 'grammaticorum equitum doctissimus' is by no means apparent.

'Loris et funibus udis.' The mention of 'lora' and 'funes' suggests a rather savage treatment of the unknown youth referred to in this line. References to the use of 'funes' for the purpose of punishment are not very numerous. Horace, however, has 'Hibericis peruste funibus latus,'[29] on which Orelli remarks that 'funes' made from the Spanish broom were used for flogging the marines. No very satisfactory explanation of the word 'udis' has ever been offered. It is not clear that savage masters sometimes used a moistened lash, or that a lash so treated would cause the victim more pain. Marx[30] quotes Petronius, 134 B, 'lorum in aqua,' as inconsistent with such explanations. It is unfortunate that the wisdom of the scholiasts was not brought to bear upon this word; their comments would certainly have been interesting.

Vss. 3–6. The changes in these three lines suggested by F. Marx have been mentioned on page 35. First he emphasizes the importance of the word 'exoratus' in the interpretation of this fragment, a word which is preserved by all the best mss. of the third class. This word, he says, may here be equivalent to 'though vainly implored for mercy,' like 'exorata' in Juvenal, 6, 415, 'vicinos humiles rapere et concidere loris exorata solet.'[31] Then reading 'puerum' for 'puer,'[32] as many earlier scholars have done, he looks about

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[28] Epp. I. 3, 9.

[29] *Epod.* 4, 3.

[30] *Rhein. Mus.* XLI. p. 552.

[31] A similar use of 'exorare,' which he might have quoted, is found in Hor. Epp. I. 1, 6, 'latet abditus agro, ne populum extrema toties exoret harena.' With this meaning of 'exoret,' 'toties' may be taken literally.

[32] An easy change paleographically.

for a finite verb of 'striking' or 'cutting.' This, he thinks, is lurking in 'udis.' which is certainly very weak and has never been well explained. The verb is probably 'ussit.' It should be noticed that the word 'udis' appears '*in ras. 3.*' and that very often in mss. the termination '–it' shows a medial 'd.'[33] For similar uses of the verb 'urere' cp. Horace. Epp. I. 16, 47, 'loris non ureris': Epod. 4, 3. 'Hibericis peruste funibus': Sat. II. 7. 58. 'virgis uri.' The conjecture 'quo melior versu est' in the fourth line he puts forward with less confidence.

Marx then refers his new reading, 'qui multum puerum . . . ussit exoratus,' to Vettius Philocomus, Cato's teacher, who was one of the first to revise the work of Lucilius.[34] This man, as being 'Lucilii familiaris.' and possibly the same person who was censured by the poet 'propter sermonem parum urbanum,'[35] may have been like Aelius Stilo and Servius Clodius, a Roman knight. His name, however, suggests a Greek origin. and in the absence of any special statement as to his rank, it is not easy to assume that he was an 'eques.'

Vs. 8. The words 'grammaticorum equitum doctissimus' are very difficult both in reference and in meaning. They would most naturally refer to the same person as 'qui . . . exoratus.' but they can hardly apply to the person who is so unfavorably compared with Cato. Schütz claims that such irony as this is quite impossible here, and failing to find any other person to whom the epithet could easily be referred, would strike out the words altogether. Apitz[36] bracketed the whole of verse 8.

Kirchner and Döderlein would refer 'doctissimus' to the same person as 'melior' and 'subtilior.' *i. e.*, to Cato.

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[33] Examples of this interchange in Horatian mss. are cited by Keller and Holder, *Epilegom.* III. p. 853. A similar list is given in Mayor's *The Latin Heptateuch*, p. 251.

[34] Sueton. *De Gramm.* 2.

[35] Quint. *Inst. Or.* I. 5, 56. taceo de tuscis et sabinis et praenestinis quoque: nam et eorum sermone utentem Vettium (Vectium?) Lucilius insectatur, quemadmodum Pollio reprehendit in Livio Patavinitatem. licet omnia italica pro romanis habeam.

[36] *Coniectan. in Q. H. F. Satiras*, 1856. p. 86.

The long separation is decidedly against this, and, besides, Cato could hardly be called an 'eques.' According to Suetonius, *De Gramm.*, 11, his social position was doubtful in his manhood and he probably never had a knight's income in his old age. To meet this last difficulty Kirchner proposed to read 'equidem' for 'equitum.'

The reading 'doctissime' has been proposed, but this is obviously suggested by the knowledge that Lucilius was a knight, and the objectionable interval is only increased.

The words 'grammaticorum equitum' are especially obscure. As they stand they would seem to imply a class of knights who were grammarians, or of grammarians who were knights,[37] but such guilds are quite unknown.

Döderlein punctuated with a comma after 'grammaticorum.' As has been mentioned above, he considered these eight verses the genuine introduction to Sat. I. 10, so that in trying to avoid one difficulty he created another almost as serious, by making Horace class himself among the grammarians—'fastidia nostra grammaticorum.'[38]

Badius Ascensis thought Maecenas was the 'eques': another old scholar thought of Laberius. Orelli came to the conclusion that the writer of these verses, whoever he was, knew no more who the 'eques' was than we do.

'Ut redeam illuc.' Cp. Sat. I. 1. 108, 'illuc, unde abii, redeo,' and Nepos, *Dion.*, 4, 'sed illuc revertor'; *Agesil.* 4, 'sed illuc redeamus.'

It is hard to find anything in the preceding lines to which 'illuc' can well be referred. As Krüger[39] remarks, it cannot refer to the promised proof that Lucilius is full of faults, for this promise is not fulfilled, or to the proof of his faults on Cato's evidence, for Horace does not return to this at all. Voss and Francke made 'illuc' refer in a general way to Sat. I. 4, or its subject.

[37] Like Juvenal, VIII. 49, nobilis indocti, 'a nobleman who is an ignoramus.'

[38] This is contrary to the sentiment of Epp. I. 19, 40, 'non ego . . . grammaticas ambire tribus et pulpita dignor.'

[39] *Drei Satiren fuer den Schulzweck erklaert*, 1850, p. 17.

It seems almost certain that these three words were inserted on account of the abrupt opening, 'Nempe etc.'[40] The preceding lines were probably written with the text of Sat. I. 10 on account of the similarity of subject, and some later scribe, mistaking them for the introduction to this satire, would add the words 'ut redeam illuc' to serve as a bridge to the lively opening 'Nempe incomposito dixi etc.,' though, as Schütz remarks, they would serve better to connect the verses with verse 2, 'quis tam Lucili fautor inepte est?' The long introduction to Sat. I. 7 (followed by 'ad Regem redeo,' vs. 9) may have suggested the expletive words that were felt necessary. Keller and Holder cite as similar interpolations the four lines once prefixed to the Aeneid and the ten lines at the beginning of Hesiod's Works and Days. It is incontestable, they add, that the satire is complete without these eight verses, and that nothing is wanting at the beginning. On the contrary, the fact that Persius, the deliberate imitator of Horace, begins one of his satires (the third) with 'nempe' speaks for the genuineness of the introductory 'nempe' here.

The external evidence that these eight verses are an interpolation to Sat. I. 10 is given in the first paragraph of this paper; a careful examination of them can only result in the conclusion that they are not the work of Horace at all. They have been assigned to different writers and to different periods.

Kirchner ascribed them to Furius Bibaculus (circ. 700 A. U. C.), arguing from Sueton. *De Gramm.* 11, that Valerius Cato, if still alive when Horace wrote this satire (A. U. C. 720), must have been over seventy years old, too old to be contemplating a revision of Lucilius. This argument was soon afterwards disposed of by Schmid,[41] who proved from the same section of Suetonius that Cato could not have been more than sixty-two years old in A. U. C. 720, and

[40] 'Scil. ut transitus ad Horatium sit.' Baehrens, *Fragm. Poet. Roman.*, 1886, p. 329.
[41] *Philol.* XI. p. 54.

was probably alive several years later.[42] C. Fr. Hermann ascribed them to Fannius. Lucian Müller, in his edition of Lucilius, 1872, says they were undoubtedly composed in the time of Horace, though their authorship is uncertain. These three scholars insisted on taking 'emendare parat' literally.

Schütz says that the writer of the fifth verse apparently knew not only Epod. 4, 3, 'Hibericis peruste funibus' and 4, 11, 'sectus flagellis . . . praeconis ad fastidium,' but also Epp. II. 1, 70, 'plagosum . . . Orbilium, etc.' This epistle is assigned by Vahlen to B. C. 14, so that these verses could not have been written by Fannius or by Furius Bibaculus. He would put the composition of the fragment as late at least as the beginning of the second century A. D. Just as Tacitus[43] says that there are men in his day who prefer Lucilius to Horace, and Quintilian[44] insists that Horace's criticism is unfair, so the unknown writer of these lines objects to Horace's treatment of his own model, appealing to the authority of Cato, who was of course not satisfied with the work of Lucilius as he found it, but still thought it worth revising.[45] The third verse, Schütz maintains, is not necessarily older than Sueton. *De Gramm.* 2. The writer may have known Suetonius' account of Cato and yet made him an editor not merely a student of Cato in his younger days, either by mistake or because he knew or thought he knew better.

Orelli remarks that the passage has 'antiquum colorem,' and assigns it to the time of Fronto. Keller would put it as late as Ausonius (circ. 350 A. D.), hinting at Tetradius who is addressed in Auson. Ep. 15, 9, as rivalling Lucilius.[46]

F. Marx, whose beautiful emendation of these lines is often referred to in this paper, says that they are important for the history of grammar at Rome and for our

[42] 'vixit ad extremam senectutem.'

[43] *Dial. de Orat.* 23.

[44] *Inst. Or.* X. 1, 93.

[45] It would be hard to show that Horace's estimate of Lucilius was any lower than this.

[46] 'rudes Camenas qui Suessae praevenis aevoque cedis, non stilo.'

knowledge of the fate of Lucilius' poems. The whole passage, he insists, suggests the philologist and reviewer, who prefers Cato's edition of Lucilius to his master's earlier one. There is a vast difference between the points of view of Horace and the author of these interpolated lines: the former speaks of Lucilius himself and his works, the latter of editors and editions.

If it once be assumed that the words 'emendare parat' do not necessarily imply that these lines were written in Cato's lifetime, it is hard to say how late they may have been composed. Whatever their age, it is quite impossible to name their author.

The fragment—and it is only a fragment, for the promise in the first verse is not fulfilled—seems to have been transferred to this satire from some source rather than composed as an introduction to it, to explain and complete it. Apart from the fact that the general sentiment of the lines (so far as this can be discovered) is not in accord with that of the satire to which they are unnecessarily prefixed, it is hard to see what Horace had to do with Cato's alleged revision of Lucilius or with the savage treatment of the unfortunate youth referred to in verse 5. Keller and Holder say that the 'Urhandschrift' of their third class of mss. was older than Priscian, and so also this interpolation, adding, however, that while Priscian quotes the spurious lines prefixed to the Aeneid, these eight verses are not mentioned by any of the ancient commentators.

STATE BANK NOTES.

By W. M. HALL.

The proposal to restore the privilege of note-circulation to banks outside of the national bank system, by removing the practically prohibitory ten *per cent.* tax, is supported chiefly by the following doctrines:

I. That the probable extinction of the national bank circulation will leave a gap in the money-supply that must be filled by notes of some kind.

II. That a well-guarded system of state bank notes would give us an "elastic" circulation, *i. e.*, one that would increase with each high tide of business, and contract when business slackened.

III. That state bank notes would give a larger permanent money-circulation to parts of the country that are now scantily supplied with money.

IV. That the present prohibitory tax on state bank notes violates the spirit of the Constitution if not its letter, and is a dangerous encroachment upon State powers or individual liberty or both.

I. NOTES TO FILL A VACANCY.

The first of these doctrines could be summarily dismissed, in view of the well-known habits of the international flow of gold, except so far as the shrinkage of the whole money-supply of the world would affect the scale of prices a little; a shrinkage that can be avoided by other means than bank notes. Yet the recent experience of the United States with money is not only an illustration of the international flow, but it is worth examination because it offers striking and encouraging proof that the substitution of coin for national bank notes is not likely to be a painful process.

From the monthly estimates made by the Treasury Department are taken the following figures, showing the amount of each kind of money in circulation on July 1 of the year named. Money held by the banks is included, for the stock they keep does not far exceed (though it does in July somewhat exceed) a reasonable reserve, which is as much a part of the needs of ordinary business as is the reserve of five dollars or fifty below which the head of a family does not permit his cash in hand to fall. Money in the Treasury is not included, because much of it is held merely to redeem certificates that are circulating outside; and because there has been a widely varying amount there, the variations of which had an unbusinesslike origin. There would be no material difference in results, so far as the purpose of this paper is concerned, if the money in the Treasury, less the backing of certificates, were included. The figures in parentheses for the true amount of national bank notes are round numbers, estimated from the reports of the Comptroller of the Currency; this needs to be distinguished from the nominal amount, because notes of surrendered circulation, being no longer an obligation upon the banks that issued them, are really certificates payable by the United States.

MONEY IN CIRCULATION.
(Millions of dollars.)

	1879	1882	1885	1888	1890	1892
Gold* (including Gold Certificates)..	126	363	468	512	505	550
Silver Dollars (including Certificates)	9	87	141	256	360	384
Greenbacks and Legal Tender Certif.	302	325	331	308	335	342
Notes of 1890..........................	......	....	....	....	....	98
National Bank Notes, nominally.....	321	352	309	245	182	167
(National Bank Notes, really)..........	(310)	(315)	(270)	(155)	(125)	(140)
Subsidiary Silver.....................	67	52	44	50	54	62
Totals..........................	825	1,179	1,293	1,371	1,436	1,603

[Copper and nickel coins are disregarded; so is paper fractional currency, which was reckoned about 16 millions in 1879 and only about 7 millions since, including the amount in the Treasury. The figures of subsidiary silver for 1879 and 1882 are too large, through including trade dollars.]

*The figures for gold, after 1879, are often disputed as too large; and probably with good reason. But it will be seen (page 56) that allowance for a smaller amount does not vitiate the conclusions of this paper.

It may here be seen clearly how the whole volume of money has responded to the needs of increased business; the growth was rapid in the revival of business following 1879, and then fell to a much lower rate, averaging little over 30 millions annually from 1882 to 1890. But the fact which now most concerns us is that the needs of business were not provided for by the creation of bank notes, nor of any kind of notes, except the note-element in the silver dollars and the notes of 1890, and a small increase in the greenbacks outside of the Treasury. More than that, the nominal bank-note circulation was reduced in the thirteen years by 154 millions, and the true bank-note circulation by about 170 millions. If 1882 and 1892 be compared, the reduction in the ten years is 185 or 175 millions; a reduction greater than the amount now outstanding. That is, we have only to do once more just what we have done since 1882, and the whole of the national bank notes will be replaced by other money.

But the matter is not quite so simple, because we shall not do just what we have done since 1882. The effect of the Act of 1890 needs to be considered, and the effect of a possible repeal of that Act. Before weighing these, it is desirable to look closely at the nature of the past additions to the money-supply, in respect to their real cost. Their cost to the country is measured approximately by the export value which the gold and silver would have had if not used for money purposes here. The cost of coinage and storage, and other such minor corrections, may be disregarded, in view of the wide allowance for error that will be used in the inquiry, and of the unequivocal result. Of course the government purchases have steadily "bulled" the silver market; how much, it is not possible to know. Against whatever such enhancement of the price of silver there has been, acting as a diminution of the cost to the country (not the government only) of the silver used for money, there is a partial offset in the increase of cost to the country of all its money-metal, through the necessity of making slightly lower average prices for exported goods in order

to send them out in place of the metal withheld and thus maintain the equilibrium of foreign trade. But to put the result of the inquiry beyond suspicion, it may be prudent to allow for the one conspicuous effect through enhancement of price of silver; and figures taken both with and without that allowance will be limits between which the truth lies.

Evidently we must include in the cost not only the imported metal, but the metal produced here, so far as either has been used for money. And for this purpose we may better allow for the Treasury holdings also, because if there is an increase of metal there the country has bought it by exporting goods or by abstaining from importing them. The increase in gold used for money, either outright or through certificates, represents one large part of our expenditure to procure new money. The other large part is represented by the gold-value for export* of the silver in the added silver dollars, *plus* the silver bought by notes of 1890. The supply of silver dollars in the middle of 1879, including the Treasury stock, was 41 millions; in 1882, 123 millions; in 1892, 414 millions. In the figures for 1879 and 1882 are counted several millions of silver bullion, destined soon to become dollars; the much larger amount of silver bullion in 1892, and the much wider divergence of its coinage-value and cost-price, are cause for considering it separately below. The increase from 1879 to 1882 was 82 millions, which cost the government as bullion about 72 millions; the international market value was a little less, but we may neglect the error. The silver that made the increase of 291 millions of silver dollars, 1882 to 1892, was substantially all bought by the middle of 1891; it cost the government about 230 millions. Using this purchase-cost as the upper limit of what the silver really cost the country, we have yet to fix the lower limit suggested above. Higher price of silver, caused by government buying, affects for this purpose not only the silver thus withheld from export, but the silver still exported.

*This is the export-value from time to time; not the present export-value of the accumulated mass.

The change of price in silver that would in any case have been used at home does not sensibly affect the cost, being a mere readjustment of domestic exchanges. The correction applies, then, to the silver bought by government, added to the net silver-export still remaining (which may be a positive or negative quantity; the latter representing an import and consequent loss by the raised price). The net export of silver from 1879 to 1892 was worth about $100,000,000; from 1882 to 1892, about $80,000,000. The government purchases from 1879 to 1890 were about a quarter of the world's product, and since 1890 more than a third, and their effect on the price must have been considerable; but it seems liberal to set three-quarters of the actual price as the lower limit of the price as it might have been with no government purchases except for subsidiary coinage. On that scale, the goods received in exchange for the exported silver* of 1879–92 may, at the lower limit, have cost the country $25,000,000 less than their apparent cost: for the exported silver of 1882–92, $20,000,000 less. Any such gain diminishes the cost of our use of silver for money, and corresponding deductions are incorporated in the following table, where the cost of the silver bought by the Treasury appears separately, with the same three-quarters rule used to deduce a lower limit of true cost.

COST TO THE COUNTRY, IN GOLD.
(Millions of dollars.)

	1879–92		1882–92	
	Lower limit.	Upper limit.	Lower limit.	Upper limit.
Increase of Silver Dollars, 373 millions	226	302	......	......
Increase of Silver Dollars, 291 millions	......	......	172	230
Silver bought with 1890 notes, and not coined	58	77	58	77
Deduction for enhanced value of silver exported	−25	......	−20	......
Increase of Gold †	418	418	157	157
Whole cost	677	797	367	464
Average cost per year	52	61	37	46½

*It would be a needless refinement of the question to take account of the diminished home production and export, due to lower price.

†Including gold coin and bullion held by the Treasury. The official estimate is 246 millions in 1879, 507 in 1882, 661 in 1892.

That is to say, the country has given full value in goods and labor for somewhere between 677 and 797 millions of its increase of money-supply since 1879; and for between 367 and 464 millions of the increase since 1882.

The money-supply itself, outside of the Treasury, increased by 778 millions from 1879 to 1892 (see table, p. 46); but the decrease of national bank notes caused the increase of the other elements of the currency to be still larger. Gold increased 424 millions, silver dollars 375 millions, "greenbacks" 40 millions, and 98 millions of 1890 notes were added; making an increase, aside from national bank notes and pieces less than one dollar (which last would have shown about the same behavior under any system of major currency), of 937 millions. This last is the amount of money that has been added in thirteen years past to meet the needs of increased business and take the place of the declining bank note circulation. But we have seen that the country earned meanwhile, *i. e.*, bought with goods and labor for which it received nothing else in exchange, between 677 and 797 millions.* That is, the note-element† in the addition of 937 millions was between 140 and 260 millions. If 1882 and 1892 be compared, the whole addition of money other than national bank notes and small pieces will appear as 599 millions, and the note-

* This comparison of increase of money-supply outside the Treasury with increase of money-metal both within and without the Treasury may seem irrational. But the former is the true measure of past additions to the money-supply and the better basis for judging what future additions are probable, and hence what the strain of making them will be; while the sacrifice in former acquisitions is better measured by the addition of metal in Treasury and outside circulation together. If any one nevertheless prefers to compare outside circulation in both cases, he will find the increase of silver dollars (table, p. 46) from 1879 to 1892 to be 375 millions, 1890 notes 98 millions, gold 424 millions, and the resulting "lower and upper limits" about 700 and 825 millions; differing from the 677 and 797 millions, reckoned above, in the direction of decreasing the note-element in past acquisitions, and therefore of decreasing the sacrifice needed in future acquisitions that may contain a less note-element or none at all. That is, it would strengthen the conclusion that in the text above is based upon a less favorable supposition.

† Not the note-element reckoned upon the present bullion value of silver, but the unearned part of the issues of silver dollars and 1890 notes as they were made.

element as between 135 and 232 millions. The following figures show, accordingly, the average annual addition:

	Whole addition. (Millions.)	Note-element.
Annually, 1879 to 1892	72	Between 11 and 20
Annually, 1882 to 1892	60	Between 13½ and 23

And the country has earned (see, also, the table on p. 49) between 52 and 61 millions annually through the longer period, and between 37 and 46½ millions annually through the shorter.

It may safely be said that our probable dealing with silver in the next few years (omitting free coinage as too improbable in the immediate future to justify the discussion, necessarily long, of its bearing on the present question) will lie within a range bounded by—

(1) Continuance of the Act of 1890.

(2) Revival of the Act of 1878.

(3) Purchase of silver, and issue of notes whose silver backing, reckoned as bullion, is kept equal to the face value of the notes; kept equal by subsequent purchase of silver, if necessary, without issue of notes against the supplementary silver.

(4) Suspension of silver purchases, except for small coins.

It is quite possible that silver legislation may combine two of these methods, or change the amount of silver to be bought under (1) or (2). But the present object is to discover whether the national sacrifice in obtaining additional money will be greater hereafter than it has been for a few years past, and that object will be sufficiently attained by taking each method separately and noting the effect in (1), (2) and (3) of different amounts of silver-purchase; for any combination will be more favorable than the least favorable method standing alone.

(1.) *Continuance of the Act of 1890.* If this happens, the addition of money will be wholly earned, except for the "lower limit" purpose a note-element due to the higher price of the silver bought and the silver exported. The

silver bought is 54 million ounces annually, which is substantially the whole amount available for Treasury purchase or export.* The domestic production of silver is increasing by about 4 million ounces per year; but supposing (to keep on the less favorable side of probability†) that the amount exported in the next dozen years should average only 6 million ounces, while 54 millions were still bought, the amount through which the higher price could operate to diminish the true cost would be about 60 million ounces annually; worth $60,000,000 at one dollar, $45,000,000 at 75 cents. Accordingly the note-element, on the three-quarters scale, lies between zero and 15 millions in the improbable event of a rise of silver that carried it to average $1 an ounce, between zero and 11 millions if silver averaged 75 cents. A smaller government purchase would not change the quantity of silver affected, but it would of course bring the note-element nearer to the zero limit through affecting the price less.

(2.) *Revival of the Act of 1878.* Taking its minimum purchase of $24,000,000 worth of silver annually, the dollars coined would be, with silver at $1 an ounce, 31 millions; with silver at 75 cents, 41 millions. The seigniorage would thus be 7 and 17 millions at those prices respectively. The other part of the note-element, by the three-quarters rule, would be (as under the Act of 1890) between zero and 15 millions at the former price, between zero and 11 millions at the latter. The whole note-element is thus between 7 and 22 millions when silver is at one dollar an ounce, between 17 and 28 millions at 75 cents. Evidently the note-element is enlarged by increased purchases or by a fall of silver.

(3.) *Issue of notes with a constantly equivalent silver backing;* a backing kept equivalent, when the price of silver declines, by purchase of more silver without issue of notes

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*The net import was about 3 millions in the fiscal year 1891; net export 6 millions in 1892.

†A larger supposed export would increase the note-element and decrease the sacrifice.

against it. The question, highly important for other purposes, whether the notes are redeemable in gold or in a gold dollar's worth of silver, has no bearing on this discussion. In either case the note-element is between zero and a quarter of the export-price of 60 million ounces, *minus* the cost of silver bought in case of falling price to keep up the backing of notes issued earlier — that kind of purchase being cost without addition to the money-supply. It would be an extreme supposition that silver should fall to 60 cents an ounce in the next ten years; that would be about 2½ cents annually. Such a fall would require, if the annual purchase for note-issue were $24,000,000, a purchase of supplementary silver amounting to less than a million dollars in the first year, to 10 millions in the last year, but averaging about 5 millions annually through the ten years; while the note-element due to upholding the price of silver would average (with silver at an average of 72 cents) between zero and 11 millions. Deducting the 5 millions of cost for supplements, we have *minus* 5 and *plus* 6 millions as the limits of the note-element; that is, it might possibly be a more expensive way of obtaining new money than importing gold would be.* On the less extreme supposition of a fall of silver to 70 cents in ten years, the average annual supplementary purchase (the principal purchase being still 24 millions) would be something less than 3 millions, making the note-element somewhere between *minus* 3 and *plus* 9 millions.† The increased expense, in the later years, of maintaining such a note-system in case of a progressive fall of silver is of course a serious objection to the system, unless it is believed that silver will not continue to fall. If silver does not change in price, the note-element is the same as under the Act of 1890; with silver at 84 cents, it is between zero and 12½ millions. Greater purchases would

*A loss of this kind, payable in future, has already been incurred by the country through its large purchase of silver, if silver does not rise again, and if the notes of 1890 or silver dollars are ever given a 100 *per cent.* backing or are withdrawn and the silver sold.

†Silver then averages about 77 cents, and the note-element, aside from cost of supplements, is between zero and 11½ millions.

probably increase the note-element unless silver declined 2 cents or more yearly.

(4.) *Suspension of silver purchases, except for small coins.* This would leave the natural movement of gold to make the necessary increase of money, and would provide no note-element.

Tabulating the effects of these methods of dealing with silver, we have the note-element appearing as follows:

	NOTE-ELEMENT LIES BETWEEN—	EFFECT UPON THE NOTE-ELEMENT—	
		of increased purchases.	of cheaper silver.
Method (1), with Act of '90 unchanged; silver at 100	0 and 15 mill.	Increase.	Decrease.
Method (1), with Act of '90 unchanged; silver at 75	0 and 11 mill.		
Method (2), with annual purchase $24,000,000; silver at 100	7 and 22 mill.	Increase.	Increase.
Method (2), with annual purchase $24,000,000; silver at 75	17 and 28 mill.		
Method (3), with annual purchase (for note issue) $24,000,000; silver declining 2½ cents yearly	−5 and +6 mill.*	Uncertain.	Decrease.
Method (3), with purchase 24 mill.; silver declining 1½ cents	−3 and +9 mill.*	Uncertain.	Decrease.
Method (3), with purchase 24 mill.; no decline of silver	0 and 12½ mill.	Increase.	Decrease.
Method (4)	No note-element.		

The past annual increase of the whole money-supply, together with the money that replaced bank notes, has included a note-element lying somewhere between 11 and 23 millions (see page 51). If we continue to extinguish bank notes at the same rate, make no change in the amount of greenbacks, and increase the whole money-supply at the same rate as before, it appears from the table above that Method (2) would involve no appreciable decrease, perhaps an increase, in the note-element; that is, the money to serve the growing needs of trade and to take the place of disappearing notes would cost us not appreciably more, perhaps less, than it has in the recent past. Under Method (1) the national expense on this score would be say 8 to 12 millions more, annually, than it has been in the recent past.

*For ten years only, and as an average; the note-element being less than zero in the later years, and going further below after the ten years, if the fall of silver continued at anything like the same rate.

Under Method (4) it would be between 10 and 20 millions more. Under Method (3) it might in a very unfavorable case be 20 millions more, but with a slower decline of silver the added expense would be nearer the 12 millions or so (between 10 and 15) which would accompany a stationary price of silver. Under Method (3) this average cost through ten years represents a smaller cost in the earlier years and a heavier one in the later years, if silver falls. It is only under Method (3) that we have to anticipate a cost, for the average annual addition of money and extinction of bank notes, exceeding by more than about 15 millions the cost that has already become habitual. Method (3), if silver should fall rapidly, would be burdensome after a few years, particularly if the annual issue of notes were much more than the 24 millions reckoned in the table; but no one wishes to see that method adopted if silver is to fall rapidly; and in any case, the extinction of bank notes within a dozen years would contribute only 12 or 14 millions annually to the burden. Under any of the more probable forms of dealing with silver the sacrifice of the country, in extinguishing the bank notes while it increased the whole circulation as usual, would not exceed by more than about 15 millions the sacrifice that is already customary; and it might not exceed that at all. Remembering that in place of an annual cost of between 40 and 60 millions for the near future, which has been the implied basis of the present reckoning because it was the average cost for the past few years (see page 19), we might have for a part of the time, as in 1879–82, a cost of 100 millions attended by great commercial prosperity—remembering, too, that it takes more than an occasional waste of 20 or 30 millions by Congress to make an appreciable difference in the course of business—it seems unlikely that the withdrawal of all the national bank notes within ten years can give a sensible check to business. Indeed, the greatest expense for new money comes just at the time when the country can best afford it, in times of rapid growth of business; and just at the time when there is need of a check upon excessive speculation.

The conclusion just reached has so wide a margin of safety that it excuses the omission, for simplicity's sake, of a number of corrections; some offset each other, some are mere differences of degree of an element common to all the years (*e.g.* a deduction from the probably excessive Treasury estimate of gold coin in private hands), and the aggregate of the corrections can scarcely swell the difference between past and future sacrifice in the enlargement of the money-supply so as to call for the retention of national bank notes, or for the provision of any other notes to take their place. If there is any need for state bank notes, it is to be found elsewhere.

II.—ELASTICITY.

In popular discussion of the repeal of the bank-note tax it is often assumed, as something near an axiom, that a bank note system may readily be made "elastic," and it seems to be implied that the easier it is for banks to issue notes the more elastic the resulting currency will be. But among economists this is so far from being a generally accepted truth that some reputable writers deny the possibility of bank notes following the needs of trade, either in expansion or contraction (except in the same way that coin would have done), so long as the bank notes are really convertible, *i. e.* are promptly and willingly redeemed by the bank. If nevertheless we grant that an expansion is possible, it is reckless to assume, without careful examination, that the bank-note currency would contract again when trade slackened. And unless it does so contract, what we have is not elasticity but a wholly inelastic distensibility.

Entering first a protest against another too easy assumption, that elasticity is an unmixed good (for much may be said for the doctrine that the evil in it exceeds the good, through removing one of the barriers against speculative excitement), we have to inquire what are the causes that may limit a note-circulation, and whether any action of the State governments upon those causes can give an increase

or an elasticity that the National government cannot give by similar action.

A bank issues notes in order to increase its receipts of interest. In a country where notes are familiar, it can usually carry notes with a reserve smaller than that which it needs to carry deposits*: and accordingly, unless taxes or other expenses intervene, its loans that can be made through notes are more profitable than its loans through book-credits of deposit. But in the United States most borrowers prefer book-credits, and the practicable note-issue is restricted to so much as can be paid out, whether in loans or in other money-payments (*e.g.* upon checks presented), without coming back for redemption faster than it is reissued. Within that limit (beyond it, if bank notes were not convertible) the bank has a motive for keeping its circulation as large as possible, unless other expenses or hindrances appear. In the United States the limit so set would be narrow for the individual bank, on account of the large proportion of credit-transactions, except for the habitually long life of the circulating notes; in fact it is so wide that a limit for the whole note-circulation, drawn on the same scale, would be impossibly large. The real limit for the whole note-circulation is of course much lower, because as the whole amount approached a point considered to be dangerous, redemptions would become more frequent, *i. e.* the limit for individual banks would shrink. Banks do not grow weary of making a profit, nor stop issuing notes without a reason for stopping. Where the note-circulation falls short of the limit (usually wide, when banks are well managed) set by the return of notes for redemption, it is certain that there are definite causes for it, either prohibiting the increase of circulation or offsetting the profit by some expense, inconvenience or dread of injury. Including redemption, we may name the following restraints upon the issue of notes, some of which are

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*Strictly speaking, deposits which are caused directly or indirectly by the bank's loans. Other deposits have no bearing on the question whether loans by book-credit or by notes are more profitable.

found at work in every bank-note system, no matter how bad:

CHECKS UPON THE ISSUE OF BANK NOTES.

1. *Return of notes for redemption.* The effect of this important restraint has been outlined above. It is thoroughly effectual only when redemption is enforced by law and not discouraged by public opinion or by bank pressure (*e. g.* refusing discounts and other bank services to persons who have presented notes for redemption). But as we shall see hereafter, unless certain artificial means are used to compel the return of the notes within a given time, the action of this check does not prevent an increase from year to year till the issue is very large, if public confidence in the soundness of the banks prevails.

2. *Fear of discredit.* That is, a belief of the bank managers that further issue of its notes would impair the bank's credit, and either directly fail of its purpose by bringing back for redemption notes equal to the new issue, or injure the deposits and other business of the bank.

3. *Fear of general injury to business.* This may include one or more of the following elements: Fear of the effect on the bank's own business of a general derangement of business caused by bad currency; fear of the effect of such derangement on the interests of the managers outside the bank; and a sense of responsibility and trusteeship towards the business community. This check is seen at its best where a single large bank issues most of the bank notes of a country (the circulation of notes of other banks being prevented or kept within narrow limits by law); the Bank of England, for instance, would doubtless be temperate in the issue of notes, even if it were not hindered by law from making any profit by an increase of circulation. The Bank of France is influenced by a similar combined responsibility and prudence. But when there are many banks, each knows that its own note-issue will be only a small part of the whole note-circulation, and that its own abstention from new issues will have little effect

towards reducing the whole; perhaps no effect at all, because other banks will issue more instead before the more rapid return for redemption comes into play, or the legal maximum, if any, is reached. And if some banks are still conservative when the motive to be so is thus diminished, other banks will surely, in such a country as the United States, be more tempted by the profit than deterred by the possible future evil, which is made scarcely more probable by their venture; unless, indeed, the number and character of the banks be so limited by law that a joint agreement is practicable (a case which will be mentioned below). The point is, that we cannot look to this check, unless supplemented by an agreement among the banks, to prevent an immediate increase of bank circulation if the latter were made otherwise profitable; for some banks would surely grasp at the profit.

4. *Legal limitation of the number of note-issuing banks*, either by requiring a special legislative charter for each, or by imposing onerous conditions for going into business under general law. This check acts upon a part of the field only, of course; leaving the banks which come within the privilege free to increase circulation, except so far as other checks interfere.

5. *A legal maximum of bank note circulation.* This, if set low enough, is perfect insurance against an undesirable enlargement. Evidently, however, it allows no "elasticity" beyond the maximum, and does nothing to prevent the circulation rising to near the maximum, so that further elasticity is impossible unless that begins by decrease; and the decrease must then be due to other checks, and not to this, unless the maximum itself is made different for different times of year.

6. *A legal minimum of reserve or of securities.* So far as this minimum is greater than the amount of reserve or of securities which the banks would hold of their own accord, it is a check upon circulation by reducing the profit. In our national banks it takes the form of an amount of

bonds costing much more than the reserve that the banks would voluntarily hold, with only partial compensation for the difference in the low interest received on the bonds.

7. *Taxes on circulation;* another mode of reducing the profit. Under the present national bank law, not only does the tax on the notes of outside banks extinguish the profit and prevent their issue, but the tax upon national banks of one *per cent.* on circulation contributes materially to the well-known unprofitableness to many of them of their notes.

8. *Legal requirements hampering the ready circulation of notes.* For instance, that they shall be of large denominations only, that they shall bear interest (this cuts into the profit also), or that they shall be of cumbrous size. Requirements that they should be indorsed on passing, that they should be on some easily damaged kind of paper, that they should be presented for redemption within a certain time or lose part of their face value, would have similar effect.

9. *Agreement among banks to set limits similar to 5, 6 and 8.* In view of the number of American banks and the difficulty of making and enforcing such an agreement, it would be a waste of time to discuss this as a possible check upon future issues here; at least till it is seriously proposed to have a few large banks monopolize the bank-note issue.

Depreciation may in times of inconvertibility act as a check upon issue, but no one now proposes to have a system subject to enough depreciation to come within the range of that check. Another check, now practically out of the field, probably acted upon the bank-note circulation in America early in the century: scarcity of capital and high rewards for its use in other ways than banking. Some men who might have been attracted by the high returns of bank-note issue were still more attracted by other enterprises that would employ their time so fully as to make inconvenient the attempt to conduct a bank.

Turning back now to see what are the checks that keep our national bank note circulation comparatively small,

we observe at once that the checks numbered 2, 3, 5 and 9 have no effect upon it. There is no fear by any bank that its credit would suffer by issuing notes up to the amount permitted by law in view of its capital, no fear of general injury to business by the issue of another hundred millions or two*, no legal maximum of circulation, no agreement among the banks. Under No. 8 the omission to make the notes a general legal tender does not in practice restrict their circulation, nor would the banks issue any more notes if they were still permitted to use denominations less than five dollars; the exclusion of bank notes from the legal reserve of national banks acts towards encouraging the return of the notes for redemption, thus setting at work check No. 1. Contrariwise, the law has by its assimilation of bank notes to government notes, in size and general aspect, done a little towards promoting their ready circulation. No. 4, legal limitation of the number of note-issuing banks, appears in the form of "onerous conditions," such as a minimum of capital, government inspection, prohibition of real-estate loans and of all loans beyond a certain multiple of the reserve held; but this is more than offset, on the whole, by the advantage which national banks receive in public opinion of their probable soundness, and it would not prevent an increase of note-circulation if such circulation offered a profit. We have left, therefore, the checks numbered 1, 6 and 7: deducting No. 1, which is not a limit but a drag upon increase (making it slower without stopping it), the true causes of the smallness of the circulation are in Nos. 6 and 7, *i. e.*, (*a*) the requirement of deposit of bonds whose market value is much higher than the amount of notes issued against them, bonds which bear a lower interest than the bank's ordinary business yields and most of which depreciate (through the effect of the approach of maturity upon their premium)

* Provided it were gradual, and the annual purchases of silver by the Treasury were small. If brought face to face with the possibility of a large increase of bank notes not so guarded, banks would doubtless recognize the danger; but, as suggested under No. 3 above, only a part of the banks would probably hold back from issue.

before the bank can sell them, (*b*) the small deposit of cash for redemption purposes at Washington, (*c*) the one *per cent.* tax on the average note-circulation of the bank.* From many banks these take more than the whole profit of the notes, and the banks tolerate the loss as a sort of advertising expense, in order to keep the advantage of being known to be under government inspection. The decline of the note-circulation before 1890 shows that on the average the loss exceeded all the gains, including the advertisement, and the slight increase† since 1890 shows that for two years past the gains have by a very narrow margin exceeded the loss, taking all banks together. The failure of the note-circulation to be elastic is due to the expense and trouble of buying and depositing bonds to secure the temporary addition, and withdrawing and selling the bonds when the time for contraction comes. Unless the bonds are withdrawn, the bank has no motive to withdraw the added notes.

Since our note-circulation is now kept small by the unprofitableness to the banks of further increase (except the trifling present increase, which may at any time fade out by disappearance of the narrow margin of gain which causes it) the way to make room for either elasticity or permanent increase is to remove part at least of the causes of the unprofitableness; to relax the checks called 6 and 7 above. Public attention seems to be given to the proposal to abolish the ten *per cent.* national tax on state bank circulation; but room for increase may be had just as certainly, though not for so great an increase, by reducing or removing the one *per cent.* tax on national bank circulation, or reducing the required deposit of bonds, or permitting the substitution of other securities bearing higher interest.

* Minor expenses (like examiner's fees), so far as they are independent of the amount of circulation, may better be included under No. 4; and some which vary with the amount, such as the cost of redemption and reissue, are small enough to be neglected in this article.

† Increase in the true circulation. Notes of abandoned circulation, awaiting redemption by the Treasury, have no place in such a comparison as this; though the government reports of circulating money include them in the figures of bank notes, which thus deceptively show a decrease since 1890.

It would take little change of these restraints to cause a marked acceleration of the increase of national bank notes that is even now visible. National banks are as ready as any others to increase their circulation when the increase pays.

IMPERFECT CONTRACTION.

Now let us postpone all the difficulties of securing a sound state bank currency, and suppose that either by adopting a system of lightly taxed state bank notes or by loosening the restrictions upon national banks we make possible the issue of 100 millions more of secure bank notes. There is no way of making the issue possible but by making it profitable. If the checks were so lessened as to let the profit exceed them before the harvest-season* came, there is good reason to believe that a part of the extra notes thus made possible would not only be sent out but kept out without waiting for the crop-season, unless certain legal requirements, hitherto unusual, were imposed upon the banks or certain new conditions attached to the notes. If this occurred long enough before the crop-season to permit the swelling of the currency to affect the amount of coin (either by increasing export or diminishing import of money-metal), it would to that extent merely substitute notes for coin lost; the remainder of the "slack" would be taken up by the growth of business at the crop-season, and this remainder only would have any value towards giving elasticity, the former part being a needless and (if extensive or often repeated) an injurious enlargement of the bank-note circulation.

But we will take a case more favorable to the proposal of a bank note elasticity, and suppose no such wasted element in the new issue; suppose the checks to be so skillfully balanced against the profit that the latter does not emerge superior till the rate of short-time interest rises at harvest-time. The new notes then go out, and the first half of elasticity is thus displayed. But what assurance

* For simplicity, the autumnal moving of the crops is spoken of alone, here and later in the discussion; but the same arguments apply to the spring trade and to sporadic maxima of business.

have we that when the crop-season is past the excess of notes will be redeemed and disappear? The apparent answer is that with slackening of business the currency will be redundant, bank reserves will be overstocked. and the banks will have no profitable use for the excess of notes, so that redemptions will exceed reissues till the whole redundancy is withdrawn. But this is only an apparent answer. The redundancy after the crop season is not identical with the preceding increase of notes; it may be far less; it usually would be far less in the United States. The growth of use for money at crop-moving time is in this country a mere wave in an ascending slope, a slope ascending so rapidly that we seldom add less than 20 millions a year to our stock of money, and sometimes 100 millions; the use is never, except through unusual business depression, so low after the wave as before it. Almost always, therefore, if we provided elasticity by means of bank notes unattended by special appliances for forcing their subsequent withdrawal, and if the elasticity were wholly by bank notes and no increase of coin shared in it, the redundancy of money after the crop-season would be less than the increase of notes during the crop-season. Conceding that notes equivalent to the whole redundancy were redeemed and retired. what motive would the banks have for retiring the rest of the recently added notes? Reserves have sunk to their normal size, for that is what the disappearance of redundancy means; the banks have no more money than they want to use. As fast as notes are redeemed, it is then the bank's interest to reissue them, *i. e.* to use the notes in place of the money paid out in redeeming them. There is thus a part of the recently added note-supply that will remain in circulation. This process would be repeated at each maximum of business, and accordingly, without such special devices as will presently be mentioned, an elasticity provided by bank notes alone would cause an increase of the bank note currency from year to year.

But it may be that a part of the increase of money at crop-moving time is in coin. By taking the effect of the

mixture upon a period of several years as a whole, we may eliminate the causes of complication and find a practical certainty that the note-circulation would show a progressive increase. For all the effects upon the coin-supply that the occasional presence of more notes can have are upon one side, in the direction of making the coin-supply less than it would otherwise have been. The issue of more notes at the crop-season will check the rise of short-time interest and make money easier; in fact, that is what it is intended for. Easier money means a better maintenance of prices of goods, and consequently less encouragement of purchases here by foreign buyers and less tendency to start an import of gold or a diminution of its export. In short, the pressure at the crop-season, so far as it is now relieved by any retention or import of gold, would force less relief of that kind because the pressure itself would be less. This, in turn, is only a part of the more general statement that whenever there is any change in our money-stock of gold, the presence of recently added bank notes in the circulation tends to make the increase of gold less or the decrease more than it would otherwise have been; for if any given quantity of these bank notes had been absent, the tightness of the money-market would have been increased or its plethora diminished. Taking any period of years, we may rely upon it that effects of this kind will have happened while the extra bank notes of the crop-season were afloat, in which case bank notes will have taken the place of the coin expelled. And these changes will be cumulative, because the presence of bank notes always acts on that side when it acts at all.

MEANS OF ENFORCING CONTRACTION.

The importance of this practical certainty that the bank note supply will increase from year to year if the checks are so balanced against profit as to permit an easy increase of notes at every maximum of business, and if no special devices are used to force all the new notes back when business slackens, lies in its strongly commending to us the use of such special devices if we undertake to make an

elastic note-circulation. That a progressive increase of notes is, on the whole, an evil, most economists will agree. It is true that there is a gain by use of cheap money when it is good money; that is, by the less sacrifice of goods involved in obtaining it, the relative diminution of the earned part and increase of the note-part. But that seems to be more than offset by the risks that attend a steady increase of bank note money—sometimes a distant risk of an amount great enough to be depreciated, always a risk of corruption of popular judgment about money through suggestion of government paper as a resource at the first pinch, and usually an enhancement (through enlarging the number of issuing banks and spreading the habit of extra note-issues) of the greater risk of encouraging riotous speculation that attends all schemes for elasticity by bank notes.

As a special device to keep an elastic note-circulation from increasing progressively, the ingenious suggestion of the late John Jay Knox * may be considered first: that the notes added at times of increase should be of a different color from the others, and should bear interest after a certain date. This proposal takes advantage of the well known fact that notes bearing interest will not circulate as money except when money is very scarce; the holders naturally treat them as a sort of bond instead. Mr. Knox's species of notes would, accordingly, almost all drop out of circulation as money soon after the date when interest was to begin. If the notes bore a low rate of interest, they would find their way to the issuing bank before long; if a high rate, part of them would be kept by the holders as an investment. In the case of a high rate, indeed, the disappearance from circulation would begin before the date arrived, and perhaps so soon as to impair the intended elasticity of the note-circulation.

While trusting to the behavior of the note-holders is reasonably safe, it seems simpler and more certainly effectual in every case to provide a legal limit acting directly

* See the *Forum*, February, 1892.

upon the banks. Suppose, for instance, that the increase of the ordinary note-circulation is prevented by fixing a maximum for it as nearly as possible at the point where it now is, admitting hereafter no new issue of notes on the ordinary terms, except in place of such circulation withdrawn;* and that extra circulation be permitted on some such terms as these—the extra notes to be issued only during two or three specified months just before and during the crop-season, and the whole circulation of the bank to be reduced to its former dimensions by, say, January 15 following. The reduction, or its equivalent, may be made certain by requiring from the bank a deposit of money on January 15 in place of any excess of notes still outstanding; the government either to undertake the subsequent redemption itself, or to return the money to the bank as fast as the redemption proceeded. The money being kept idle by the government meanwhile, the effect on the money-supply of the country is the same as if the excess of notes had been wholly retired before January 15; and the contraction has begun some time before the date, as the bank, besides its actual redemption, began to accumulate money for the final alternative redemption or deposit with the government. The extra notes could then have precisely the same appearance as the ordinary notes, and have the same validity as between the bank and the holder; it would make no difference whether the notes redeemed and retired were the same individual notes that went out in the extra issue; all that is necessary is the retirement of a certain amount of the notes of that bank, drawn indifferently from the old and the new ones. The whole matter would be settled between the government and the bank, and no one else would be troubled with any discrimination among the notes he handles. A second period of possible issue may be provided for the "spring trade": but there should be at least one space of a month or two in each

*Giving preference to newly organized banks in allotting this substituted ordinary circulation; if any were left, old banks that wished to enlarge their ordinary circulation could have it.

year, and better two such spaces. when the extra circulation is wholly absent, so that it may have no opportunity to become permanent circulation and displace coin.* The hard-and-fast limiting of the ordinary note-circulation need not exclude new banks from the privilege of note-issue, for if there are not enough old banks withdrawing ordinary circulation to make room for them in the ordinary circulation, the new banks may nevertheless, by coming under the inspection-rules, be allowed to share in the extra issues.

The question how the extra notes should be secured is of some importance. If deposit of securities is required, it will be found to impair the readiness of the banks to issue extra notes; the nearer the formality and expense of making the issue approached that of the ordinary issue, the less elasticity the system would give. On the other hand, if the original security were no more than enough for the ordinary issue, it would not be enough for a perfect safeguard to the increased amount.† A first claim upon all the property of the bank, reinforced by a safety fund and perhaps by a personal liability of the stockholders, would be very strong, but might imaginably fail in some one or two instances at last; though the government might protect the note-holder by assuming that risk itself through guarantying the notes, or transfer it to the other banks in the system through making them responsible beyond their share of the safety fund. All these comments upon the mode of securing the notes apply equally to Mr. Knox's plan and to the second plan described. So do all the following questions: What securities should take the place of United States bonds when those are extinct; whether a maximum should be set to the

* *I. e.* displace coin progressively. There would doubtless be a slight displacement in the first year or two, and then the coin-supply would go on a little less than it would otherwise have been, but less by a nearly constant amount.

† But the deposit of securities for the ordinary circulation might be made to exceed the face-value of the ordinary notes by a margin large enough to cover a considerable extra circulation. Just now, national banks whose deposited bonds are 4 *per cents* could issue 30 *per cent.* more circulation without exceeding the market value of their bonds plus their redemption-deposit of cash.

extra notes also; whether all banks in the system should have the extra-note privilege, or banks in certain cities only; what features of the law should be left subject to variation by the Comptroller of the Currency or some other executive officer. There ought to be an arbitrary maximum of increase, lest in an occasional time of speculative temptation the note-issue should run wild; for serious harm might be done by a rapid increase long before it reached any automatic check by redemption. Both of the plans need free criticism and working out in detail by practical bankers before either can be trusted; but the point I wish to urge is, that some such special provision must be made to insure a return of the note-circulation to its former volume, or it will probably increase.

In any bank note system, the whole volume of notes below the lowest point reached in the duller times is dead and useless for purposes of elasticity. It is only the flow and ebb that give the elasticity; if the notes never fall below 100 millions, that 100 millions might as well be coin.* We may accordingly, if we please, rid ourselves of practically all the present ordinary note-circulation, substitute coin for it, and still have the desired elasticity by using the extra notes of one of the two plans described above. Of course the reduction ought to be made gradually, and by the weight of conditions that do not affect the extra notes, *e. g.*, by a higher tax.

OBJECTIONS TO STATE BANK NOTES.

It is hard to see why the national bank system, as such, should be blamed for its failure to be elastic. Any state bank system must be subject to the same natural laws. The transfer of supervision to the State governments makes no change whatever in the rule that the increase or decrease of note-circulation will be determined by the excess of the profits over the checks or of the checks over the profits; that rule is, from the nature of the case, unchange-

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* Except that the change might introduce some additional difficulty into the arrangements for putting good security behind the extra notes, and the notes might become unfamiliar enough to be a little less convenient as money.

able. All that can be done is to vary the nature of the checks; and wherever that can be done by the state governments it can be done by the national government. If it is desired to increase the note-circulation, that end is reached as certainly by removing taxes from national bank circulation as by removing them from state bank circulation. If it is desired to make temporary contractions easier, by reducing the expénse and trouble of withdrawing deposited bonds and redeeming the notes, or to adopt the Knox plan or the second plan described in this article, every detail of the improved process can be applied by the national government as easily as by the state governments, and indeed with less expense. Anything that would be dangerous in a national bank note system would be as dangerous (and usually more dangerous) in a state bank note system. And any imaginable gain under state supervision can be had as well or better under national supervision.

The difficulty of framing a good system of bank-note law and supervision by forty-four distinct legislatures needs no detailed discussion, unless by a competent humorist. There is no hope that a jumble of various state-systems would "average up" into a good bank-note circulation. If in ten states, five states, one state, the system is bad, there will appear bank notes imperfectly guarded by securities or dependent upon too small a reserve, in which case it is only a question of time when some bank will let its notes go to protest; to say nothing of the moral certainty that the wish of the legislatures to see the new system well started would cause the checks upon profits to be set too low, so that (taking the country through) there would be a large unnecessary original issue of notes, coming before any need of trade called for them, and merely crowding out better money.* In whatever states the notes were insecure, those states would suffer annoyance and occasional loss by circulation of such notes; some of the notes

*This point has not received the attention it deserves. Its practical importance is great.

would pass into other states and carry annoyance with them; and the repute of good notes would be injured outside of the state of their origin, because many men would not take the trouble to remember which states were sound, and would look with distrust upon any notes of a distant state, perhaps of any state but their own. Unless all the states agreed upon the same paper and similar printing, the detection of counterfeits would once more become a fine art, and the unskilled man and woman would be daily exposed to loss. It is not enough to answer that there is no danger of the return of the absurd and dangerous bank money of the first half of the century; if there is any failure to redeem notes, any discount upon some notes of distant origin or of tarnished fame, the opportunity of petty fraud upon the ignorant and careless is thrown wide open, and every man must choose between possible loss and the vexatious precaution of examining every note he receives. Moreover, the workman in taking wages, the retail dealer in taking payment from a customer he is anxious to retain, is often reluctant to give offense by objecting to money on the mere chance that it is depreciated, and will sometimes take it when he knows there will be a small depreciation, rather than raise the objection. The presence in any money system of elements that are distrusted, that must be looked for and chaffered over or subject the recipient to loss, is so wearisome an addition to the friction of trade that a clear case of beneficence in other directions (and no small beneficence) must be shown to give such kinds of money any claim to consideration. And when such beneficence as state bank notes are capable of can be had without the friction and annoyance by using national bank notes instead, the attempt to substitute the former seems a ludicrous folly. Even if we imagine the prodigy of wise concurrent action of all the states at first, what guaranty is there against the appearance from time to time hereafter of those legislatures whose pride it is to despise experience, to brush aside the sophistries of prejudiced conservatives, and to open short cuts to

prosperity for the oppressed plain people? Many of the advocates of restoration of state bank notes see the difficulties distinctly enough to propose that there shall still be some national restriction and supervision. The best that can be said of such plans is that a safe built partly of iron and partly of wood will resist fire better than a safe built wholly of wood. The advocates of an admixture of wood may fairly be asked what good is expected from the change.

If we are to attempt elasticity, then, common prudence requires the preference of national bank notes to state bank notes; for any tolerable system of so-called state bank notes must include national control so nearly complete that it is really a national bank note system.

III.—INCREASE OF SCANTY LOCAL CIRCULATION.

So far as it deals with the increased demand at crop-moving time, this argument for state bank notes is of course mere repetition of the argument from elasticity; but a separate place may be given to it because some persons have seriously urged that the permanent stock of money could thus be increased in the regions where a scarcity is now felt. Taken in this sense, even, it is completely answered by the answer to the elasticity argument; for if there is any way by which state bank notes could give a larger permanent money circulation to parts of the country, the same way can be provided for national bank notes by national law and supervision. And if this were not so, the proposed advantage of a larger local circulation is imaginary. Money is sent away from districts where capital is scanty because goods are so valuable to the farmer, the trader and the manufacturer, that they prefer having more goods to keeping a comfortable cash reserve in hand or in bank; indeed they usually borrow besides. Double the money in such a district, and within two months there would be little more than at first, if it were money that could be used in outside payments; in the long run there

would scarcely be more, unless it depreciated, than if the addition had never been made. A district within a country can no more enlarge its permanent circulation by issuing paper than the country itself can; whatever nominal enlargement may appear will be caused by depreciation. Even a nominal permanent enlargement could be had only through notes that were objectionable outside and were issued in such volume as to exceed* the formerly normal amount of money in the district; until they reached that volume they would merely displace other money. With a circulation thus enlarged, the district would enjoy all the blessings of a depreciated paper currency and a varying rate of exchange with the outside world; and unless the district had a well defined frontier line, which is improbable, there would be a strip of country all around it where the notes would be the cause of daily disputes and frauds. If we want this state of things, we can have it; but we can give the local banks free rein just as easily by national law as by state law.

IV.—EVILS OF THE TEN PER CENT. TAX.

The objection to the tax on state bank circulation appears in two forms. One of these, urging that individual liberty is unfairly dealt with by a prohibitory tax on bank notes that fail to conform to legally set rules, must be the product of hasty writing without second thought. Surely the creation of money is, like marriage, "affected by a public interest," and subject to the right of the whole community to protect its interest by prescribing conditions for every such transaction. Does any one (except those who follow their truth wherever it leads, with a Tolstoi-like disregard of consequences and of other truths) propose that every mushroom bank or factory store shall be left free to circulate its notes or "orders" as money where it can? If not, some power must prohibit, and prac-

*Strictly, of course, allowance must be made for the scattered remains of the former money, and conversely for the increasing needs of trade. And to cause the inflation the notes must become practically inconvertible.

tically the power must be either the Nation or the State. Here rises the other form of the argument: that the prohibitory tax goes beyond the constitutional powers of Congress. Such a claim can be maintained only as a protest against the decision of the Supreme Court, like the still existing dissent from the Legal Tender decision. There is no room for the claim of "moral" unconstitutionality, such as there might have been if the Court had merely sustained the law in its formal aspect of laying a tax, for the decision was not reached by ignoring the prohibitory nature of the tax, but by affirming the power to prohibit. The words are: "Congress may restrain by suitable enactments the circulation as money of any notes not issued under its own authority." Ought Congress to retreat before constitutional scruples which have (to state the case mildly) nothing near unanimity of legal opinion to support them and have already been overruled by the Supreme Court, scruples which at best affect merely the question whether Congress or the State legislatures shall use a power that one or the other must use; when the abandonment of the field by Congress would expose the country to the inevitable evils of disjointed management of note-circulation by the separate States? If any one holds that decision of the Supreme Court to be erroneous, he might better aim to cure the error by a constitutional amendment than to put upon the country the needless and ridiculous embarrassments that state bank paper would inflict.

Between national bank notes and state bank notes, then, the choice seems too easy to be called a problem. The real bank-note question is whether we need any bank notes at all; and if we do, how, under national control, they can be kept secure and their volume can be made elastic. The latter part of the subject has had a fairly full public discussion, but too little attention has been given to the possibility of doing without bank notes. That the withdrawal of the present stock of bank notes would not probably be

a troublesome process, has been shown in the earlier part of this paper. And if it is thought necessary to secure for the money-supply an elasticity of volume, in addition to the existing virtual elasticity by change of speed in circulation and by additions of metal, it is possible to have

ELASTICITY WITHOUT NOTES.

Our choice of means is not restricted to notes. The Treasury itself is a ready-made apparatus. Whenever Treasury receipts exceed Treasury payments, the circulation outside contracts;* whenever the payments predominate, the circulation enlarges. When such a change comes without any corresponding need of trade, the disparity is set right by the usual automatic methods, *i. e.* by a quickening of the average speed of circulation or an increase of bank reserves in the cases of needless contraction and expansion respectively, followed (if the disparity is large enough and lasts long enough) by a change in the rate of short-time interest and ultimately by a change in the export or import of metal. When the expansion or contraction happens to fit a need of trade, we have a true elasticity imparted by the Treasury action. In fact, several important outpours and absorptions of money by the Treasury occur every year; if they could be made to fit the needs of trade better than they do now, the circulation would be made elastic *pro tanto*. For instance, nearly all the interest on the public debt is paid in April and the corresponding quarter months, now that the interest-bearing debt is mostly the 1907 four *per cents*. If the interest were made semi-annual, and payable in March and September, about 11 millions† of interest-money would be cast into the channels of business at crop-moving time, and another 11 millions in time for the spring trade, while there would be a corresponding contracting influence at work through

*Contracts relatively; that is, if greater forces are just then expanding it, Treasury action merely diminishes the expansion. The word "enlarges," in the next phrase, is used in the same relative sense, though the enlargement is usually absolute also.

†Instead of the present arrangement of 5½ millions in October, and 5½ more in January, when it is not wanted.

the summer and the winter, when it is wanted. Again, the pensions are paid quarterly, but are distributed in groups upon different sets of quarter-days. These might be so grouped, probably without serious inconvenience to the pensioners,* that the payments in two months of spring should be about twice as large and in two months of autumn about three times as large as at other times. With pensions amounting to 160 or 180 millions annually, such an arrangement could be made to "plump" an extra 30 or 40 millions in the autumn and half of that in the spring. Without recommending these particular methods, one may take them as illustrating the possibility of such adjustments. Other means, some of which are applied by the Treasury, are the purchase of bonds at the end of summer, the increase of Treasury deposits with the banks, the arrangement of contracts so as to make the principal payments for supplies in the autumn and spring.

It is a fair question for debate whether this paternal-looking behavior of the Treasury would be good policy in the long run. The objection to it as government interference is sound. But as compared with the present Treasury methods, it would be only a better arrangement of ebbs and flows that exist already in considerable degree and often at the wrong times.† Whatever evils it has, the Treasury method of obtaining elasticity is perhaps better than any known bank-note system, and is far better than any possible state bank system. In the long run, among fallible men, expensive money is the most economical; money that cannot be duplicated without a liberal expense of labor; metal, or certificates known to have their full equivalent of metal behind them. In the sacrifice a country necessarily makes to obtain such money, as compared

*Crudely, by making a part of the pensions semi-annual, with the autumnal payment larger than the other; an inequality that fits tolerably well the needs of indigent pensioners.

†See illustrative figures in Mr. Kinley's article on the Independent Treasury in the *Annals of the American Academy of Political and Social Science*, September, 1892. The figures are for operations at New York, but are a sufficient indication of what happens in the whole country.

with watering it by a note-element, it is buying business security and a charm against popular delusions. Something might be said, even if no project for elasticity through Treasury payments is accepted, for the belief that our best policy is to prefer safety to elasticity, to arrange the checks on profit so as to insure the gradual extinction of the bank notes,* to substitute nothing for them, and so let coin flow in; when that is done, to increase the coin reserve against greenbacks by 15 or 20 millions annually till it equals the whole volume of greenbacks and they can be converted into coin certificates. When a generation has grown up that has never seen a bank note or a government note, the seed of many a folly will be dead in the popular mind.

The principal conclusions we have reached may be summarized as follows:

1. The extinction of the national bank notes requires no creation of other money. The coin to take their place will come in without perceptibly greater national sacrifice than has attended the extinction of a still larger amount of notes in the past few years.

2. An elastic bank note system would probably cause progressive increase in the amount of note-circulation, unless restraints hitherto unusual were applied. Such restraint might be given by causing the notes added in the temporary expansions to bear interest after a set time, or by expressly requiring the issuing banks to reduce their circulation within the old limits before a set time.

3. Nothing is to be gained, either towards elasticity or towards permanent enlargement of local circulation, by substitution of state control for national control. The increase or diminution of bank notes is determined by the relation between the profits and certain well defined checks, and the state governments cannot apply the checks better than the national government can.

* Which of course does not imply a disappearance of the national bank system.

4. The relegation of control to the states would almost certainly cause the checks to be set too low at first, and cause a large initial increase of notes.

5. Insecure notes from a single state would diminish the practical convenience of the whole note-circulation.

6. Security of all the notes under separate state management is almost incredibly improbable; and if attained in one year, might easily fail in the next.

7. The prohibition, by tax or otherwise, of circulating notes that fail to conform with national law, is not an injustice to individuals. The objection on constitutional grounds touches only the question whether state or national legislatures shall impose the necessary restrictions; and if sound at all, is rather an argument for a constitutional amendment than for a bad money system.

8. A means of elasticity, probably safer than any bank note system, exists in the irregularity of Treasury expenses.

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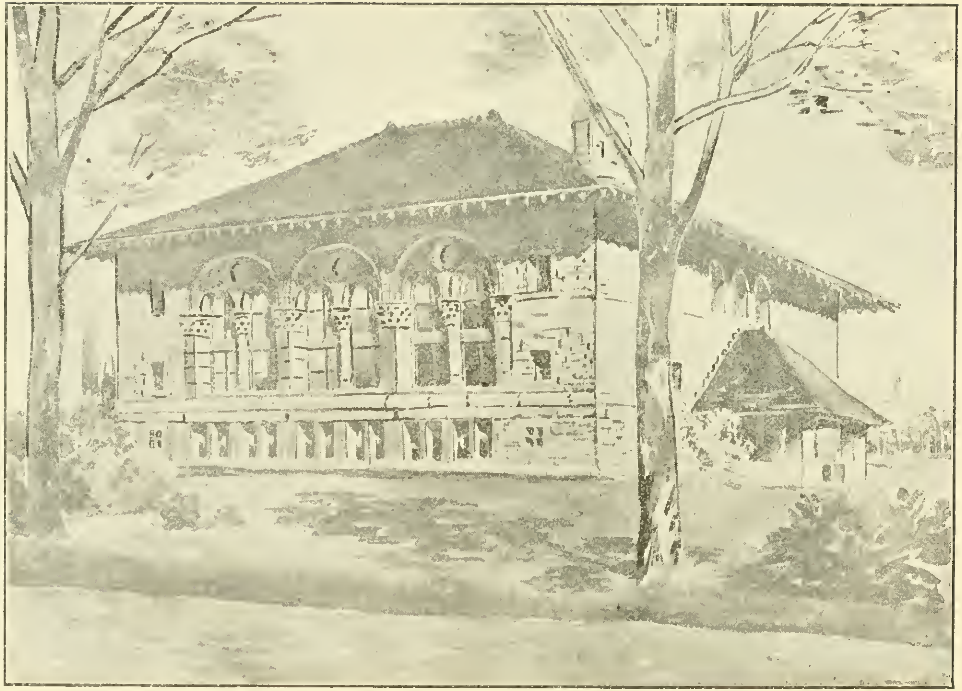

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THE ETHICAL PROBLEM OF THE PUBLIC SCHOOLS.*

BY WILLIAM FREDERICK SLOCUM, JR.

The political instincts of the people of the United States have led them to seek the best possible system of public schools, and the supreme motive for the expenditure of the vast sums of money that have been voted with great willingness for their foundation and their continued support has been the education of the youth of the country for citizenship. The final test of all citizenship must be an ethical one; and especially is this true in a democracy where the stability of its life depends upon the character of its citizens. With this fact in view, it is pertinent to ask whether the public schools are fulfilling the mission for which they were founded.

There has been for some time an increasing interest in the moral aspect of the public school problem. One indication of this is seen in the appearance during the last two years of seven rather notable text-books upon ethics, especially designed for schools of lower grade. The question that is now asked, however, does not find its answer in any reply given to the query raised as to the wisdom of publishing these books, for it seeks to go behind the inquiry, Should ethics be taught at all to boys and girls of the age of those in the public schools? It asks whether the problem of public morals is involved in the very nature of the system as such.

No one denies that the education of the thirteen million children in these schools has much to do with the destiny of the republic, nor that the country has placed its future, for good or evil, in the hands of the public school teacher.

The church may have the capacity for the moral training of the youth of the country; but, great as is its influence, the ethico-religious movement is not at present far reaching

* Reprinted from THE ATLANTIC MONTHLY, May, 1894.

enough to fashion even the majority of these thirteen million pupils into citizens in whom righteousness shall be the controlling element; and there is no reason for thinking that it will be in the immediate future.

The home comes much nearer meeting the need; but doubtless Mr. G. H. Palmer's statement is correct, in his article Can Moral Conduct be Taught in Schools? "The home," he says, "which has hitherto been the fundamental agency for fostering morality in the young, is just now in sore need of repair. We can no longer depend upon it alone for moral guardianship. It must be supplemented, possibly reconstructed." It still does, and always will, train the choice few for leadership; but after enumerating the homes in which the best that was in Puritanism still is the controlling element, and those that develop morality by means of the self-respect engendered by intellectual and æsthetic culture,—in fact, all those in which high ideals predominate,—there is still left a vast number where self-seeking is the main principle of life. If to the number of children in these latter homes are added the thousands who exist with scarcely any trace of home life to shelter them, we shall be forced to admit that there would be a moral crisis if the public school were not doing its beneficent work.

The question still awaits us, however, What is the public school system achieving for public morals?

Just at present there is a movement in various quarters to introduce instruction in the theory of morals into even the lower grades of the schools; but no one seems to be sure that this will not produce self-conscious prigs, or encourage morbid introspection rather than sturdy morality. But all are agreed that it is the function of the public schools—not to say of all schools, for that matter—to produce what some one calls "unconscious rectitude" in these thirteen million children. All appear to believe that development of morality is essential, and few that the teaching of mere ethical theories will be of much value.

The problem involves, then, the study of the system as a system from the standpoint of practical morality, to see if it

is a moral force in and of itself. Its power for righteousness depends upon what it is by virtue of its plan, purpose, and scope; upon its spirit, genius, and the manner in which it is realizing the ideal that has brought it into existence.

It is not possible at present to make a comprehensive and accurate study of the moral value of the public school system. The method of examination must be inductive, and the conditions vary so greatly in different communities that it is exceedingly difficult to reach conclusions that are drawn from a sufficiently large number of facts to make one's deductions satisfactory. The literature upon the subject, and in fact upon the general subject of the public schools, especially from a sociological and economic point of view, is exceedingly meagre. A good illustration of this point is the article in the ninth edition of the Encyclopedia Britannica, where, in over one hundred of its large and closely printed pages upon the United States, less than quarter of a page is devoted to this institution, and even what is written is of no special value. Such papers as the articles of Dr. Rice which have lately appeared in *The Forum* will furnish the basis of other work, and encouragement should be given to such critical examinations of the system; but much more work of this nature must be done before a comprehensive and discriminating thesis can be written upon the real influence of the public schools upon the morals of the country.

Certain conclusions, however, in regard to their power can be reached, and these ought to be stated in an article attempting to give a judicial opinion of their ethical merits. First there should be indicated the points both of direct and of indirect ethical value, and then the lines of weakness or of positive failure.

Modern psychology, leading to the study of the objective manifestations of mind, tells us that "habit covers a very large part of life;" that instincts are simply habits to which there is an innate tendency; and that these habits are due to what is characterized as the "plasticity" of brain matter to outward influences. Whether, for example, as one of our distinguished writers upon psycho-physics has told us, the

habit of putting one's hands into one's pockets is mechanically nothing but the reflex discharge, or not, the fact remains that "the walking bundle of habits of later years" does spin his fate for good or evil in that plastic state which covers the time when the child is usually a pupil in the public schools. If this is true, there is reason for saying that there is ethical value in the systematic order and discipline that are found in the majority of these schools. The constant and punctual attendance, the orderly arrangement of pupils, together with strict requirements in connection with these matters, fit one for successful business life, and create a sense of responsibility in regard to the use of time. The system of the public schools tends to make the pupil systematic, and helps to produce the accurate and methodical man or woman of later years. The testimony in regard to this is incontrovertible.

More than this, however, there is ethical value in the very conception from which the movement started, and the idea along which it has developed. The notion of self-improvement for a high end has in itself moral worth; for it demands that the youth of the country shall be upright not only because excellence of character is a good in itself, but because it promotes the good of the state. The expenditure of such a large proportion of the public revenues, the erection of so many buildings, the employment of such large numbers of high-minded persons, the creation and constant support of such an elaborate scheme, for the one purpose of producing good citizens, are object lessons that must have great influence upon the public.

What has been said indicates some of the lines in which the schools exert a direct moral influence; but in addition to this a large amount of testimony shows that, especially where there is a compulsory school law, a sense of responsibility has been developed in parents, making them recognize their own obligations. This, the reflex influence of the public schools upon the communities in which the system is at its best, is shown in many ways. Parents whose education has been meagre and faulty have become learners themselves, and have been led for the first time to consider seriously the

duties and future of their children; and this thought for the welfare of others has had a wholesome reaction upon their own lives.

In naming the elements that give moral value to the public schools mention should be made of the indirect good accomplished by keeping large numbers of children from the haphazard companionship of the streets, and from idleness and degrading influences. Especially in the larger towns and cities has this been true. To this negative protective good should be added the positive advantage derived from the acquisition of habits of neatness, personal cleanliness, and, in many schools, good manners.

After enumerating these things, which are more or less incidental to the system, and others that might and ought to be considered, it still remains to be said that the greatest ethical value in the public school system is, and must ever be, the intellectual work that is accomplished by it. There can be no doubt that there is a great amount of teaching that is not only unmoral, but positively immoral, in its direct or indirect influence. Recent publications demonstrate this fact, and show that the public schools will be at their best as a moral force when their work is thoroughly scientific.

Their success, then, in achieving the purpose for which they have been created depends primarily upon the character of the instruction that is given in them. It may be true that "pupils will not learn their lessons in arithmetic if they have not already made some progress in concentration, in self-forgetfulness, in acceptance of duty;" but it is equally true that mental exactitude and thoroughness of work, under the influence of a teacher whose method is scientific and whose spirit is earnest, will develop the elements that produce concentration, self-forgetfulness, and dutifulness. The tendency of mechanical, unscientific instruction is towards immorality. Schools that are under the control of selfish officials, with incompetent supervision and antiquated methods of teaching, have no power to quicken those springs of action which are the sources of morality. On the other hand, ethical capacity and moral strength can and ought to be produced by a high-

minded instructor in and through the very process of teaching arithmetic, grammar, and geography. Mental activity and intellectual self-respect are important factors in the truest morality. Habits of attention and observation may be developed into self-control, and the power of judgment into capacity for distinguishing between right and wrong. The ability to hold one's self uninterruptedly to any task may be power for resisting wrong or for the performance of duty.

In this connection mention should be made of a certain force of character which may be produced by the element of continuity in the courses of study through which the pupils are required to pass. So far as these are fitted to the normal, natural method of mental growth in the pupil they have ethical value. Obedience to the laws of mental development is essential to the highest type of manhood, and abnormal, restricted, unnatural mental growth is apt to produce immorality.

The things that have been mentioned lie on the hopeful side of this problem, and on the whole they make the outlook encouraging. They lead, however, to the question, How can an institution that is fraught with so much good, and which is necessary to the life of the state, be still further improved, and how can certain evils within it be eradicated? To do a little in the effort to answer this question, and also that this statement of the moral problem of the public schools may not be one-sided, an examination must be made of the evils that at least modify their usefulness.

Dr. Rice says, in his last article on Our Public School System: "One half the work of placing the schools upon a healthful foundation has been accomplished when the members of the boards of education become endowed with the desire to improve the schools." To accept as final the opinion that they are perfect always results in the evil elements becoming conspicuous. The most dangerous official is the one who regards no criticism as valid simply because it is uttered against the public schools. Neglect of such an essential institution is not worse than bigoted satisfaction with it and all that pertains to it.

The pride of its friends is that it is a great *system* of education. Mention has already been made of the value of the element of continuity in a course of study, but there is also a difficulty connected with it that cannot be ignored. The fixed schedule of study is fixed for all; the long courses are, with few exceptions, unmodified for the slow or the quick minds. The only reply the writer has been able to secure to the question, "*What can be done to remedy this?*" has been, "There is no escape from it, except in a few cases where very unusually bright children are promoted more rapidly than the others." The time taken for many children of more than average ability to complete a subject is unreasonably long; but the nature of the child must bend to the system, the system little or not at all to the peculiarities of the pupil. Now, nothing is more important, in creating and preserving "unconscious rectitude," than the element of spontaneity, and there can be no doubt that many children who pass through the long years spent in the public schools lose in this respect rather than gain. The kindergarten is obviating this danger somewhat; but wherever there is a suppressed mental life there must exist, in some degree at least, a suppressed moral nature: there is a logical connection between the inflexible system that holds a responsive, sensitive child in its grasp for years, and mental reactions that too often develop into moral weakness, and occasionally into vice. This tendency is, no doubt, not entirely the fault of the system, as a hard-and-fast system, but in a large degree of those unscientific methods which merely tax the memory, stunt rather than develop the reasoning faculty, and usually make the child unhappy, and sometimes morbid. President Eliot has shown that there is a waste of time in the student life by keeping pupils too long on subjects that should be covered in a much shorter period. But this loss of time has a more important bearing than the one which he considers. The attempt to save time is important: the attempt to save the moral nature is far more important. The destruction of interest and enthusiasm in a child has more than an intellectual significance: it interferes as well with his moral development. If one believes that there are certain definite

laws for the growth of the soul, which have been discovered by the world's great teachers, he ought also to believe that the violation of these laws in the training of children must react on the moral as well as on the mental life of those who can least afford to pay the penalty. The destruction of individuality brutalizes a nature, and there is constant danger of this where mere system is conspicuous and becomes the controlling element. It is exceedingly difficult for an instructor to hold the interest and develop the enthusiasm of a pupil after an appropriate amount of time has been given to any one subject; and although it is true that the teacher is the most important factor in connection with the system, and that sing-song recitations and pure memorizing will, under any condition of affairs, produce unscientific results, yet the best teacher is influenced by the system under which he teaches. There can be no doubt that many children who pass through the long years of continuous school life lose in some degree the quality of spontaneity, and that the loss of it is accountable for the lack of some of those finer sentiments that have always been the glory and the beauty of human life.

No discussion of the moral problems of the public school system would be satisfactory if reference were not made to what has, perhaps somewhat exaggeratedly, been called "the pauperizing tendency of the public school system." Free tuition has led to free text-books, until the principle has been clearly laid down that the state must furnish, without charge, to all its children whatever education they desire. Especially in the West has this been carried to its logical extreme, and the state university is asked to provide the highest special education not only without charge for tuition, use of buildings and apparatus, but in some cases with free rooms that are furnished and warmed at the expense of the state. In other words, it is claimed that no money equivalent should be given for the benefit received and the service rendered. Parent and pupil can take from the state, but, except what the pupil may return through his better preparation for citizenship, nothing is to be given for that which has been bestowed; and with large numbers of persons there is no

sense of obligation whatever in the matter. It is said by those who oppose the extreme form which this theory has taken that it carries the paternal feature of government to a dangerous extent; that it makes the citizen selfish and grasping; that it may, and in many cases undoubtedly does, minister to that spirit which characterizes much of our American life,—the spirit that ever asks, What shall we have? and seldom, What shall we give? and which is the bane of our present social order. It is further claimed that the results of this are already apparent in our national life; that the spirit which made our pension system is encouraged and developed by the "pauperizing tendency in the public school system."

Although it has been difficult to secure accurate information in regard to the results of this "free element" in education, it has become only too evident that many parents look upon the teachers as if they were servants; demanding everything from the school without any idea that they owe anything in return. Such facts as these—and there are many others which might be cited—indicate some of the evil results of the plan, and make it very clear that here is an actual danger to the higher ethical conditions. We should carefully guard our national life at this point.

There seems to be no escape from this free element and its logical results. All that can be done is to ward off the possible danger by constantly holding before the pupils the idea that they must repay the state in good citizenship.

Impurity may not be a greater evil in public than in private schools; but there are certain conditions in the democratic commingling of children in the former which make it more than a possible evil. There can be little or no social distinction except that growing out of the location of the school buildings. There is the "up town" and the "down town" school; but if a pupil is admitted into the schools at all, there can be no law requiring him to be in one building rather than in another, except the regulation that arises from residence in a particular locality; and even this is not enforced in some cities and towns. The very idea of the public school makes any classification upon social and ethical grounds an

impossibility. There are localities where this evil of impurity is nothing more than a potential danger; but there are very many others where it is a real evil. On the part of teachers there is a growing intelligence concerning it, and a greater vigilance in guarding against it. Those who do realize its enormity, and meet it aright, have secured results that ought to encourage all others; but there should be a most stringent requirement in this matter in defining the teacher's duties. In some of the best normal schools the students have the plainest and clearest instruction upon this subject. They are told the habits for which they are to watch, and the best ways to meet the evil of impurity in whatever form it is present among children. But such preparation is far from universal. Not many years ago, a graduate of one of these schools said that the teacher who gave her class instruction on this subject asked its members how many of them had not known of at least the existence of a vile vocabulary among their schoolmates. All but two of the large class replied that during their early life in the public schools they had heard what they could never forget, though no words could express the longing they felt to blot it from their memories; and in looking back from their more mature standpoint, it seemed to them that the teachers must have felt no special duty in the matter. These were young women from the public schools of one of the older States. There is no doubt, however, that each year our public school teachers have an increasing sense of responsibility for purity in thought and word of the children under their care.

The difficulties with which they have to contend are very great. The two or three children who, with an air of mystery, bring information in regard to forms of impurity have great power for mischief, especially when they put a base interpretation upon things that are in themselves pure; and the quick imagination of a child, together with the fact that this information is not guarded, as it would be if it came from an older and a wise person, makes it doubly dangerous. The testimony of one teacher, which has been repeated by many, is to the effect that the large majority of children in

the public schools know, theoretically, as much about the forms of impurity at twelve and fourteen as they ever will. Thus the situation calls for teachers wise in heart and head, watchful in regard to this danger, and skillful in meeting it; for the sense of disgrace that comes to many children from the mere acquisition of this information is a blow to that peculiar delicacy of feeling which exists with the highest morality. In many cases the inherent force of home training preserves the child from radical injury; but some children never escape the wrong that is done them, others are led into practices that seriously modify their usefulness, while still others are ruined.

The public school is a normal outgrowth of our social and political order, and its tendencies are the logical outcome of this order. Its dangers are those that exist in this democratic state, but it lies in the power of the schools to eradicate much of the evil in the state. It is difficult to say how this is to be accomplished, but certainly the most effective method will be along the line of the general improvement of the system.

This improvement will be brought about by the divorce of the control of the schools from partisan politics; by the appointment of teachers for merit only, merit in which force of character should be regarded as a *sine qua non;* by the introduction of scientific instruction to the exclusion of mechanical methods; and by constantly making prominent the idea that the pupils are being fitted for citizenship and actual service. Something could also be said in regard to the necessity of a larger number of teachers, in order that the element of personal influence may be greater and more immediate.

As this paper is only a statement of the ethical problem of the public schools, and not an attempt to solve it, it is not within its province to discuss the many possible remedies that have been suggested by teachers and others who are studying this question. Few hesitate to say that there are defects in the system, and possible moral dangers associated with it, against which our national life should be guarded with great wisdom and persistence.

The public school stands in close relationship to every moral problem in the republic. The problem of municipal government is pressing upon thoughtful citizens to-day, and many schemes are devised to make it impossible for dishonest politicians to practice their dishonesty and selfishness; but a radical cure of this and all other evils in the body politic can be effected only by the creation of upright citizens. A majority of the voters receive their only training in the public schools. If low and selfish aims rule their conduct; if they lack the possibility of enthusiasm or a high purpose; if, in short, their lives are wanting in principle, it is not enough to say that demoralizing influences overthrow the good wrought within the schools, because the business of the schools is so to establish morality that it cannot be overthrown by evil circumstances in after life. For, as has already been pointed out, the church and the home of the present day are not able to perform this work, and therefore the schools, because of the very idea which underlies their foundation and secures their continued support, and because of the amount of time which the child necessarily spends in them, must be held largely responsible for the foundation of character; in other words, for the training of upright and patriotic citizens. This, as has just been said. is their *business*. School boards and teachers are needed who realize this important fact. and who are willing and able to make the development of principle the central point in their work.

No one who examines carefully the present political and social order can fail to notice that there is a spirit of self-seeking abroad that is destructive of the noblest virtues and the highest ethical conditions; that vast numbers of citizens are controlled by the passion for getting rather than for giving. This is the dangerous element in the social problem. It is the bane of that partisanship that is ever willing to sacrifice the state for party supremacy; it is the moral obliquity of the pauper and the criminal, who are ever seeking to get something without rendering a fair and just equivalent. Is the public school laying its foundation deep enough? Has it struck its roots into the moral nature of these thirteen

million children? These are the questions that serious and earnest people are asking. There is a striking similarity between the excellencies in our national life and the excellencies in our public school system. There is also a striking similarity between the evils in both. Can it not then be said that the eradication of the evils in the public schools will have very much to do with their eradication in the life of the state?

To touch the springs of action in these pupils is to touch the very sources of power in the national life: and there is no opportunity to be compared with that offered by the public schools. The institution is so sacred, so far-reaching in its influence, that it must be rescued from political strife and partisan narrowness.

THE ORIGIN AND USE OF THE NATURAL GAS AT MANITOU, COLORADO.

BY WILLIAM STRIEBY.

The waters of the mineral springs at Manitou owe their sparkle and piquancy to the carbonic acid with which they are naturally surcharged. The beautifully clear water as it issues from its subterranean channels is accompanied in several of the springs by a considerable flow of exceedingly pure carbon dioxide. When dipped from the springs it continues to effervesce for some little time, and the agreeable flavor of the gas makes the drink very palatable in spite of the large quantity of alkaline mineral matter which it holds in solution.

For many years these springs have been locally esteemed as furnishing both a refreshing beverage and a valuable medicinal agent, but it is only within the last six or seven years that the bottled waters have been put upon the market to supply a wider circle of admirers. In order to give to the bottled waters the charm of the original effervescence, it was necessary to re-charge them with carbon dioxide: and this was done at first in the manner usually employed in bottling seltzer, and some other gassed beverages. This process, briefly, consisted in the preparation of carbon dioxide from sulphuric acid and marble-dust, and the absorption of it by the mineral water by agitation under pressure in strong iron cylinders. Then it was bottled in the common way, or sometimes was enriched by the addition of ginger syrup and flavorings, and put up as ginger-ale, or as the Manitou Mineral Water Company felicitously called it, "ginger champagne." About the year 1889 the desirability and feasibility of using the natural gas from the springs to re-charge the mineral water and champagne, was suggested to the Company, and this plan, proposed by the writer and carried out under his supervision, was soon put into practical operation.

It may be of interest to discuss the origin of this natural gas, and then to describe the means by which it is made to again impregnate the mineral water taken from the springs.

In seeking to trace the sources of this abundant supply of natural gas, it will be helpful to consider certain of the prominent geological and topographical features of the district in which the springs occur. Manitou lies in a mountain gulch or valley at the base of Pike's Peak, and just at the entrance to the Ute Pass. In fact the valley is but the eroded and widened outlet of the Pass — itself a narrow water-carved channel through which the drainage of an area some seventy-five to one hundred square miles in extent finds its way to the eastern base of the mountains. The stream which drains this tract obtained the name "Fontaine qui Bouille" from the bubbling or boiling springs on its banks at Manitou. On the south and west of the picturesque little city rise the granite slopes of Pike's Peak, while to the north are piled up the huge masses of stratified rock which flank the other side of the valley. Thus Manitou lies at the junction of the old archaean rocks, with more recent sedimentary formations. Just west of Manitou, however, the limestone beds are found for a short distance on both sides of the Ute Pass. The archaean rocks at Manitou consist almost exclusively of highly feldspathic red granite which disintegrates very readily under atmospheric agencies. Further up the Pass are found patches of syenitic gray granites containing soda feldspars. There are no igneus rocks in this neighborhood save a few narrow dikes, and on the south slope of Pike's Peak, about two miles from the summit, one exposure of phonolite, but not many miles distant at Cripple Creek, large areas of eruptive rocks are exposed to view. A number of rock slips or faults in the granite are to be seen in the neighborhood of the Peak.

At Manitou and to the northwest, only the paleozoic series of sedimentary beds are exposed. *e. g.* silurian, carboniferous and juratriasic, and these are mainly represented by limestones and sandstones. The granite slopes of the Front Range, which extend in a generally north and south line, are

depressed in a sort of recess or bay at Manitou and to the northward, and these older sedimentary beds rest upon them, and outcrop in their regular order, showing themselves for several miles up the Ute Pass. To the northwest of the Ute Pass and continuing in the same direction, extends a long, narrow strip of the same series of sedimentary formations. Probably these two areas were once continuous and have since been separated by erosion. At the lower end of the latter, or Manitou Park, area, a prominent fault having a slip of three or four hundred feet follows the junction of the granite with the southern side of the sedimentary beds,* and at Manitou, near the Rainbow Falls, there is also a fault with a slip of twenty feet or more, which takes about the same direction. It is probable that these faults form a continuous fissure over the short interval between the two areas, and quite possible that the influence of this fissure or slip determined the course of the stream which cut the Ute Pass so deeply in the rocks. From the prevailingly slight dip of the strata it is probable that the sedimentary beds at Manitou do not extend to very great depths beneath the surface, and the same observation applies also to those at Manitou Park. At Manitou the general inclination of the beds is to the southeast, though there are several folds which steeply incline some portions of them just west of the city. There are, however, in the vicinity of the springs at Manitou, a couple of small flexures of strata, indicated mainly by rock exposures on the north side of the valley which should be noticed. They run transversely across the valley, and may be described as being only more abrupt or sudden inclinations of the southeastwardly dipping strata. It is at the crests of these low folds that the springs appear. The oldest limestones are very silicious, and in places just west of Manitou where they are shattered and bent, often contain cavities filled with argillaceous red oxide of iron. The "Caverns" so much visited at Manitou occur in the limestone, and show the results of a former great chemical activity at that point. When first discovered, very considerable beds of the same ferrugin-

*See the Pike's Peak Folio, edition of 1892, U. S. Geological Survey.

ous matter covered the floors and filled some of the passageways of the caves. The springs are found at three points in the valley, and these places are at the apices of a triangle roughly equilateral. The group including the Navajo, Manitou, Cheyenne and Shoshone, which lies in the center of the town, and forms the eastern end of the triangle, has the largest flow of water and gas. The eastern of the two transverse rock flexures occurs at this point. West of this group the valley widens and divides, and the Ute Iron spring is found in Engleman's Cañon, a short distance above its mouth, and almost in the granite. The third position, that of the Hiawatha group, is in the general line of the valley, near the limestone rocks, and not far below the entrance to the Ute Pass. It is said by old residents that the largest spring was not many years ago found at this place, but that it was deeply covered and quite obliterated by deposits of gravel brought down the Pass during a severe freshet. In the neighborhood of each group of springs the gravel and wash has been cemented by a tufaceous substance deposited from the waters and called locally "soda-rock." The Navajo group and the Ute Iron spring are near to the line of contact of the granite with the sedimentary beds; the Hiawatha group is near the line of the Ute Pass fault. In all cases the springs lie close to the streams, and in the creek bed near some of them, numerous small vents are shown by rising bubbles of gas.

The rain and snow descending upon the earth bring with them matters washed from the air, and after reaching the ground the waters dissolve portions of all the soil and rocks over which and through which they make their way. It is therefore natural to seek in the waters of these springs for evidence concerning the rocks they have traversed, and thus, if possible, trace nearer to its origin the accompanying gas. The following analyses of water from several of the more important springs were made by Prof. Elwyn Waller, Ph. D., of New York, from samples taken from the springs by the writer in the summer of 1891. These results have been confirmed by analyses made in the laboratories of Colorado College on

several different occasions, when the samples were also taken in the summer season.

CONSTITUENTS FOUND.	IN PARTS PER 100,000 OF WATER		
	Navajo spring.	Manitou spring.	Ute Iron spring.
Sodium	53.959	52.176	41.370
Potassium	7.650	8.219	6.018
Lithium	0.201	0.230	0.035
Calcium	47.560	47.382	21.747
Magnesium	7.859	8.173	3.939
Iron	0.032	0.028	0.326
Aluminium	0.092	0.069	0.140
Chlorine	24.625	24.781	17.520
Sulphuric anhydride	18.410	18.232	20.703
Carbon dioxide	195.366	193.545	104.159
Silica	4.230	4.280	6.200
Oxygen in bases	39.296	38.910	23.203
Water in bicarbonates	39.962	39.588	21.305
Totals	439.243	435.613	266.665
Totals in grains per gallon	256.157	254.041	155.514

The elements found in the Hiawatha group of springs are the same as those shown to be in the other groups.

Below are tabulated the salts contained in the waters, calculated according to the conventional methods, from the preceding figures.

SALTS PROBABLY PRESENT.	IN PARTS PER 100,000 OF WATER.		
	Navajo spring.	Manitou spring.	Ute Iron spring.
Sodium chloride	40.803	41.061	29.030
Potassium sulphate	17.067	18.335	13.425
Sodium sulphate	18.749	17.397	25.792
Sodium bicarbonate	115.804	110.518	78.540
Lithium bicarbonate	1.946	2.235	0.336
Calcium bicarbonate	192.613	191.900	88.076
Magnesium bicarbonate	47.812	49.719	23.965
Iron oxide (ferric)	0.046	0.040	1.037*
Alumina	0.174	0.130	0.264
Silica	4.230	4.280	6.200
Totals	439.244	435.615	266.665

* Ferrous bicarbonate.

The temperature of the Navajo and Manitou springs was tested on two or three occasions and found to be about 15° C. In July, 1894, it was also the same. On December 31, 1894, after five days of quite cold weather (the thermometer registering nightly −18° C. or lower), and again on January 10, 1895, after a continuous period of cold weather, the temperature of several of the springs was tested, and samples of water from some of them also taken. The temperatures observed, and the residues obtained by evaporation of the samples of water, are given in the subjoined table. The residues are given in grains per gallon.

DATE OF TEST.		NAME OF SPRING.					
		Navajo.	Manitou.	Ute Iron.	Ute Chief.	Hiawatha.	Hiawatha No. 2.
Dec. 31	Temperatures C.	14°.5+	14°.5	12°.0	11°.0	†11°.5+	11°.5−
	Residues	178.40	....	*109.52	....	119.16	51.10
Jan. 10	Temperatures C.	14°.5+	14°.5−	7°.0	....	†12°.0	11°.0
	Residues	177.57	176.12	113.66	....	117.86	50.85

*A small precipitate of an iron compound had formed and settled before the analysis was begun—the result is therefore a little low.

†The Hiawatha spring is covered with an iron cap, cemented to the curbing, and the water tested on December 31 was obtained from the overflow and caught in a gallon measure, and as the vessel was cold the temperature obtained was probably too low. On January 10 the thermometer was held in the water escaping from the overflow pipe. This overflow pipe is buried under four or five feet of gravel and is some twenty or thirty feet in length.

The gas emitted from all of these springs is pure carbon-dioxide. The quantity given off from the different springs is quite unlike, varying from a few hundred cubic centimetres per hour to three or four hundred litres or more in the Navajo and Cheyenne. The flow from each spring is, generally, during the greater part of the year, very uniform in quantity, though it is said that after heavy rains or when snow is melting the quantity given off is increased. This increase may arise from greater hydrostatic pressure of the swollen stream upon small vents in the creek bed and unseen vents along the banks near the springs, thus forcing more gas

through the main channel. The foremen of the bottling department of the Manitou Mineral Water Company's plant report that a diminution of the flow of both water and gas occurs in the Navajo group of springs during the coldest weather of winter. It is to be observed that the spring waters never become turbid or roily after storms and freshets, although the surface streams are especially affected in this way. One of the springs of the Hiawatha group (the covered one) is slightly turbid from insoluble salts which probably result from reactions occuring in close proximity to the spring. Some of the spring waters on standing, quickly deposit a precipitate, while others, even from the same group of springs, remain clear for a much longer period. Considerable pressure is developed by the escaping gas when the waters are immediately bottled and hermetically sealed as soon as drawn from the springs. Large glass vessels are apt to be burst by the pressure of gas if filled quite full before sealing.

The surface waters in the district under examination contain relatively very small quantities of mineral matters in solution; nor are there springs or pools in the area which drains through the Ute Pass that are known to be highly mineralized. In the upper part of the Pass between Cascade and Green Mountain Falls there are springs and small marshy spots where a notable quantity of hydrated sesquioxide of iron is liberated from ferruginous waters, and these waters come apparently from the granites on the south side of the Pass. No thermal springs are found in this vicinity other than those at Manitou.

Taking the three analyses given above as a basis of discussion, and using also the later results as data, certain points among the many possible inferences will here be noted.

1. All the springs hold the same salts in solution, a fact which seems to point to a common origin.
2. The waters of the Navajo and Manitou springs are almost identical in mineral contents, while the Ute Iron spring contains a much smaller quantity of dissolved salts. There is also a difference in the weight of the residues

from the Hiawatha springs. It is very probable that percolating waters from the streams or from local seepage channels, make their way into the springs—such influx being greater in some springs and smaller in others. In the Ute Iron spring calcium and magnesium are low, and silica, chlorine, iron, sulphuric-anhydride. soda and potash relatively high. The proximity of this spring to the silicate rocks on the south, and to the very broken silicious silurian limestone and cambrian quartzites on the north and west, suggest reasons for a possible change in this spring water, on the supposition that its main source is the same as that yielding the waters of the other groups.

3. The presence of so large quantities of the bicarbonates of calcium and magnesium points to a prolonged contact of the waters with the limestones.

4. The almost total absence of iron salts indicates either a source quite free from ferruginous minerals, or more probably the oxidation of dissolved iron and its previous precipitation as hydrated sesquioxide by the carbonated alkaline matters with which it comes in contact as the waters move toward the springs.

5. The high percentage of chlorides may be derived from the silurian rocks or with less probability from the more distant juratriasic beds, since no saline deposits in them are known in this vicinity.

6. The large percentage of sodium bicarbonate probably indicates an origin among silicate rocks, whence the soda (and potash) may come as carbonates formed by the decomposition of the rocks by atmospheric waters containing carbonic acid, or as alkaline silicates, which react upon the limestones (calcium carbonate) before reaching the surface.

7. The sulphates may come from unseen gypsum beds such as are found two or three miles away, and lower down the Fountain creek, but it is more probable that they come in greater part at least from the oxidation of sulphides in granites, igneus rocks or even sedimentary

beds. The oxidation of ferrous sulphate, such as was described as occurring in the Ute Pass, gives rise to sulphuric acid, and by subsequent reaction with carbonates to sulphates of the alkalies. The ferruginous deposits in the broken silurian limestone indicate such a reaction.

8. The concentration of the solutions—that is, the large quantity of mineral matters contained in the spring waters, comes evidently from prolonged contact with rocks, such as would arise from percolation, and probably also from an increased solvent power of the water, due to heat or pressure, or both combined.

9. The difference in temperature of the several springs is remarkable as showing that either the waters come from different sources, or if coming from the same source have been cooled in an unequal degree by passing through diverse strata, or through the influx of cooler foreign waters. It was assumed under (2) above that the most probable view was that the same water is made to vary its content of dissolved mineral matters by the admixture of other waters; and the variation in temperature between the springs will be found on inspection of the table above to be in striking harmony with this supposition. The temperature in general is lower when the mineral content is lower, but it should be remembered that the inflowing foreign waters may also pass through strata so deeply buried as to be much warmer than mere surface waters.

10. The variation of the springs between summer and winter, in their contents of mineral matters dissolved, in the quantity of water flowing from them, and in the volume of gas yielded, together with the remarkable uniformity of temperature throughout the year in some of them, are evidently significant phenomena. The causes producing them will be discussed in a subsequent paragraph.

Before proceeding to deduce from the foregoing statements a theory of the derivation of the carbon dioxide of the

Manitou springs, it may be helpful to recall to mind some of the explanations most frequently given for the production of natural gas in large quantities beneath the surface of the ground. First, then, may be mentioned the slow, natural distillation of the buried or fossil organic matters that are found in many strata, notably in the shales. The volatile products evolved in this manner contain large quantities of marsh-gas and similar combinations of carbon and hydrogen, with but little carbon dioxide. The wide distribution of gas wells over the country attests a very general chemical action of this sort yielding combustible gases. At Colorado City, three miles distant from the Manitou springs, a small flow of combustible gas has been obtained from the two wells already bored. A second cause, obviously a true one where the proper conditions exist, is found in the heat of lavas or igneus rocks where they come directly in contact with limestones. It may be doubted whether much gas is ever produced by actual rise of temperature in this way without the aid of moisture, but unquestionably superheated waters carrying dissolved mineral matters and accompanied by vapors and gases produced by such igneus rocks, would effect the liberation of this gas. Thirdly, limestones deeply buried under later deposits of rock will suffer an increase of temperature due to the rise of isogeotherms, and heated mineralized waters will then bring about chemical reactions with the limestones. Simple rise of temperature alone will not suffice in such enclosed zones to liberate the carbon dioxide, though with the aid of moisture it may metamorphose the limestones to marble. A last cause to be noticed here is the chemical decomposition of limestones effected, with or without high heat, by acid waters or by salts which react with the carbonates of lime and magnesia, forming a new series of salts and setting free carbonic acid gas. This last cause has been in part anticipated in the two preceding theories of the derivation of the gas.

As the development of a theory of the formation of carbon dioxide probably turns in no small degree upon the action of substances in solution derived from the rocks, it is appropriate here to consider some of the results of investiga-

tion along this line. The disintegration of surface rocks by atmospheric agencies is very apparent to the eye, and the products of this resolution of the rocks have been the subject of careful study. Carbonic acid from the atmosphere, or that derived from the oxidation of decaying organic matters in the soil, plays a very important part in the disintegration of silicate as well as of carbonate rocks. It produces in the former class carbonates of the alkalies and alkaline earths, and these salts are carried away along with some silica and metallic carbonates, etc., in the percolating waters. It is believed that all rocks, at least to very great depths, are permeated with water which has made its way from the surface downward, and which exerts an action like that shown on rocks near the surface of the ground. As these meteoric waters descend they gradually lose the more active elements, dissolved oxygen and carbonic acid, with which they began their journey—being exhausted in short distances when they percolate slowly through the rocks, and carrying them to great depths when they pass through porous or shattered rocks, especially when they find channels or fissures in which to flow: but as the waters reach more deeply buried zones a new resolvent power and chemical activity is developed in greater and greater degree by the increasing heat and pressure to which they are subjected. Under the influence of these stimuli the metamorphism of rocks proceeds at a vastly increased rate, kaolinization, solution, chemical combination and crystallization working a silent, ceaseless change in many kinds of deeply buried formations. Evidence of this heightened action of heated waters is afforded to us in the hot springs of all countries; as a rule, waters which issue from the earth at high temperatures bring with them excessive quantities of dissolved mineral matters. The modern theories of the filling of fissure veins is largely based upon the greater solubility of silicious and calcareous compounds, metallic sulphides and other vein matter, in the heated waters at the deeper parts of the earth's crust to which such crevices extend, or to waters heated by bodies of igneus rocks in process of cooling: although of course at any depth, small or great, the solvent action of water takes place, and solution

may proceed more rapidly in one kind of rock than in another. The application to be made of the foregoing observations depends upon the fact that waters thus highly charged with mineral matters in rock fissures would be well adapted to produce reactions upon limestones if they should chance to come in contact with them. It should not escape notice that the flow of gas at Manitou takes place within a very limited area, and that no other similar springs are found in this immediate region. It is a very local phenomenon. Whatever the causes which give rise to the gas, they can have no general application to the similar series of rocks extending for many miles along the mountain slopes, else the evidences of their action would be more widely distributed. Hence some local cause is to be sought at Manitou as the producer of the gas.

Having thus examined the situation at Manitou and briefly reviewed some theories of the evolution of carbonic acid gas from the earth, it is now possible to more definitely assign a reasonable cause and source of the flow of gas in question. It seems to be generally agreed that the carbonates formed through the decomposition of silicate rocks by atmospheric waters will not by any known reactions in those rocks alone account for a large flow of gas. Also the evolution of carbon dioxide by the slow, natural distillation of carbonaceous matters deeply buried in strata, is an explanation not applicable to the present case, because of the great purity of the Manitou gas. The disengagement of the gas from limestones subjected to heat from rising isogeotherms appears improbable because the strata at Manitou seems not to be very deeply buried, and because the salts found in the water of the springs point to another class of rocks as a source, and also because the springs are confined to so limited an area.

The presence of masses of igneus rocks near the limestones may be considered a possible explanation of the phenomenon, since though no such eruptions outcrop near Manitou, there may yet be intrusive masses of them buried more or less deeply beneath the gravel or sedimentary beds. The waters of the springs would probably have a much higher temperature than they now possess if masses of igneus rock retaining

still sufficient heat to cause the water to dissolve its large percentage of mineral matter, occurred in this immediate neighborhood. Altogether, in the absence of some positive evidence of the presence of such heated rocks this explanation must be quite doubtful.

It remains then to discuss the probability of the formation of this gas by chemical reaction of dissolved substances in a flow of water which reaches the limestones of Manitou, and to examine whether there are at hand sources whence this saline solution might reasonably be expected to come. This explanation differs from the preceding one mainly in that the source of the reacting salts is more remote. Is there such a source of active chemical solutions present at Manitou, and if that be asserted, what evidence can be adduced in support of a theory of this sort? It must be admitted that positive proof is not now attainable, but there are some considerations which may be urged in favor of such a theory.

Reference has already been made to the faults and rock-slips at Manitou, and to the probability of the presence of an extensive and profound rock-fissure extending from Manitou continuously along the general line of the Ute Pass up into Manitou Park. In view of the presence of an extensive outflow of igneus rocks not many miles distant at Cripple Creek, and of an outcrop of phonolite nearer still on the south side of Pike's Peak, it is altogether likely that a deep fissure such as that at Manitou, if not in some way connected with those disturbances and the loss of interior liquid matter under this region, yet penetrates to depths strongly affected by these heated zones. If a fissure occurs at Manitou which penetrates the earth's crust sufficiently to reach highly heated rocks, the natural action observed in fissure-vein filling will be very actively induced, and highly mineralized solutions will result as before explained. And if further the fissure at Manitou extends to Manitou Park or communicates with fissures of the Cripple Creek district or with those in some other elevated region, the waters which everywhere find exit in such rock crevices, would, in seeking their level, according to physical laws, naturally emerge at the lower point where the

springs are found. Heated waters, or waters even slightly heated, on coming into contact with the superincumbent limestones of Manitou would promptly set up chemical reactions caused by the presence of soluble matters such as the silicates of the alkalies and other metals, silicic acid, sulphides, sulphates, etc., derived from the heated rocks. The reactions which would take place need only be generally indicated here since they are not peculiar to this theory. Alkaline silicates would change to carbonates; alkaline bicarbonates, in so far as these reactions were possible, to sulphates, or,—if chlorine combinations of lime and magnesia exist in the silurian limestone—to chlorides, in both cases—at even a very moderately elevated temperature—with the evolution of carbon dioxide; iron salts would first become carbonates and then peroxidize, setting carbon dioxide free and forming a ferruginous precipitate. If the waters were only slightly hot, silicic acid would form insoluble calcium silicate with the liberation of carbon dioxide. In the case of hot waters the basic carbonates of magnesia and probably also of calcium would be formed instead of normal or acid salts by reactions of the salts of the alkalies, etc. These, with other known and possible reactions, account for the generation of the gas. The salts contained in the spring waters may also be fairly explained in part by these reactions, and in part by reactions produced, and other salts introduced, through the accession of seepage waters in the passage to the springs. The concentration of solutions, or the high percentage of salts in the water, is well explained by this derivation. A large fissure like the one assumed to exist at Manitou must receive meteoric waters by seepage along its whole course, and such additions bring with them each their small quantity of dissolved salts. The waters which emerge from the fissure at or above Manitou must also be considerably changed in mineral contents by the accession of seepage waters from the local rocks, and these changes are probably greater at some points than at others. The difference in temperature between the various springs, and also their difference in mineral matters dissolved in the water, may thus be credibly explained. The surface waters do not readily make their way through the "soda-rock" into

the springs and hence the springs do not become roily, and the temperature of the water remains quite constant, retaining still enough heat to be designated as "thermal" in the U. S. Government Reports.

The coincidence between the diminution of the salts and that of the water and gas during the coldest winter months, probably arises in this manner. The feeders of this fissure-flow are in large part the seams and cleavage cracks in the rocks adjacent to the main fault or its branches, and they in turn derive their supply from the surface waters which percolate downward. In the very cold weather the seepage waters—as is well known in mining regions at high elevations—are greatly diminished, being held in check by frost, so that the supply is lessened in the main fault. In like manner the slopes of the Ute Pass and the neighboring hills which may be supposed to furnish the seepage waters of the sedimentary beds at Manitou are restraining (by frost) their quota of the supply for the springs. Especially would this be true if, as appears probable, the more elevated and comparatively shaded south side of the Pass furnishes the bulk of the seepage waters which make their way to the springs. Under these conditions the temperature of the springs would not generally greatly vary in summer or winter, since the colder surface waters of the latter season which mingle with the decreased fissure-waters would also be much diminished in volume. With the decrease in water would come a decrease in salts, and so also of the gas produced by the reactions previously outlined.

The salts coming from the assumed fissure beneath the limestones naturally tend to follow the seams and bedding planes of the stratified rocks and thus to make their way down the easterly slopes without coming to the surface. The resistance to this flow resulting from friction and perhaps from sharp folds and faults east of Manitou, causes the waters to force an exit through the broken and folded strata at the western side of the city. The two small flexes of surface rock before mentioned appear to give in their crevices the opportunity for a final escape to the surface of the water and gas.

All the circumstances, therefore, connected with the position and flow of the springs, and the mineral contents of the water, etc., etc., seem to be consistent with, if they do not favor this explanation.

Briefly, then, the theory advanced of the origin of the natural gas at Manitou may thus be summarized: Water percolating through silicate rocks and becoming highly mineralized under favorable conditions of temperature and pressure, makes its way through cracks and profound rock-fissures by the action of gravity and the ascensional power imparted by heat, to the limestones west and north of Manitou. It is here increased in volume and in dissolved salts by the numerous additions of seepage waters from the local rocks, and also lowered in temperature at the points where these influxes occur. By chemical reactions some of the dissolved salts are changed, and the carbon dioxide originally held (almost entirely) by the limestones is liberated from that combination but dissolved in the water on account of the great hydrostatic pressure. As the waters rise through the irregular channels enlarged from cracks and seams, the pressure decreases, and more and more of the dissolved gas escapes from the water, until at last when the surface is reached at the various springs the gas emerges with the rythmic flow due to the irregularities in the channels of exit.

Of the many temptations to comparisons and generalizations growing out of his study upon the origin of the natural gas at Manitou, the writer yields to the two following: (1) The caverns at Manitou mark the scene of a former considerable chemical activity, possibly induced by the same causes now at work in the lower strata in the manner mentioned above. If the theory advanced in this paper is true, caverns of like kind may now be in process of excavation which will in time rival or eclipse those so much admired in the now drained and fragmentary parts of strata on the west side of Williams' cañon. (2) The data in the hands of the writer concerning the carbonated mineral springs of other localities are too meagre to permit of any very general comparisons or deductions, but it would appear from published descriptions that

in some notable instances, at least, the flow of carbon dioxide, just as at Manitou, occurs where there are no igneus rocks, but at points where the older rocks have been faulted or fissured below overlying limestones. A case in illustration occurs at the Saratoga springs in New York. The silurian limestone is there faulted by a fissure which extends down into the archæan rocks below, and no other visible cause appears for the generation of the gas. Also the conditions of the Cañon City, Colo., carbonated springs seem, from an examination of the geological maps, to be very similar to those at Manitou—but the writer has no positive knowledge as to whether or not the rock fissures in that region extend to the vicinity of the springs as is the case at Manitou. The published analyses of the water from the springs at Cañon City show that in respect of both the kinds of salts and the quantity of them present, they very closely resemble the springs at Manitou.

The second part of this paper concerns the use made by the Manitou Mineral Water Co., of the natural gas which comes from the springs. A description of the steps taken and the apparatus devised to accomplish this end may be of interest. In the early days the Navajo group of springs bubbled and fizzled in a peaty morass, and could be approached with difficulty. The Navajo spring which gave the most gas and water was then curbed with cement and stone and the swamp filled up with earth. From this spring was drawn the water put up by the Company in 1889, at the time the proposition was made to use the escaping gas to recharge the bottled water. In the accomplishment of this plan three problems required solution, namely: (1) to ascertain the quantity of gas available for use: (2) to devise means for catching and storing the gas: (3) to obtain a gas-pump which would continuously and practically compress the gas to the degree obtained in the old gas-generators, *i. e.* some 60 to 80 lbs. per square inch.

The measurement of the gas was effected as follows: A large tin funnel, stiffened at the wide opening with heavy wire, and made very short from large to small end, was sunk

mouth downward under the water of the spring as deeply as possible. The mouth of the funnel was bent after several trials to conform to the irregularities of the spring, and the funnel when thus fitted was held in place by wooden supports, because pressed upward with much force by the rising gas. A large bell-glass of a capacity of 7 or 8 litres was used to make the measurement. This bell-glass when filled with water by immersion in the spring was held mouth downward over the small opening of the funnel whence the gas now escaped, and as the gas entered it the water was displaced. The bell-glass was gradually raised out of the water as the gas accumulated, until when it was full of gas the mouth just dipped beneath the surface of the water. In this way the gas was measured at the then prevailing atmospheric pressure. Owing to the very rapid flow of the gas, the time required to fill the bell-glass could not be very accurately determined, though by repeating the experiment many times and taking an average of the time records, the number of seconds required to fill it was ascertained with sufficient accuracy for practical purposes. The quantity of gas evolved daily was now easily computed, provided the flow was constant as observation had seemed to indicate. With these figures it was possible to calculate the number of bottles of water that could be recharged with this natural gas per diem. Roughly speaking, water will absorb its own volume of carbon dioxide whatever be the pressure to which the water and gas are subjected. Assuming a pressure at which the company would bottle the waters, the reduction in volume of the gas was obtained by the use of the formula $v' = \frac{vp}{p'}$. The rise of temperature, due to compression, could practically be neglected since the water sufficiently cooled the gas. The number of bottles which could be filled from this supply of gassed water was $\frac{v'}{A}$ if A represented the capacity of each bottle. Thus the Company was assured that the supply would be much more than sufficient for the then daily output of the works.

The second problem involving the construction of apparatus for catching and storing the gas was somewhat more

troublesome. The conditions were (1) that the waters of the spring, which were used for bottling, must be preserved from contamination by metallic salts; (2) that the gas must be forced by its own pressure to an appropriate gas-holder; (3) that the spring must be accessible for cleaning and adjustment of the overflow pipes which carry the water to the bottling works. Another point which had some influence in determining the character of the apparatus to be used, was the wish of the Company to have all parts of it open to inspection by visitors to the works,—a policy the wisdom of which has been amply justified by subsequent experience. To meet these requirements it was decided to immerse a bell-shaped vessel in the spring to catch the gas. From this bell the gas could be carried in pipes to the works, two or three hundred feet distant. In order to give the gas sufficient pressure to force it through the conducting pipe, and send it into a receiver or gas-holder, it would be necessary to depress the bell in the spring water sufficiently to allow for the difference in level between the surface of the water on the outside and that of the water within. At the gas-holder this conducting pipe would have to dip into water several inches in order to make a water-seal connection, to prevent loss of gas in case of accident to the pipe or bell, and in the gas-holder itself, a slight pressure would be needed to send the gas to the pumps. In preparation for the reception of this bell the spring was cleared out and somewhat deepened, and upon the rocky bottom, a short distance above the vents whence issue the gas and water, a shelf of cement, circular in plan and about six inches wide, was built within the inclosing walls of the spring, which were also made cylindrical. On this shelf the bottom of the bell was to rest. An incident in the work of deepening the spring and constructing the shelf and walls is worthy of passing note. The volume of gas was so great that workmen could not remain a moment in the excavation without apparatus to supply fresh air. This was provided by the use of a dentist's gas-inhaler connected with a piece of common garden hose reaching above the curb of the spring. Even with this inhaler it was necessary to stop the nostrils of the men with plugs to keep out the gas. When,

as often happened, one of these nasal stoppers become displaced, the workman would precipitately bolt for the surface with a shout of pain from the sharp sting of the gas in the nose and air-passages. Tears would flow from the eyes of those engaged in working at the bottom of the spring.

The material of which to make the bell was important from the fact that the water of the spring must be used for bottling. Iron rust would destroy the clearness of the water; lead, copper and zinc would add poisonous salts, which, although present in very minute quantities, would yet cause distrust in the minds of users of the water; silver was the ideal metal, but its then high value barred its use. Block-tin was accepted, though the difficulties in the way of its use were quite serious. The form of the bell adopted is shown in sectional elevation and plan in the annexed plate. It was built by The Hartt Manufacturing Co., of Chicago, after plans of the writer. The sheets of block-tin (No. 12 American wire-gauge thickness) were held in place and stiffened by a skeleton frame made of heavy iron wire encased in tin pipes. The frame as thus made has been found in subsequent use to be too light, and the bell requires the most careful handling to prevent distortion and cracking; otherwise it has well fulfilled its purpose, and now (Jan. 1895) is almost as good as when first set in 1890. In order to show to the many visitors the flow of gas in the spring, the top of the bell was made of plate-glass, and just below the glass hung an electric incandescent light. The bell is held firmly in place by iron stays fastened to the curbing.*

The temperature of the springs (about 60° Fahr.) is so high that the gas is loaded with moisture, and condensation in the conducting pipe results, especially in cold weather. In order to prevent a stoppage of the pipe a drip-trap (see plate) made of gas pipe was inserted at the lowest point in the line, and an escape for the accumulating water provided.

* Before closing the description of apparatus for catching the gas, it should be added that at one of the Hiawatha springs the owners have caught the gas by the use of an iron dome cemented upon the curbing of the spring. There is no way to get into the spring to repair pipes, etc., except by breaking the cement sealing. It would appear to be a wise measure to have provided a manhole with a movable cover in this dome to obviate that difficulty. A couple of iron bells placed in other springs are rusting rapidly and render the waters turbid.

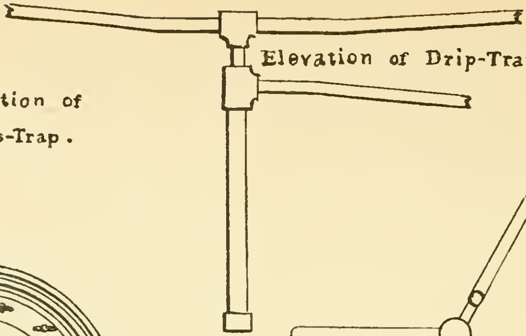
The conducting pipe was buried under ground to prevent freezing.

The gas-holder is designed to hold twenty-four hours yield of gas from the springs. It contains no strikingly novel features. The bell or holder is supported by three chains which run over pulleys attached to heavy timbers set for this purpose in the Company's building, and bucket weights are loaded to give the required pressure to send the gas to the carbonating apparatus. The holder dips in water contained in a cemented cistern which can be conveniently drained and cleaned. A large manhole in the top of the holder provides access to the cistern and to the enclosed pipes. An electric incandescent lamp suspended just beneath the arched top of the holder permits inspection of the pipes for the ingress and exit of the gas; and for this purpose also a number of small glass plates are cemented in a perpendicular line on one side of the holder at intervals of about two feet. The pipe which brings the gas from the spring, and that which leads it to the carbonators, and that which provides for the escape of an excess of gas, rise from the cemented bottom of the cistern and are supported upon an iron tripod, as shown in the accompanying plate. The pipe for the entrance of the gas curves in the form of a goose-neck and dips under the water some five or six inches, to effect a water seal against back-flow. The overflow pipe is capped by a metal bell which dips into the water of the cistern so long as the holder is not yet full of gas, but when the holder is full the cap of the overflow pipe is raised from the water and gas is allowed to escape. This cap or gas-trap (see plate) is connected by a metal chain with the top of the holder, and is therefore raised from the water when the holder rises above a certain height. This apparatus has sometimes caused trouble by the kinking of the chain connecting the holder with the metal cap. The chain should be made of small links so that it may readily coil in the little dish at the top of the metal cap, and uncoil again without kinks. A small rod suspended from the roof of the holder, which would lift the cap by an arm, would perhaps obviate any difficulty of this sort.

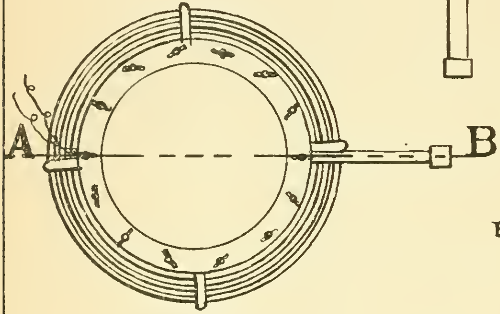
The problem of the compression of the gas in order to re-charge the water with it, was one of vexation and difficulty. The plan at first attempted was to pump the gas into the old iron gas-generator, whence it could be drawn and used in the same manner as was the artificially-made gas. It was not successful, because the pump bought for the company would not continuously, nor even for an hour, compress the gas without destroying the packing of the piston-head. The heat developed by the compression and the strain upon them, in a few moments reduced rubber washers to shreds, and leather ones lasted but little longer. About this time the writer, while in New York city, heard of a new form of carbonating apparatus invented by the Wittemann Brothers. Three of these machines were then employed in practical work, and an inspection of the one operating in New York left no room for doubt as to their merit. The principle of the machine was one to tickle the fancy of a man of science. The gas was pumped into a small cylinder of glass or metal and the water to be carbonated was also pumped into the same chamber by the action of the same piston-rod. The water entered the cylinder of compressed gas through a smaller interior cylinder pierced by a great number of very small holes. The water was thus sprayed through the compressed gas, and was therefore most advantageously disposed for quickly absorbing the gas; in fact the absorption was instantaneous. The carbonated water now flowed to a balanced reservoir which operated to shut off the supply of gas or water or both, if necessary, to maintain a proper pressure and a constant supply for the bottling tables with which it was connected. Thus the machine was automatic and continuous-working in its operation. The success attending the use of this pump induced the Company to put in several larger machines of the same sort as their business increased. As a result of the experiences in the operation of the apparatus described in this part of the paper, several slight modifications might be made if new ones were to be constructed, but it is fair to observe that in practical working the results have been on the whole very satisfactory.

The writer wishes to acknowledge the courtesy of the Manitou Mineral Water Company in assenting to the publication of many details pertaining to the successful operation of their business.

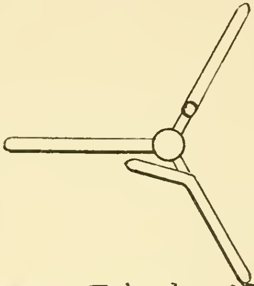
Section of Gas-Trap.



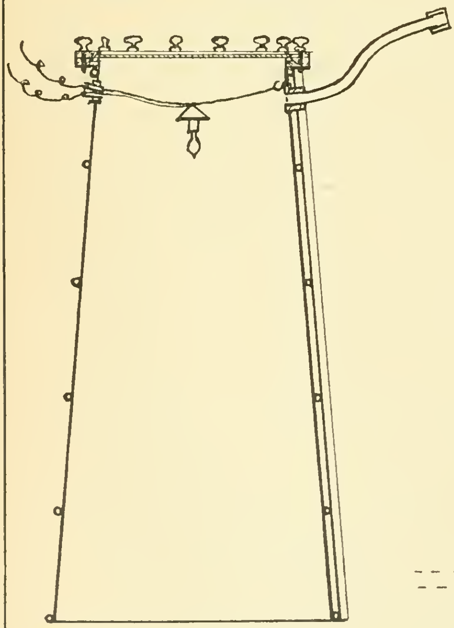
Elevation of Drip-Trap.



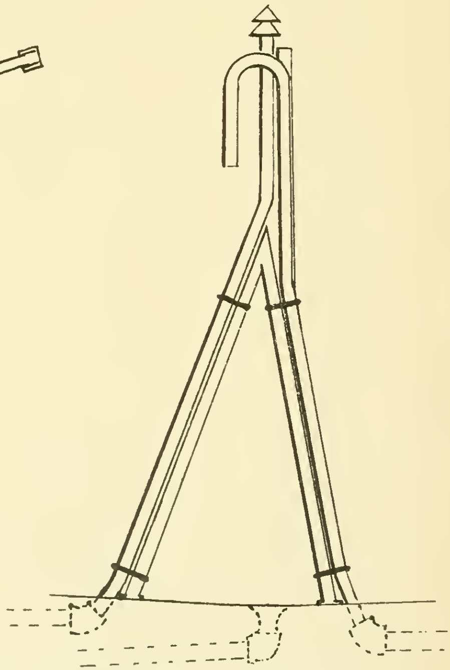
Plan of Gas-Bell.



Plan of Tripod and Pipes.



Sectional Elevation on A.B. of Gas-Bell.



Elevation of Tripod and Pipes.

Scale. $\frac{1}{2}'' = 1'$.

HERPETOLOGICAL NOTES FROM KANSAS AND TEXAS.

BY F. W. CRAGIN.

Terrapene ornata, Ag., var. nov. *cimarronensis*.—I would propose this name for what appears to be merely a color variety of *T. ornata* in which the ordinarily yellow parts on the limbs and neck are replaced by bright red. This variety, with which I have been acquainted for some ten years, is common over most of the western half of Kansas and Oklahoma. In west-central and northwestern Kansas, it partly, and in the southern tier of Kansas counties from Barber westward almost if not quite supplants the typical variety. I have observed a single specimen in southeastern Colorado, about midway between Kansas and the mountains. The peculiar home of the variety is apparently the "Red beds" country of the Cimarron basin, upon whose red terranes it may possibly have had its origin, afterwards extending its range by migration.

Apropos of the western range of *Terrapane*, it would seem that it falls a little short of the eastern base of the Rocky Mountains in Colorado. Prof. Geo. H. Stone states that, in his twelve years of residence here, he has never observed it in the vicinity of Colorado Springs. Prof. William Strieby, who resided for two years at Santa Fé, states that he has observed a single specimen of the box-tortoise in New Mexico, though he does not now remember the exact locality.

The Three-toed Box-tortoise, *Terrapene triunguis*, Ag., seems not to have been recorded from Texas, though it is known from Indian Territory and Louisiana, so that its occurrence in Texas is not unexpected.

I am indebted to Mr. G. H. Ragsdale for several specimens of this species which are now before me from Gainesville, Texas, which is apparently the most westerly locality known for its occurrence. That it will be found yet a little further west in the valley of the Red river, is not improbable.

At Gainesville, as doubtless in Indian Territory, the range of this species and that of *T. ornata*, Ag., overlap. The *T. triunguis*, however, keeps mainly to the more or less wooded bottom-lands, as Mr. Ragsdale informs me, while *T. ornata*, chiefly an open prairie species, is sometimes found in the timber also.

The specimens of *triunguis* from Gainesville are larger than those of *ornata* from the same place; and the high, rounded carapace, with broad, flat, yet abruptly elevated carina on the third and fourth vertebral scutes and bones (usually seen also on the posterior part of the second), the much less transverse second to fourth vertebral scutes, the greyish olive-brown color, sometimes relieved by ornata-like, dark and yellow subradial markings especially upon the excentric place of origin of the concentric imbrications, the more depressed skull and orbit and consequently more obliquely placed jugal bone, the possession of three claws only on each posterior foot, and the essentially woodland habit, are all peculiarities which distinguish the former species, as from the latter. Both the fore and the hind limbs of the only present example in which the skeleton is preserved have the phalanges 2, 3, 3, 2, 1, those of the fourth and fifth digits of the hind limb being rudimentary and these digits without claws.

I am also indebted to Mr. Ragsdale for a shell and skull of the Keeled Mud-turtle, *Goniochelys carinata*, Gray, and the same of *Chrysemys elegans*, Wied., which he collected in Elm Fork of the Trinity river at Gainesville.

Gerrhonotus infernalis, Baird.—Cope has recorded this species from Helotes creek, near San Antonio, and from Wichita county, Texas. It is common in Hays and in the western part of Travis county. In the summer of 1893, I observed it in the Cross-Timbers, between Roanoke and Lewisville, extending its known range somewhat eastward.

The little burrowing snake, *Stenostoma dulce*, B. & G., almost the sole representative of the family *Typhopidæ* in the United States, was also taken in the Cross-Timbers, at a

point about five miles west of Lewisville, considerably northeast of the most northeastern record with which I am acquainted (Bexar and Erath counties: Cope).

A red and black example of *Contia episcopa isozona*, Cope, was captured in the road a few miles west of Sun City, Kansas, on the point of the divide between Medicine Lodge river and Elk creek, and was sent to me several years since by Mr. Wm. A. Sherrill. Of the several phases described from western territories by Cope in the *Proceedings of the Philadelphia Academy of Natural Sciences*, this specimen most resembles, and indeed almost precisely tallies with that from Utah. I may remark in this connection that, of a small collection of reptiles which I received a few years ago, through Mr. John Pym, from southwestern Utah, *all* were, to my surprise, species common in Kansas.

In September, 1886, coming down the valley of the Cimarron river from New Mexico, I first noticed the little Sonoran toad, *Bufo debilis*, Girard, near the Z H ranch in the Public Lands (now Beaver county, Oklahoma), at a point thirty-five or forty miles west of the southwest corner of Kansas. The species was observed a few days later in great abundance and activity (during rainy weather) in Morton county, Kansas, and in the southern part of Hamilton county. I have collected a single specimen in the western part of Barber county, Kansas, also.

THE CHOCTAW AND GRAYSON TERRANES OF THE ARIETINA.

BY F. W. CRAGIN.

The "ram's horn oyster," *Exogyra arietina*, F. Roemer, is the characteristic fossil of a column of sediments, the so-called Exogyra arietina marl, that in Hays, Travis and Williamson counties, Texas, consists mostly of calcareo-argillaceous, and more or less ferruginous marl, and attains a thickness of sixty to eighty feet, occupying the interval between the top of the Washita limestone of Shumard and the base of the Shoal creek limestone of Hill. This column was recognized by Dr. Shumard as an important member of the Cretaceous section of Texas, and was named by him, the *Exogyra arietina marl*: and this name, either as it originally stood or under such slight variations of form as "the Exogyra arietina clay," "the Arietina marl," etc., has been used for it by most later writers. The same formation outcrops, with more or less interruption by faulting, mantling, etc., and with more or less variation in thickness and lithological character southwestward to the region of El Paso. Its detail in this direction being less known than that of its northward extension, will not here be discussed. From Austin to the Red river valley in Cooke and Grayson counties, the Arietina becomes, as Taff has shown, gradually reduced in thickness and decidedly more calcareous. For this calcareous northern phase of the Arietina which, in the Red river valley, occupies the entire interval between the summit of the Pawpaw clays of Hill and the base of the Dakota sandstone, Hill has recently proposed the name, Main-street limestone.*

The Main-street limestone, however, consists of two members. Its dual character has been independently determined in the field by the present writer. But the members that compose it were first recognized as terranes by Taff in his second Report on the Cretaceous Area North of the Colorado

* Bulletin of the Geological Society of America, V., 330.

River,* who correlated the upper member with the Shoal creek (Vola) limestone.

THE CHOCTAW LIMESTONE.

The lower of these two members, the *Choctaw limestone*, is the limestone that, in Cooke and Grayson counties, rests upon the Pawpaw clays (and sands) of Hill, and a typical locality of which is at the quarry owned by Mr. J. T. Munson *et al.*, on the summit of the Pawpaw creek bluffs, southeast of the Union depot of Denison. Thence it extends along the brow of the creek-bluff to the northeastward, to a point north of the Denison-Bonham road. It also outcrops extensively on Choctaw creek, from which it is named, and thence westward on most of the tributaries of Red river, and on the bluffs of the river itself to Gainesville, just west of the Cross Timbers, where it takes a southerly course. It is well developed in Denton county also, on Denton creek, near Roanoke. Though softer and less durable than the so-called "quarry-limestone" of Denison (at base of the Pawpaw clays), it is a true quarry-limestone, and has been quarried for building purposes not only at the quarry above named, but also at many other localities. It is the same limestone that is called by Taff, "the Exogyra limestone," in the report above cited (pp. 281, 282, 283, etc.)

The thickness of the Choctaw limestone in Grayson county is ordinarily about five feet, though locally it becomes a few feet greater. In Cooke county the same limestone attains a maximum thickness of ten or fifteen feet.

The paleontologic aspect of this terrane may be seen in the following list, which includes most, if not all, of the fossils that have been obtained from it:

FAUNA OF THE CHOCTAW LIMESTONE.

Hemipedina charltoni, Cragin.	*Cucullæa recedens*, Crag.‡
Holectypus charltoni, Crag.†	*Cardium* sp. (distinct from *texanum*)
Enallaster texanus, Roemer.	*Cardium* (*Protocard.*) *texanum*, Con.

* Pages 275-283.

† In the small peristome and posteriorly placed periproct, this species is closely related to *Holectypus castilloi*, Cotteau, from which it differs chiefly in the form of the periproct.

‡ This species is apparently distinct from *C. terminalis*, Conrad, of the Glen Rose beds; but the question can perhaps hardly be settled till specimens of *terminalis* can be found showing the details of the hinge.

Terebratella wacoensis, Roem.
Ostrea franklini, Coquand.
Ostrea subovata, Shum.
Ostrea quadruplicata, Shum.
Exogyra arietina, Roem.
Lima wacoensis, Roem.
Vola texana, Roem.
Trigonia clavigera, Crag. (?)*
Roudairia denisonensis, Crag.
Pholadomya ragsdalei, Crag.
Pholadomya sancti-sabæ, Roem.
Homomya washita, Crag.
Pachymya austinensis, Shum.†
Turritella denisonensis, Crag.
Turritella seriatim-granulata, R.

All of the above-listed fossils were obtained by the writer in this limestone at Denison, excepting the *Holectypus*, which was recorded from the same limestone at Cedar Mills (Taff) in the writer's "Contribution to the Invertebrate Paleontology of the Texas Cretaceous."

Perhaps the most marked paleontological characteristic of the Choctaw limestone is the plentiful occurrence of *Terebratella wacoensis* in association with *Exogyra arietina*. In Grayson and Cooke counties, a good guide-fossil is *Pholadomya ragsdalei*, whose peculiar ornamentation usually suffices, even in fragments of casts or moulds, for its recognition. This fossil has hitherto been found in the Choctaw limestone only, and is by no means rare.

In the Arietina marl of more southerly counties, as Williamson, Travis, and Hays, the Choctaw terrane is represented by the calcareous transition from the marl of the medial, culminating horizon of *Exogyra arietina* to the Washita limestone below, and its fauna is poorer and less peculiar. There, as in northern Texas, *Terebratella wacoensis* and *Exogyra arietina* are the leading members of its fauna, the others being such as are common in the upper part (Denton beds‡) of the Washita limestone, such as *Vola texana*, *Turrilites brazosensis*, etc.

* The only specimen of *Trigonia* that I have seen in the Choctaw limestone is a cast of a small one, that does not show the character of the ornamentation, but agrees in size and form apparently with *T. clavigera*, and is probably that species. *T. clavigera* is the common Trigonia of the Denton marls, and resembles *T. emoryi*, with which Hill has apparently confused it, only in badly worn specimens. Comparing, in numbers, finely preserved specimens of *T. clavigera* from the Denton marls of Cooke county, Texas, and the Chickasaw Nation, with such of *T. emoryi* from the Walnut clays, confusion of the two species is impossible. *T. crenulata*, cited by Taff from the Denton marls, does not occur in rocks of the Comanche series.

† Not "*Pachyma*," as it is often spelled and pronounced.

‡ The *Denton marl* was clearly defined as to position, paleontology, and typical locality by Taff, by means of a detailed section, in the Fourth Annual Report of the Geological Survey of Texas, Part I, p. 272, and, though he failed to apply it elsewhere in the same report, this name has priority over the later name, "Marietta" proposed by Hill for the same terrane.

THE GRAYSON MARLS.

The Choctaw limestone is conformably succeeded by the *Grayson marls*, passing up into them through a horizon of sometimes quite abrupt, sometimes more gradual transition, the marls being unconformably overlaid by the Dakota sandstone. In their northeasterly exposures, the Grayson are chiefly yellow, highly calcareous, sparingly arenaceous marls, of such consistency as to crumble readily under the hand or break into rough lumps under light blows of the hammer, but which contain, especially in the lower parts, irregular concretionary tracts of cement-like hardness and texture. They yield rapidly to the weather and are hence often absent or reduced in thickness by erosion, both Predakota and recent. Again they occur only as slopes, grass-grown or mantled with debris from the overlying Dakota. In Denton county the Grayson terrane consists of alternations of marl and limestone,* and attains a thickness of thirty to forty feet. In Travis county it contains, in the upper part, beds of hard limestone charged with *Gryphæa mucronata*, and there, as elsewhere south of the Brazos, it is overlaid conformably by the Shoal creek limestone, into which it seems to graduate through the basal part of the latter.

In Northern Texas, where the thickness of the Arietina is reduced, the Grayson marls have a thickness of fifteen to twenty-five or thirty feet.

A typical locality of the calcareous yellow marl phase, showing its basal transition to the underlying limestone, may be seen at the cut and fill of the once projected D. B. & N. O. railway, between Pawpaw creek and a right-hand branch of the same immediately southeast of the quarry above cited as typical for the Choctaw limestone.

The Grayson marls were considered by Taff to be the equivalent of the Shoal creek, or Vola, limestone of the Austin district, and were referred to by him as the Vola limestone and marl in his second "Report on the Cretaceous Area North of the Colorado River," which was published in the Fourth Annual Report of the Geological Survey of Texas.

*See Taff's Elm creek section, in Fourth Ann. Rep. Geol. Surv. of Texas, Pt. I, p. 280.

(See p. 277.) In the "Contribution to the Invertebrate Paleontology of the Texas Cretaceous," published in the same report, the present writer recorded a few fossils from beds referred to, in doubtful terms or in quotation-marks, as of Vola equivalency (*l. c.*, p. 147, etc.). He there purposely implied doubt or placed the word, "Vola," in quotation-marks, quoting from the labels accompanying the specimens collected by Mr. Taff's field-party, feeling doubtful concerning the supposed discovery of the Vola in that region and never having examined the region personally. In the summer of 1893, however, after the publication of his "Contribution," he visited Denison, in the course of an extended expedition, and studied the matter in the field, and so independently arrived at the same opinion that has been announced by Hill in the *Bulletin of the Geological Society of America* (Vol. V) viz., that no sufficient evidence of the existence of an equivalent of the Vola limestone in the Red river district has as yet been found.

Paleontologically, the Grayson and the Shoal creek terranes have little in common. In his "Mesozoic Echinodermata of the United States" (Bul. 97 U. S. Geol. Surv.), Clark gives, presumably on Hill's authority, the Shoal creek limestone as the source of his *Hemiaster calvini*, the latter being apparently synonymous with the *Epiaster hemiasterinus*, nob., an echinoid common in the Grayson marls in northern Texas. It is possible that the *Gryphæa mucronata* of the Grayson limestone beds of Travis county may extend up into the basal part of the Shoal creek limestone; but to the best of the writer's recollection, he has never observed such an occurrence.*

The following list includes all of the fossils that the writer has identified from the Grayson terrane, nearly all of which have been collected by the writer in Grayson county:

* In a letter received from State Geologist Dumble of Texas, since this was written, and just as this MS. is going to press, speaking of the 'Vola,' he writes that the *Gryphæa* extends "up into the solid limestones to a height of four or five feet." But from his use of the word, "limestones" (in the plural), I am in doubt as to whether he here refers to the lower part of the *main stratum* of the Shoal creek limestone, or to the separate bed or beds of solid limestone underlying it. In either case, it would seem to indicate only a transitional horizon such as occurs at the limits of most of the terranes of the Comanche series.

FAUNA OF THE GRAYSON MARLS (AND MARLY LIMESTONES).

Cyphosoma volanum, Crag.
Holaster completus, Crag.
Holaster nanus, Crag.
Holaster supernus, Crag.
Enallaster inflatus, Crag.
Epiaster hemiasterinus, Crag.*
Ostrea perversa, Crag.
Gryphaea mucronata, Gabb.
Exogyra arietina, Roem.
Exogyra drakei, Crag.
Lima wacoensis, Roem.
Vola texana, Roem.
Avicula dispar, Crag.
Modiola pedernalis, Roem.
Cardium (Protocard.) texanum, Con.
Roudairia denisonensis, Crag.
Tapes dentonensis, Crag.
Cyprimeria texana, Roem.
Homomya washita, Crag.
Turritella seriatim-granulata, R.
Turritella denisonensis, Crag.
Nautilus texanus, Shum.
Turrilites brazosensis, Roem.
Sphenodiscus pedernalis, Roem.
Hoplites texanus, Crag.

The Grayson fauna, like the Choctaw, is much poorer in southern Texas than in northern. The more common forms occurring in it in Travis county are, in the writer's experience, the following:

Ostrea perversa,
Gryphæa mucronata,
Exogyra arietina.
Vola texana,
Plicatula dentonensis, Crag.

Hill has recorded from the beds of this terrane at Austin, the following also: *Terebratella wacoensis*, Roem. *Diplopodia texanum*, Roem. *Protocardium texanum, Con.*, *Pachymya austinensis*, Shum., and *Astarte robbinsi*, White; in case of the first and last, only a single example of each.

In both northern and southern Texas, the Grayson marl is especially characterized by that peculiar form of Gryphæa that, by Hill, Taff, and other recent authors, has usually been regarded as a variety of *G. pitcheri*, Morton, and has been called by Taff, "the Vola phase of *G. pitcheri*," but has recently been recognized by Hill as the truly distinct *G. mucronata*, Gabb.

* This name has priority over that of "*Hemiaster calvini*," a name subsequently applied to the species by Clark, whose specimens are "from the Shoal creek limestone . . . . in Travis county." Owing to the feeble development of the peripetalous fasciole, the writer referred this species to the genus *Epiaster* it being, in fact, intermediate between that genus and *Hemiaster* in the one important character that distinguishes these genera. Since, however, the peripetalous fasciole is *present*, though imperfectly, and in some specimens *very* imperfectly, developed, it would probably be more consistent to refer the species to *Hemiaster*, as Clark has done. But if this be done, it is questionable whether the same generic disposition should not also be made of the well known "*Epiaster*" *elegans*, Shum, which, in the most perfectly preserved examples, is also seen to have a feebly developed peripetalous fasciole.

With others, the writer formerly supposed that this shell was a mere variety of *G. pitcheri*, but now fully confirms the original opinion of Gabb and the revised opinion of Hill that it is specifically distinct. While the Grayson Gryphæa often more or less nearly approaches the so-called "Kiamitia variety" of *G. pitcheri*, with respect to the elongated triangular outline of the shell, it presents important differences, such especially as the truncate anterior border, the sharp angulation of the sulcus, and the very prominent and compressed anterior and posterior folds of the left valve; and these features are so constant under all the variation in shape of the Grayson shell that, not only for stratigraphic purposes and convenience of expression, but even on the strictest biological grounds, this shell must be regarded as distinct from *G. pitcheri*, while closely related to it.* Gabb's *G. mucronata* and Conrad's *G. navia* are not considered by the writer as synonymous, but the latter is regarded as merely an arched and imbricated example of *G. pitcheri*, lacking (as do all of the stratigraphically low-occurring Gryphæas of the Comanche) the compressed folds (especially the anterior compressed fold) and truncate anterior border which are the fundamental characteristics of the left valve of *G. mucronata*.

At the above-given typical locality of the yellow marl phase of the Grayson, there is, in the lower part of the marl, a horizon of profuse occurrence of typical *Exogyra arietina* and, just below this, one of *Turrilites brazosensis* and *Holaster completus*. In the upper part of the marl at the same locality, the *Exogyra arietina* is small and not abundant, the *Turrilites* plentiful; but the *Holaster completus* has not been obtained, and, if it occurs there, must be rare.†

The fauna of the Grayson marl presents some affinities to that of the Choctaw limestone, but far more is peculiar to either fauna than is common to both.

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* Strong incurvature of the beaks and the conspicuous and intervalled margins of imbrication are also constant or nearly constant features of *G. mucronata*, but as these are features exhibited by some examples of *G. pitcheri*, they are not of equal diagnostic value with the characteristics above mentioned.

† It is to be noted, however, that the latter species, like *Holaster simplex* frequently occurs in groups, so that what has appeared rare or absent in a horizon may, by some fortunate find, become locally abundant.

The relationships of the two faunæ, as far as now known, may be seen in the following table:

TABLE OF OCCURRENCE OF FOSSILS IN THE TERRANES OF THE MAIN-STREET ARIETINA.

	Choctaw	Grayson.		Choctaw	Grayson.
Diplopodia texanum	...	x	*Vola texana*	x	x
Hemipedina charltoni	x	...	*Avicula dispar*	...	x
Orthopsis sp.	...	...	*Modiola pedernalis*	...	x
Cyphosoma volanum	...	x	*Cucullæa recedens*	x	...
Holectypus charltoni	x	...	*Trigonia clavigera (?)*	x	...
Holaster completus	...	x	*Astarte robbinsi*	...	x
Holaster nanus	...	x	*Cardium sp.*	x	...
Holaster supernus	...	x	" *(Protocard.)texanum*	x	x
Enallaster inflatus	...	x	*Roudairia denisonensis*	x	x
Enallaster texanus	x	...	*Tapes dentonensis*	...	x
Epiaster hemiasterinus	...	x	*Cyprimeria texana*	...	x
Terebratella wacoensis	x	x	*Pholadomya ragsdalei*	x	...
Ostrea franklini	x	...	*Pholadomya sancti-sabæ*	x	...
Ostrea perversa	...	x	*Homomya washita*	x	x
Ostrea quadruplicata	x	...	*Pachymya austinensis*	x	x
Ostrea suborata	x	...	*Turritella denisonensis*	x	x
Gryphæa mucronata	...	x	" *seriatim-granulata*	x	x
Exogyra arietina	x	x	*Nautilus texanus*	...	x
Exogyra drakei	...	x	*Turrilites brazosensis*	x	x
Plicatula dentonensis	...	x	*Sphenodiscus pedernalis*	...	x
Lima wacoensis	x	x	*Hoplites texanus*	...	x

From this table it will be noted that, of forty-two species found in the two terranes, only ten, or less than one-fourth part, are common to them both, and that, of the eleven species of sea-urchins, not a single species is known to occur in both. The absence of *Cephalopoda* in the Choctaw terrane is also noteworthy. Of *Turritella denisonensis*, common in the Choctaw, the Grayson has thus far yielded only a single example. The *Gryphæa mucronata* is not known in the Choctaw, nor does *Ostrea suborata* seem to extend up into the Grayson terrane. The latter observation essentially applies also to the association of *Terebratella wacoensis* with *Exogyra arietina*, although, as above noted, a single specimen of the *Terebratella* has been found by Hill in the Gray-

son at Austin. If *Ostrea quadruplicata* extends up into the Grayson terrane, or beyond the basal transitional part of it, such occurrence is certainly rare.

A few of the species now apparently confined to one of these terranes, will doubtless be found, sooner or later in the other: but, even so, many of the forms that occur in both are extremely rare in one while common in, and so, in a sense, characteristic of the other: and it is obvious that the faunal aspects of the two terranes, like the lithological, are—and are likely to remain—far more conspicuous for their differences than for their resemblances.

DESCRIPTIONS OF INVERTEBRATE FOSSILS FROM THE COMANCHE SERIES IN TEXAS, KANSAS AND INDIAN TERRITORY.

BY F. W. CRAGIN.

The material described in this paper has been derived chiefly from the Arietina beds of northern Texas, a small part of it coming from rocks of lower horizons and elsewhere, as noted under the "Occurrence" of the species. The specimens have been collected by the writer, except as otherwise stated below.

The types of the Kansas specimens are in the paleontological collections of the museum of Colorado College; those of the others, in the private collection of the writer.

The designation, *Kiowa shales*, is proposed for the inferiorly dark-colored and superiorly light-colored shales that outcrop in several of the counties of southwestern Kansas, resting upon the Cheyenne sandstone in their eastern, and upon the "Red-beds" in their middle and western exposures, and being overlaid by brown sandstones of middle Cretaceous age, or Tertiary or Pleistocene deposits, according to locality.

The Kiowa shales are a locally modified northern extension of part of Hill's Comanche series, cut off from the main part by erosion. They are named from the place of their typical occurrence, Kiowa county, Kansas; and in that county they outcrop only in those southern townships which once formed the northern part of Comanche county. The fossils of these shales are chiefly those which, in Texas, are most common in the Fredericksburg division; but a few of them are such as are most characteristic of the Bosque division, and a few others are such as either culminate in or are peculiar to rocks of the Denison beds.

For explanation of the terms, *Choctaw limestone*, and *Grayson marl*, see the preceding article.

Astroecœnia nidiformis, sp. nov.

Stock massive, broad and low, its breadth increasing more or less from the base upward, its summit excavated, the prominent, narrowly-rounded border-region of the summit being irregularly lobed; cells united by rather thicks walls, calyces small, irregularly polygonal or slightly rounded-polygonal; columella short; septa rather stout, their free margins apparently a little uneven, their summits moderately depressed below the level of the calyx-borders, the primary and secondary septa six each, short septa of the third order also appearing.

Measurements.—Maximum breadth of polyp-stock 102, breadth at right angle to line of maximum breadth 95, height 29; average diameter of calyces, including half of the inter-calycular walls 2, thickness of inter-calycular substance .4–.7 (average about .5) mm.

Occurrence.—The type was collected, several years since, from the platform of arenaceous shell-conglomerate that caps the Cheyenne sandstone and forms the base of the Kiowa shales at Belvidere, Kansas. (No. 5 of the writer's Belvidere section.) A second specimen was recently collected by the writer, but was so poorly preserved as to be of little use for purposes of study.

Hemipedina charltoni, sp. nov.

Test small, round-pentagonal, arched above, concave below; apical disc and periproct moderately broad; peristome considerably smaller in proportion to the test than in *Pseudodiadema texanum*, Roemer; pore-belts straight, the pores somewhat elongated in the direction of the belt, the pore-pairs simple throughout and somewhat oblique; ambulacral areæ half as broad as the interambulacral, each ornamented with two rows of primary tubercles that are somewhat smaller than those of the row of largest tubercles on the interambulacral areæ, each primary tubercle being subtended by an irregularly polygonal string of smaller (secondary) ones; interambulacral areæ with six rows of primary tubercles, these being much larger in the middle row of each semiambu-

lacrum than in the others, the primaries subtended by secondaries for the most part in polygonal strings; surface of test closely granulated in the intervals between tubercles on both ambulacral and interambulacral areæ.

Measurements.—Height of test 11, breadth 25, greatest breadth of periproct 3.5, breadth of peristome (about) 7 or 8 mm.

Occurrence.—The type of this species was obtained from the Choctaw limestone, south of the Denison-Bonham road, near the lime-kiln about a mile east of Denison, Texas. It was associated with *Exogyra arietina*, *Ostrea quadruplicata*, *O. subovata*, *Lima wacoensis*, *Pholadomya sancti-sabæ*, *P. ragsdalei*, etc.

The writer takes pleasure in naming this interesting echinoid after his friend and sometime fellow-traveller, Prof. O. C. Charlton.

Besides *Orthopsis occidentalis*, nob., this is the only representative of the section of *Diadematidæ* with perforated and non-crenulated tubercles known from North American rocks.

Although a single *recent* species of *Hemipedina* is known (*H. cubensis*, A. Ag.), this genus is known as a *fossil*, from Jurassic and lower Cretaceous rocks only; and its occurrence in the upper part of the Washita division therefore confirms the conclusion which the writer has previously drawn from the similar occurrence of *Holectypus*, that the Washita division should be referred to an epoch not later than late Neocomian.*

PECTEN INCONSPICUUS, sp. nov.

Shell small, thin, subcircular, a trifle higher than long, slightly truncated anteriorly and posteriorly, right valve gently convex, its outer surface smooth except for faint concentric striæ and a few remote, subimbricate growth-lines; umbonal angle sharp at apex, nearly a right angle; anterior ear (imperfect in the type) reëntrant below, as indicated by the direction of the striæ upon it, outline of posterior ear

*See "Contribution to the Invertebrate Paleontology of the Texas Cretaceous," in the Fourth Annual Report of the Geological Survey of Texas, Part II, p. 159.

making an obtuse angle. its posterior margin rather more than one and a half times as long as its dorsal. Left valve unknown.

Measurements.—Height 9.5, length 8.75, convexity of left valve 1 mm.

Occurrence.—On slope of Pawpaw creek, east of Denison. Texas, in red ochraceous shell-conglomerate of the Pawpaw clays. The associate fossils are *Ostrea quadruplicata. Tapes dentonensis, Yoldia microdonta, Turritella seriatim-granulata, Sphenodiscus, Turrilites,* etc.

VOLA FREDERICKSBURGENSIS, sp. nov.

This name is proposed for the species of Vola described by Roemer from Fredericksburg, Texas, in his *Kreidebildungen von Texas,* as "Pecten quadricostatus, var.," and is based on his description and illustrations (pp. 64. 65; Pl. VIII, fig. 4 *a, b, c*).

The species is the ordinary one of the Fredericksburg division of the Comanche Cretaceous series, being very common in the Comanche Peak limestone of Texas and Indian Territory. and in the lower parts of the Kiowa shales of Kansas.

It is easily distinguished from *V. texanus.* Roemer. by its much narrower and more elevated ribs, more triangular form, and usually (in adult examples) by its larger size.

It has been referred to by authors under various names; but it is distinct from any of the species to which it has hitherto been referred. It is. however, closely allied to *V. alpina,* D'Orb.

AVICULA DISPAR, sp. nov.

Shell small, semicircular, radiately ribbed, compressed, very inequivalve, the left valve being feebly arched, its convexity greatest in the basal region and its basal margin more or less overhanging that of the smaller flat-concave right valve; anterior ears sharply delimited. nearly equilateral triangles; posterior ear feebly developed, narrow, rounded off above on the distal part. that of the left valve not abruptly separated from the body of the valve, that of the right valve

scarcely observable; beaks inconspicuous, the entire umbonal region strongly compressed, the beak of the left valve higher than that of the right; valves thin; ligament double, the outer part placed in an open groove, the inner in a triangular pit under the beaks; hinge-plate of left valve broader than that of the right; radial ribs of left valve large, broad, flat-convex, separated by narrow, abruptly impressed intervals, the intermediate ribs in the type-specimen not extending over the somewhat strongly convex basal region, which is smooth or finely concentrically striate and pearlaceous, but suddenly truncated; ribs of right valve feebly developed, narrow, irregularly tuberculated, obsolete on the ventral and anterior parts; radial groove separating anterior ear from body of valve, in the left valve, deeply impressed, and having its counterpart on the right valve in a low ridge.

Measurements.—Height 20, length 33, breadth 5 mm.

Occurrence.—The type of this species is a finely preserved shell found by the writer in the Grayson marl, on a draw of Pawpaw creek, half to three-quarters of a mile southeast of the Union depot at Denison, Texas. It was associated with *Exogyra arietina*, *E. drakei*, *Turrilites brazosensis*, etc., in the lower part of the marl.

The ribs of the body of the left valve are fourteen in number, and adjacent to either ear is a radially striated segment about as wide as the widest rib. The anterior ear of the left valve shows two or three rather coarse ribs on the posterior part, the posterior ear being ornamented with radiating raised lines. In the narrow intervals between several of these lines and between several of the broad ribs of the posterior part of the body of the left valve, the hand-lens reveals more or less regularly arranged elements that present the appearance of cross-threads or obliquely compressed tubercles.

I am not acquainted with any species that is closely analagous to *Avicula dispar*.

Inoceramus comancheana, sp. nov.

Shell equivalve, obliquely and broadly rhombic-ovate, more gibbous than that of *I. labiatus*, Schloth., the axis of

greatest dimension diverging from the hinge-line much more widely than in the latter species; alar outline rounded; anterior margin descending steeply in a nearly straight line for a considerable distance in front of the beaks, then curving rather suddenly away toward the somewhat prominently convex distal part of the basal outline, anterior and posterior margins making nearly a right angle with each other; beaks placed opposite the anterior extremity of the hinge, moderately inflated, and moderately elevated above the hinge-line, anteriorly flattened, but not abruptly so; hinge-plate rather short, broad, the ligamental grooves crowded, more numerous and longer than in *I. labiatus*, though ample and shallow; valves thin, ornamented with numerous concentric rib-like folds, which, on the discal and ventral parts, are quite strongly elevated and much narrower than the intervals between them.

Measurements.—Height 82, length 95, breadth 54, axis of greatest dimension 107 mm. In some examples, the species attains considerably larger dimensions.

Occurrence.—This is the common *Inoceramus* of the Duck creek (lower Washita) limestone, or basal part of the Washita division of the Comanche series. The types were collected by Mr. J. T. Munson and the writer one to two miles northeast of Denison, Texas. The species occurs also in the Chickasaw nation, three miles north-northwest of Marietta, in the same limestone. At both localities it is associated with *Pachydiscus marcianus*,* *Schloenbachia peruviana* and *S. serrtaescens*, *Hamites fremonti*, *Epiaster elegans*, var. *prænuntius*, etc.

In the upper part of the Kiowa shales of southern Kansas, occur imperfect casts of an Inoceramus which I provisionally refer to this species. In Clark county, Kansas, these are associated with *Ostrea quadruplicata* and *Ostrea franklini*.

**Ammonites* [*Pachydiscus*] *brazoensis*, Shum. (1860), is a synonym of *A.* [*P.*] *marciana*, Shum. (1854); but I am somewhat doubtful about the propriety of using the older name, as the description and figure accompanying it are those of a very young specimen. It seems a strange circumstance that Dr. Shumard failed to refer to the two names as synonymous. This circumstance, however, may indicate either that he did not recognize in "*brazoensis*" his "*marciana*" of 1854, or that he merely considered the older name invalid, as based on a young specimen and one that did not show the suture.

In Marcy's Red River Report (p. 193, Pl. 6, fig. 2), Shumard describes and figures an Inoceramus that "occurs rather abundantly at Camp No. 4. Cross Timbers, Texas." referring it to *I. confertim-annulatus*, Roem. That his description and figure relate, not to the upper Cretaceous *confertim-annulatus*, but to the species which I have here named *I. comancheana*, is indicated not only by the figure that he gives but also by the fact that the stratigraphic source of all of the fossils that he described from "Cross Timbers. Texas," was evidently the middle part of the lower Cretaceous; *i. e.*, the Comanche Peak limestone and the Duck creek (lower Washita) limestone. Nearly all of these fossils are common in, and chiefly characteristic of, the Comanche Peak limestone; but the association of *Schloenbachia peruviana*, Von B., with *Pachydiscus marcianus*, Shum.,* which he mentions, is a feature of the Duck creek horizon and indicates the presence of this horizon there also, and hence the probable presence there of the common Duck creek fossil. *Inoceramus comancheana*.

INOCERAMUS MUNSONI, sp. nov.

Shell (?inequivalve), elevated, obliquely triangular-ovate, of moderate convexity; hinge short; beak of left valve elevated high above the hinge-line, strongly flattened on its right (inner) anterior quarter, its apex incurved as in *I. sulcatus*, Park.; valves extremely thin, moulded with small, feebly expressed, concentric undulations and striæ and, on its anterior third, with three large and prominent, wave-like, radial folds, the foremost and shortest one of which forms a sort of shoulder in advance of which the border of the shell is strongly inflected to form the flat ante-umbonal area, the part of the valve posterior to these three folds, presenting two broad, obsolescent, low-convex, radial segments, the posterior one of which may be subdivided into two narrow folds.

Measurements.—Of a left valve: height 57, length 53, convexity 23, axis of greatest dimension 67 mm.; of a smaller left valve, height 36, length 33, convexity 11 mm.

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* His *Ammonites acuto-carinatus* and *Ammonites marciana*. See l. c., p. 197.

Occurrence.—In the Duck creek limestone, on Duck creek, between one and two miles northeast of Denison, Texas.

This fossil is much less common than the *I. comancheana*.* The specimens occur as casts, with occasional remnants of the shell.

The species is named after Mr. J. T. Munson. of Denison, in recognition of the generous aid which he has shown himself every ready to lend to scientific research.

NUCULA CHICKASAENSIS, sp. nov.

Shell small, of moderate convexity, though more convex than that of *N. catherina nobis*, ovate-triangular, the anterior side descending directly and subvertically from the low, nearly terminal beaks, giving the anterior part of the profile an obliquely truncate appearance, posterior cardinal side slightly convex, posterior extremity somewhat narrowly rounded, base long, gently convex, and suddenly curving upward at its anterior end; shell divaricately ornamented with fine and close raised lines and striæ, each of which has the apex of its V-like angulation directed toward the beak, the anterior arms of the V's being gently concave toward the anterior, or supero-anterior part of the shell, the posterior arms presenting a gentle concavity upward and backward, the angle of divarication, at apex, somewhat less than a right angle.

Measurements.—Height 11.5, length 14, breadth 8 mm.

Occurrence.—The type of this species was collected by my wife from the Comanche Peak limestone, overlooking Little Hickory creek south of Overbrook in the Chickasaw

* The writer would here note that the species, "*Inoceramus multistriatus*," which he published in his "Contribution to the Invertebrate Paleontology of the Texas Cretaceous," is invalid, having been based on a mutilated specimen of *Avicula pedernalis*, as he discovered when, on his expedition of July to September, 1893, he collected specimens of the latter fossil, no specimen of which, sufficiently well preserved to be easily recognized by one not previously acquainted with the species, had been available for his study in the museum of the State Geological Survey prior to his leaving Austin. Not suspecting the former presence of the large wing, which had been broken off, on the imperfect specimen examined, the writer was led to refer this specimen to the genus *Inoceramus*, in view of its transversely fibrous shell-structure and of its degree of resemblance in form to *Inoceramus sublævis*, overlooking the fact that the shell of *Avicula* also has such structure.

Nation. It was associated with *Holectypus planatus*, *Exogyra texana*, *Turritella seriatim-granulata*, etc.

The species belongs to the subgenus *Acila*.

CARDIUM QUINORDINATUM, sp. nov.

Shell of rather small size for its genus, rounded-triangular, subequilateral, the anterior and posterior sides of the outline slightly concave in the upper and convex in the lower part, the basal margin being evenly rounded, the part of the outline formed by the arched umbonal summit being very obtuse; beaks subcentral, large, arched strongly, the arch rising high above the hinge-line: surface ornamented with something like twenty broad, prominent, rounded, spinigerous costæ separated by narrow plain-bottomed intervals, the costal spines being of the squamous type, having the form of caret-shaped hoods, and being closely set in five radial ranks on the same number of slightly elevated rays on each costa, the rays being separated by striæform grooves. The intercostal valleys are a little less than two-fifths as wide as the costæ. Of the five spinigerous rays on each costa, the middle one is relatively larger and more coarsely spined than those adjacent to it, the two outermost being the smallest of all.

Measurements.—Height 21, length 19, breadth 16 mm.

Occurrence.—There is a little doubt as to the source of the two type-specimens of this species. They were, however, almost certainly collected at a ledge of the Washita limestone a little east of Georgetown, Texas, in association with *Ostrea roanokensis*, *Pleurotomaria robusta*, and *Schloenbachia anatina*.

ROUDAIRIA DENISONENSIS, sp. nov.

Shell among the larger representatives of its genus, elevated-triangular, the lateral profile of its cast approaching an isosceles triangle with convex base, but the supero-posterior part of that profile presenting, as in *R. quadrans*,* a low

* See *American Geologist*, Vol. XIV, Pl. I, fig. 14. *R. quadrans* should be compared with the shell described in the writer's "Contribution to the Invertebrate Paleontology of the Texas Cretaceous," as *Trigonia securiforms*, with which it is quite likely that it will prove to be identical. There can be little doubt that the latter species belongs to the genus *Roudairia*.

shoulder (the angulated summit-line of the feeble keel made by the apposed edges of the valves) a short distance from the beaks; valves of quite moderate convexity; beaks placed a little in advance of the middle, narrow, their tangent apices curved inward and downward, but inclined forward in only a very slight degree; anterior and posterior slopes more or less flattened, the posterior separated from the discal slope, in the cast, by a faint angulation; discal and anterior slopes ornamented with unequal concentric lines, and the posterior slope with 35 or more radial raised lines, which are relatively much finer than those of *R. quadrans*.

Measurements.—Height 78, length 69, breadth 50 mm. A second specimen is relatively longer, measuring, height 71, length 67, breadth 46 mm.

Occurrence.—The types of this species are several casts, two of which show the character of the ornamentation. They were collected by Mrs. F. W. Cragin, Mr. J. T. Munson, and the writer, chiefly from the Grayson marl, at the old D. B. and N. O. railway cut about half a mile southeast of the Union depot of Denison, Texas. A single example, somewhat smaller than those from the marl, was obtained from the Main-street limestone on the summit of the creek-bluff adjoining.

Among the considerable number of fossils found associated with the *Roudairia denisonensis*, are *Exogyra arietina*, *E. drakei*, *Vola texana*, *Avicula dispar*, *Sphenodiscus pedernalis*, *Turrilites brazosensis*, etc.

PHOLADOMYA RAGSDALEI, sp. nov.

Shell large, triangular-ovate, compressed, multicostate; umbonal region somewhat elevated, but beaks not strongly arched; much the larger part of the shell ornamented with straight to slightly curved radial costæ, which are narrow like the intervals between them and more or less crossed and interrupted by raised, concentric growth-lines; anterior area of shell ornamented with a system of concentric, strongly curved, loop-like to more or less nearly quarter-circlet costæ which are quite independent of the concentric, elevated growth-lines and are severally continuous at their attenuated posterior

ends with the similarly attenuated supero-anterior ends of the radial costæ, forming with the latter, at the place of meeting, a series of beakward-pointed V-like angles.

Measurements.—(Approximate.) Height 70, breadth 10, length 100–105 mm.

Occurrence.—Several molds of this species were found by Mr. J. T. Munson and the writer in the Choctaw limestone, with *Ostrea quadruplicata, Exogyra arietina, Terebratella wacoensis*, etc., on the Pawpaw creek bluffs east and southeast of Denison, Texas. For an example from Cooke county, Texas, showing the shell itself and the major part of both valves, the writer is indebted to Mr. G. H. Ragsdale, after whom the species is named.

A loan-collection now in hand contains one example of this beautifully and uniquely marked species from a county in Texas south of the Red river tier, but it is not now possible to ascertain its exact source.

As all of the specimens of this fossil that the writer has seen are more or less imperfect, there remains some doubt as to the generic position. But the species is so striking and unique in the character of its ornamentation that there will rarely be difficulty in recognizing it, even in the imperfect specimens in which it commonly presents itself.

HOMOMYA WASHITA, sp. nov.

Shell large, curved-oblong, closed or nearly closed anteriorly and closed along the dorsal margin back of the beaks, obliquely truncated and gaping posteriorly, the breadth usually a little greater than the vertical dimension from hinge-margin to ventral margin, the length equal to somewhat more than one and a half times the breadth, the greatest breadth being about half way between the beaks and the mid-region; beaks nearly terminal, low, swollen, obtusely tangent, their bases long in the direction of the length of the shell and rising at a very low angle from their posterior origin to their rather broadly rounded summits; surface marked only with concentric growth-lines and undulations, and sometimes showing distally two or three broad growth-zones or stages.

Measurements.—Height (to summit of beaks) 92, vertical dimension from base to hinge-line 76, length 120, breadth 80 mm.

Occurrence.—With *Cyphosoma volanum*, *Enallaster inflatus*, *Ostrea roanokensis*, *Exogyra arietina*, *E. drakei*, numerous large *Vola texana*, *Roudairia denisonensis*, and many other fossils, in the Grayson marl in the abandoned D. B. and N. O. railway cut about half a mile southeast of the Union depot of Denison, Texas; also in the Choctaw limestone, with *Ostrea quadruplicata*, *Exogyra arietina*, *Terebratella wacoensis*, *Pholadomya ragsdalei*, etc., on summit of Pawpaw creek bluff southeast and east of Denison. The species is apparently more common in the limestone than in the marl, but the specimens thus far observed from the former are usually smaller and more poorly preserved than those from the latter.

Tellina subæqualis, sp. nov.

Shell subæquilateral, elongate-elliptical, the height contained twice in the length, the outline a little narrowed toward either extremity, slightly more toward the posterior than toward the anterior, the dorsal outline gently declivous and nearly straight for some distance on either side of the beaks, the anterior outline evenly rounded, angulation of the posterior slope scarcely appreciable in the cast, valves of moderate convexity, the curvature across the discs being remarkably uniform from a little below the umbonal summits to the basal margin and from the anterior to the posterior extremity; beaks low and obtuse; pallial sinus extending forward a little beyond the point directly opposite the beaks, its fundus rounded.

Measurements.—Height of cast 14.5, length from anterior extremity to a point opposite the beaks 13.5, from posterior extremity to the same point (about) 15.5, convexity of cast of right valve about 3 mm.

Occurrence.—On Pawpaw creek, east of Denison, Texas, in ochraceous shell-conglomerate of the Pawpaw clays; with *Yoldia microdonta*, *Turritella seriatim-granulata*, etc. Occurs chiefly as casts with fragments of shell, and as molds.

This *Tellina* is closely related to *T. equilateralis*, M. and H., of the Fox Hills division of the upper Cretaceous; but comparison of the Denison specimens with Meek's figures of that species shows that they represent a decidedly more elongate form than the latter.

Corbula crassicostata, sp. nov.

Shell triangular-ovate, gibbous, nearly as broad as high, short; gaping posteriorly by a short, conically inflated, gently truncated rostrum, which is placed high above the base of the shell; umbones placed in advance of the middle, that of the right valve only moderately high arched, its summit obtuse; surface ornamented with very coarse, flattish-topped, concentric ribs, separated by abrupt, deep, narrow intervals. There are seven or eight of the ribs on the basal half of a right valve the same number of millimeters high.

Measurements.—Height 7.5, length 10, breadth about 7 mm.

Occurrence.—In arenaceous limestone bands of the Kiowa shales, at Belvidere, Kansas; in Nos. 2–4 of the writer's "Belvidere Section."

So far as the writer can judge from material now in hand, the similar *Corbula* that abounds in the condition of casts and molds in the ochraceous shell-conglomerate of the Denison beds, at Denison, Texas, presents no differences of specific value from the Kansas shell above described. The casts show that the pallial line is very sharply impressed.

Margarita brownii, sp. nov.

Shell of moderate size and thickness, the postlabial region becoming thicker at maturity, depressed-conical, deeply umbilicated; whorls four and a half, somewhat depressed, rounded, smooth, the body-whorl obtusely angulated below, becoming much thickened and tumid on the part near the aperture, the tumid portion, however, flattened a little on the quarter between base and umbilicus; aperture round, much smaller than the nearly oblong and sharpened peristome, the latter being continuous except on about one-fifth of the circuit, and

the beveled zone between the aperture and the peristomial edge being continuous around the aperture, broadest where it rests against the whorl above, narrowest on the basicolumellar quarter: inner lip free opposite and below the umbilicus, adnate above it, the free portion everted so as to encroach slightly upon the umbilicus and to overarch about one-third of it; umbilicus somewhat narrowed, but deep, striated or costellate, each costella terminating inferiorly in a slight enlargement, the series of narrow tubercles thus produced giving the angulated border of the umbilicus a crenulated aspect; umbilico-basal angle not extending over the tumid labial region. The fine growth-lines of the shell are directed obliquely backward from the suture above, on the upper part of the whorls, becoming transverse on the lower part of the body-whorl.

Measurements.—Height 10.5, breadth 14.5 mm.; divergence of slopes 109 degrees.

Occurrence.—In the Caprina limestone of Travis county, Texas, in the south bluffs of the Colorado river, west of South Austin; associated with *Requienia patagiata*, *Monopleura marcida*, *M. pinguiscula*, *Lucina acute-lineolata*, *Nerinea pellucida*, and other fossils of the Barton creek and Deep Eddy bluff fauna.

For the type-specimen, I am indebted to Prof. B. S. Brown, after whom the species is named. The specimen, like others of this fauna, is beautifully preserved in calcite, and has a bright red tinge, due to a thin incrustation of iron oxide.

NERITOMA MARCOUANA, sp. nov.

Shell small, of moderate thickness, depressed-subglobose, oblique, consisting of apparently three and a half whorls; spire sublateral, small, eroded; body-whorl large, ventricose, evenly rounded, nearly smooth, its upper part with feebly elevated costellæ, extending obliquely upward (that is, toward the suture and at the same time somewhat toward the aperture), and separated by round-bottomed, groove-like intervals of about the same breadth, that begin, in part abruptly, at or just above the periphery; periphery and base of body-whorl

smooth, or marked only with ordinary growth-lines; aperture obliquely and rather narrowly ovate; inner lip with a callous, strongly flattened, and without teeth; outer lip with a moderate, shallow, broadly rounded excavation just below the peripheral line. No umbilicus. The uneroded portion of the spire rises but little above the body-whorl.

Measurements.—Height 10, breadth 10.5 mm. Costellæ of body-whorl, about 3 in 2 mm. A Belvidere example collected a few years ago, and not at hand for measurement, is somewhat larger than the type-specimen from which these dimensions are taken.

Occurrence.—In the Kiowa shales of Kiowa county, Kansas, in No. 3 and the upper part of No. 4 of my Belvidere section. In the past ten years, the writer has collected several specimens of this shell, of which two or three were obtained at Belvidere; and one, of probably—though he cannot say positively—the same species, in limestone-bands in these shales, just west of Windom, McPherson county, Kansas.

This adds another shell of *Jurassic* type to the fauna of the Pre-Washita part of Hill's Comanche series.

The species is named in honor of Mr. Jules Marcou.

SOLARIUM CHICKASAENSE, sp. nov.

Shell rather small, discoidal, slightly concave above, more so below; whorls five, all exposed above and (probably all) below, in contact and slightly embracing, *inequilaterally* rounded-subquadrate in cross-section, broader than high, flattened above, the upper surface of each whorl dropping a little lower than that of the whorl next outside of it, the superior suture distinct, inferior surface of body-whorl raised into a somewhat prominent rounded shoulder, periphery of body-whorl rounded; cross-section of cavity near aperture oblique, inequilaterally ovate, or rounded-triangular, transverse (that is, broader than high), but far less so than in Roemer's *Solarium planorbis*, outline of the cavity and that of the exterior of the shell in the cross-section not parallel with each other, the shell being relatively much thickened at the corners; upper side of whorls marked with ordinary

growth-lines which are crossed by a series of subremote revolving striæ, the upper centripetal border of the whorls being puckered so as to give it a crenulated appearance; ornamentation of lower surface of shell unknown.

Measurements.—Greatest breadth of shell 13.5, height (that of the body-whorl) 4.3, height of cross-section of cavity near aperture 2.5, breadth of same 3 mm.

Occurrence.—A single specimen of this shell, *in situ*, was found by the writer in the Comanche Peak limestone on a south branch of Little Hickory creek, a few miles north and a little west of Marietta, Indian Territory. It was associated with *Enallaster texanus*, *Vola fredericksburgensis*, *Turritella serialim-granulata*, *Tylostoma tumida*, and *Schloenbachia peruviana*.

Viewed from above, this shell bears close resemblance, even in the oblique and slightly wavy apertural border, to *Solarium planorbis*, Roemer, as figured in Palæontologische Abhandlungen, Band IV, Tafel XXXI; but in apertural view (a section immediately back of the aperture), the body-whorl is seen to be quite different from that species as figured, with respect to shape, thickness of different parts of the section of the shell, and form of the cavity. The apertural view of the present shell does not exhibit that very strongly and equilaterally compressed form of whorl, with parallel-outlined and uniformly thick-walled cavity, that Roemer's figure represents (fig. 14 c, *loc. cit.*). Indeed, that view of the present shell may be described as intermediate between the figure just cited and that of *Euomphalus subquadratus*, M. & W., given in Volume V of the Geological Survey of Illinois, Pl. XXIX, fig. 13 b, so that it would seem not an unreasonable question to ask, whether the species could not be referred to the genus *Straparollus* (which survived till the Jurassic, according to Zittel), quite as well as to the genus *Solarium*.

It may be of interest to know, in comparing Austin rocks with those of the Red river region, that in the same limestone, and not far away from the locality that yielded this close ally of *Solarium planorbis*, the writer has found one of the specific members of the peculiar Barton creek fauna of

the Caprina limestone that yielded the *S. planorbis;* viz., *Natica avellana*, Roem.

TURRITELLA DENISONENSIS, sp. nov.

This name is proposed for a large, tall-spired, species of *Turritella* that seems to bear considerable resemblance to *T. leonensis*, Con., and is possibly only a variety of that species, but which, instead of having all of the whorls convex and evenly rounded, as they are understood to be in *leonensis*, has only the spire-whorls so, the body-whorl being enlarged and angulated or shouldered, presenting a strongly flattened, sloping face on its upper (posterior) part, a broader flattened face on its middle part, and a less strongly flattened face on its lower part. The casts show weathered remnants of raised, narrow, cariniform lines, separated by broad, depressed intervals, in each of which is a similar but feebler line: lines of the body-whorl apparently irregularly turberculated, those of the spire-whorls not obviously so on the only shell-fragment observed. The shell has evidently consisted of not less than ten or eleven whorls.

Measurements (of cast).—Height about 107, breadth of body-whorl 40 mm.; angle of spire-slopes 21 to 24 degrees.

Occurrence.—The casts of this shell abound in the Choctaw limestone, at the top of the bluff of Pawpaw creek east and southeast of Denison, Texas. A single example was found in the Grayson marl, in the cut of the abandoned D. B. and N. O. railway, southeast of the Denison Union depot.

VANIKORO PROPINQUA, sp. nov.

Shell rather small, depressed-subglobose, thin or of moderate thickness; whorls four, convex, those of the spire not prominently so; body-whorl greatly enlarged, rounded, somewhat narrower and more elevated than in *V. ambigua*, M. & H.; spire rather low, proportioned almost exactly as in *V. ambigua;* suture not deeply impressed; axis (? perforate); aperture rhomboidal-ovate, angular above, obtuse below; ornamentation unknown.

Dimensions.—Somewhat smaller than *V. ambigua*, M. & H., the exact dimensions not mensurable owing to the imperfec-

tion of the labial region. Angle of slopes of spire a little less than 90 degrees.

Occurrence.—In Nos. 3–4 of the Belvidere section of the Kiowa shales, near Belvidere, Kansas. I have seen but one specimen.

This shell bears a striking resemblance in form to that of *V. ambigua* as figured by Meek in Vol. IX of the Hayden U. S. Geological Survey, differing from it chiefly by the relatively somewhat more elevated body-whorl and aperture and the smaller size. The surface of the shell is somewhat weatherworn in the type and does not reveal its original sculpture.

Anchura kiowana, sp. nov.

Shell small, consisting of six convex whorls; spire elevated; suture impressed; wing of moderate size, consisting of a proximal flange-like part, continued posteriorly across half or more of the first spire-whorl, and a carinated falciform process; carina gradually arising at the base of the falciform process and traversing the latter to the extremity; falciform process much shorter and less upturned distally than that of the somewhat similar species, *A. ruida*, White, not rising to the lowest level of the suture between the body-whorl and the first spire-whorl, but having its point directed outward and somewhat upward, so as to make a large angle with the axis of the spire, extero-inferior outline of wing rounded and the border between this and the canal sinuous, margin of upper (flange) part of wing describing a slightly concave to sigmoid outline and more or less thickened and reflexed; inner lip provided with a moderately broad and prominent callous; canal short and obliquely truncated; spire-whorls and posterior half to two-thirds of body-whorl ornamented with narrow, curved, subvertical ribs, or folds, of which there are about twenty-four on the first spire-whorl, and with numerous revolving striæ, the latter ornamentation gradually becoming prominent and superseding the ribs on the lower third to half of the body-whorl.

Measurements.—Height 19, breadth of body-whorl, including excursion of wing, 15 mm.; angle of spire-slopes about as in *A. ruida*, White.

Occurrence.—This handsome little shell is common in the Kiowa shales of Kiowa county, Kansas, especially in the lower part of No. 3 and the upper part of No. 4 of the writer's "Belvidere Section." (See Nos. 9 and 11 of the *Bulletin of the Washburn College Laboratory of Natural History*, and *American Geologist* of January, 1891.)

Some of the commonest of the many fossils associated with it are *Gryphæa pitcheri*, *Cardium kansasense*, *Cyprimeria texana*,* *Tapes belviderensis*, *Turritella serialim-granulata*, and *Schloenbachia peruviana*.

The *Anchura* also occurs in the arenaceous platform of shell-conglomerate forming No. 5 of the same section, the examples from this horizon, like the *Turritellæ* from the same, being larger and more coarsely ornamented than those from Nos. 3 and 4. The flange of the wing also extends further upward in specimens from No. 5, sometimes reaching to the lower part of the second spire-whorl.

The writer has also collected this fossil in the lower third of the Kiowa shales in Clark county, Kansas, in a draw of Bluff creek nearly opposite the mouth of Hackberry creek.

The species is distinguished from *Anchura ruida*, White, by the vertically costate body-whorl, by the shorter, differently directed falciform process, and by having the alar carina confined strictly to the wing and nearly to the falciform process, instead of being common to the wing and part of the body-whorl.

NAUTILUS WASHITANUS, sp. nov.

Shell large, compressed nearly as much as in *Nautilus neocomiensis*, D'Orb., the size, form and ornamentation being essentially as described by Shumard for his *Nautilus texanus*,† the siphuncle, however, placed between the middle and the *dorsal* (outer) side of the septum and (sometimes, if not always) nearer to the latter side than to the middle.

Occurrence.—Common in the Washita limestone of Texas.

**Cyprimeria gradata*, Cragin, which will probably prove to be the same as *C. texana*, Roemer, when the hinge-details and form-variations of the latter shall be fully discovered.

†Transactions of the St. Louis Academy of Science, I, 590.

Two species of *Nautilus* are common in the rocks of the Washita division, viz., *N. texanus*, Shumard, and *N. washitanus*, nobis, which have hitherto been confounded, owing to their general external resemblance and the fact that, occurring more commonly as fragments than as complete specimens, they often cannot be easily distinguished except by the position of the siphuncle, which is frequently not shown.

All of the critically available specimens of *Nautilus* that the writer had observed from rocks of the Washita division, before the publication of his "Contribution to the Invertebrate Paleontology of the Texas Cretaceous," had the siphuncle in the dorsal (outer) half of the septum, and the writer therefore stated in that report that Shumard was in error in alleging its position to be ventral. The writer is now acquainted with *two* types of *Nautilus* from the Washita division and now admits the accuracy of Shumard's description. So far as observed, however, the *common* species of the Washita limestone is *N. washitanus*, the *Nautilus* of the Denison beds being ordinarily *N. texanus*, as shown by its having the siphuncle nearly at the limit between the *ventral* (inner) and the middle third of the septum. The latter species has been found by the writer in the Denison beds of Denton and Grayson counties, Texas, and ranges up into the highest terrane of these beds, viz., the Grayson marl.

VERTEBRATA FROM THE NEOCOMIAN OF KANSAS.*

BY F. W. CRAGIN.

The "Belvidere section," "Blue Cut Mound section," and "Bluff Creek section," mentioned below are the geological sections which the writer published several years ago in the *Bulletin of the Washburn College Laboratory of Natural History* (No. 11), and in Vol. VII of the *American Geologist.*

The first and second are in the southeastern part of Kiowa county, Kansas, the third in the northeastern part of Clark county.

The shales in which all of the described fossils were found belong clearly to the Fredericksburg division of the Comanche series, as shown by the occurrence in them of *Sphenodiscus pedernalis*, Roem., *Schloenbachia peruviana*, Von B., *Holectypus planatus*, Roem., *Exogyra texana*, Roem., and many other invertebrate fossils of that division.

That the Fredericksburg division corresponds to a part of the European Neocomian, is clearly shown by its echinoderm fauna, which, barely represented in Kansas, is well developed in Texas.

PLESIOSAURUS MUDGEI, sp. nov.

Plate I, figs. 1-3 (? and also fig. 1).

About the size of *P. neocomiensis*, Campiche, as tested by comparison of the type (Kiowa county) and Clark county dorsal vertebræ with the measurements which Lydekker gives of the casts of dorsal vertebræ of that species presented to the British Museum by M. Campiche, but differing from *P. neocomiensis* in the form of the vertebræ; dorsal vertebræ moderately cupped, their centra having the three dimensions (length, height, and breadth across ends) nearly equal; constriction as shown in figs. 2 and 3 of the plate.

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*A private edition of this article, without the plates and with a different pagination, was published May 12th, 1894. The shales herein referred to are the *Kiowa shales* of the later-written articles on geology and invertebrate paleontology of the Comanche series, published herewith.

Femur and humerus (as indicated by a Belvidere, Kansas, specimen belonging with little doubt to this species) proximally enlarged so as to form a sort of head, slightly deflected, or asymmetrical, but without any distinct neck and trochanter such as are seen in *Trinacromerum bentonianum*, Crag.; flattened and laterally expanded at the distal extremity, and provided with two distal facets.

Measurements.—Dorsal vertebra of type-specimen from the Blue Cut hill: length of centrum 45, height of same 44, height of vertebra to floor of neural canal 46, breadth of centrum at either extremity 45 mm. Humerus (or femur) from about one mile south of the Belvidere railway station, provisionally referred to the same species: length (minus epiphyses) 167 (as restored 171), girth at point of greatest constriction 120, breadth at same point 41, greatest distal breadth (estimated) about 70, thickness at distal part about 35 mm.

Occurrence, etc.—Remains of *Plesiosaurus mudgei* are common in the Fredericksburg (Neocomian) shale of Kiowa and Clark county, Kansas.

My announcement, several years since, of the occurrence of Plesiosaurid remains in the Neocomian of southern Kansas was based upon the discovery of two vertebræ of *Plesiosaurus* in No. 5 of my "Blue Cut Mound section," one of which vertebræ, attached to a specimen of *Gryphæa pitcheri* (the vertebra illustrated on Plate I of the present article and constituting the type of *P. mudgei*) was turned over to the writer by the collector, Mr. A. L. Diamond.

A year or two later, I examined a vertebra, then in the possession of Mr. Henry Fares, which agreed closely with the former one and which had been obtained with others in central Clark county.

Last autumn, fragmentary remains of what is probably the same species were found by the writer at several points near Belvidere, Kansas, in No. 3 of the "Belvidere section."

The best of the last-mentioned remains, a femur or humerus, is figured on Plate I with the type-vertebra, and indicates paddles whose size is quite consistent with that of the animal indicated by the Kiowa and Clark county vertebræ.

Plesiochelys belviderensis, sp. nov.

Plate II, figs. 1-8.

Shell of moderate size, that of the type-specimen considerably smaller than that of *P. solodurensis*, Rüt., anterior costal bones spearhead-shaped, or triangular, pointed at the distal end, their breadth contained twice or a little more than twice in their length, the articulation for the first marginal bone being relatively much shorter than in *P. solodurensis* – that is, equal to one-fifth of the length of the costal itself; neural bones narrow and elongate, the posterior lateral articulation for the costal bone much longer than the anterior and nearly straight (at least not distinctly of exteriorly concave outline in the one well-preserved neural of the type-specimen), anteriorly emarginate, the emargination in the type-neural being ample, deep and trilateral; vertebral isthmus of type-neural expanded beneath the middle of the bone so as to form a thick, nearly rhombic, plate-like structure; upper surface of shell (costal bones) ornamented with delicate vermicular grooving and pitting.

Measurements.—Length of anterior costal bone 82, its greatest breadth 38; minimum breadth of narrowest preserved costal bone 24, maximum breadth of broadest 40; length of one of the largest costals (nearly entire) 111; length of neural bone 35, greatest breadth of same 24, posterior breadth of same 14; length of a dorsal vertebra 32, anterior breadth of same 23, posterior breadth of same 17 mm. If proportioned to the anterior costal bone as in *P. solodurensis*, the carapace of the type-specimen would have a length of some 300 millimeters, or about a foot.

Occurrence.—The type of this species, consisting of the two figured anterior costal bones, several other more or less complete costals, the figured neural bone, and the figured (in figs. 6 and 7 inverted) vertebra, were found by the writer with the vertebra of (*? Lamna*) *quinquelateralis* about half a mile south of the railway station at Belvidere, Kansas, at the upper limit of the black shale that constitutes No. 4 of the "Belvidere section."

Mesodon abrasus, sp. nov.

Plate II, figs. 18 and 20.

This name is proposed for certain pycnodont teeth of low, rhomboidal form and feebly convex upper surface which occur in No. 3 of the "Belvidere section," southwest of the Belvidere railway station, and seem to agree with the larger mandibular teeth of *Mesodon*. The specific name refers to the occurrence in the type-specimen (see fig. 18) of two small oblique facets produced at one end by attrition.

Measurements.—The type has a height (above root) of 3, a length of 13, and a breadth of 6 mm.

To the vomerine set of the same species may belong the rotund-oval, or nearly hemispherical teeth of similar height but smaller size which occur not uncommonly at the same locality and horizon, the largest now available example of which (see figs. 15–17) measures about 6 and 7 mm. in minor and major horizontal diameters.

(? Lamna) quinquelateralis, sp. nov.

Plate II, figs. 9 and 10.

The specific name, *quinquelateralis*, is applied to a species of shark whose vertebræ differ from all others of which I have any knowledge. The type-vertebra is short, much broader than high, shallow-cupped, and more or less sharply pentagonal ended.

Measurements.—Height 20, length 12, breadth 28 mm. The two upper angles measure each about 130 degrees; either lateral angle about 105 degrees. The lower angle is broad and rounded.

Occurrence.—A single vertebra of this form was found by the writer at Belvidere, Kansas, with the above-described remains of *Plesiochelys*, in the upper part of No. 4 of the "Belvidere section."

Hybodus clarkensis, sp. nov.

Plate II, figs. 11–14.

Fin-spine large, gently recurved, laterally compressed, the sides being nearly flat, the anterior border subacute, or forming a sort of keel, the posterior part beveled on either side

of the plate which bears the denticles, its general surface minutely and unevenly striate and punctate-striate, there being on the anterior part, a little back of the border (on one side only as preserved in the type-specimen) one or two narrow, low, longitudinal folds, or carina-like ridges; the greatest breadth of the cross-section of the spine in the posterior part of the denticuliferous portion contained twice in the length of that section; denticles proportioned, arranged, and recurved somewhat as in *H. reticulatus*, Ag., as figured by Zittel, their compressed convex border with delicate but salient carina, their broader, concave border rounded.

For minor details, see illustrations, of which figs. 11–13 are in part restored, and fig. 14 is a cross-section. The smaller and more compressed denticles in fig. 12 represent the average size of the denticles more accurately than the large ones.

Measurements.—Greatest breadth of the preserved part of spine 13 mm. The entire length of the spine (of which a length of a little more than 60 mm. is preserved) cannot have been much less than a foot.

Occurrence.—The type-spine of this species was found and presented to the writer by a member of the family of Mr. W. E. Brown, in Clark county, Kansas, in the area of the outcrop of the Fredericksburg Cretaceous shale, at the south end of the "Amphitheatre" mentioned in connection with the above-cited "Bluff Creek section."

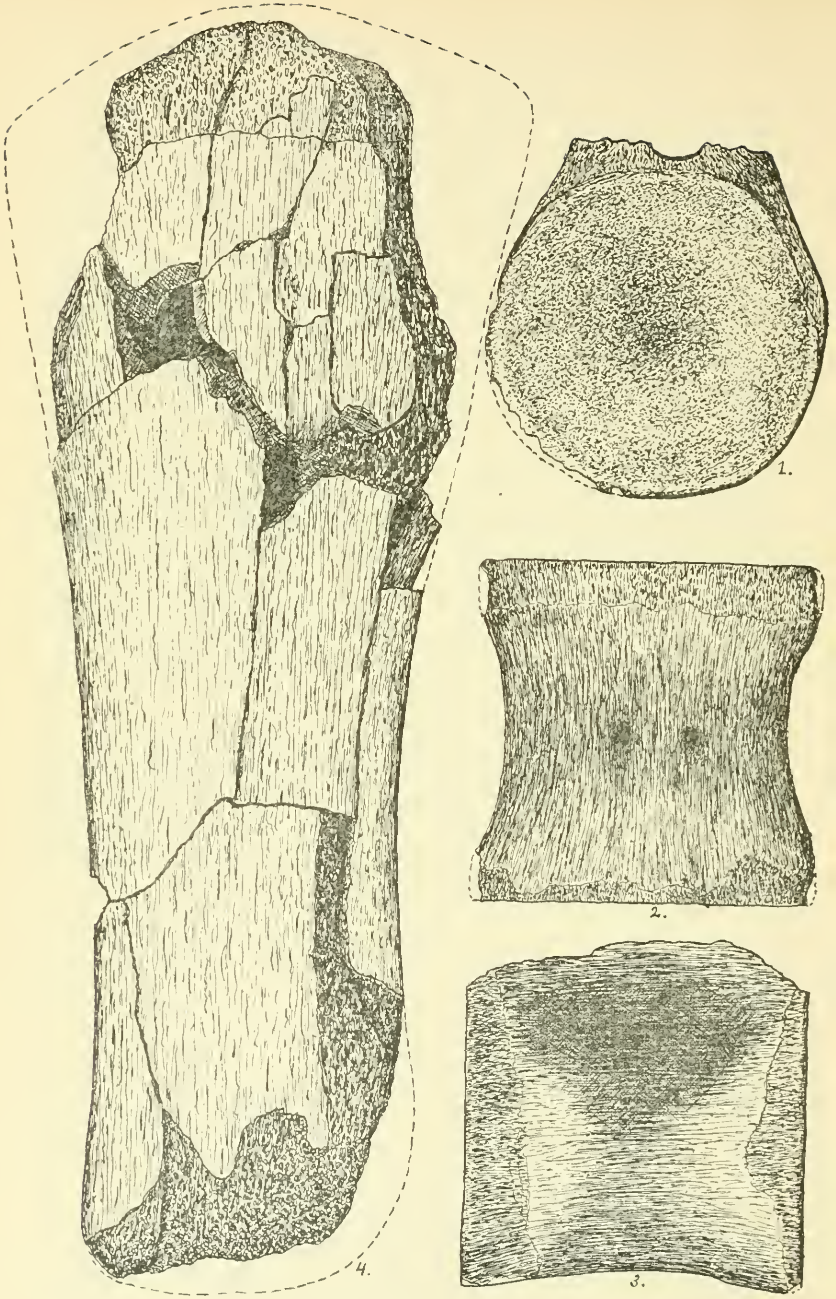

Plate I.

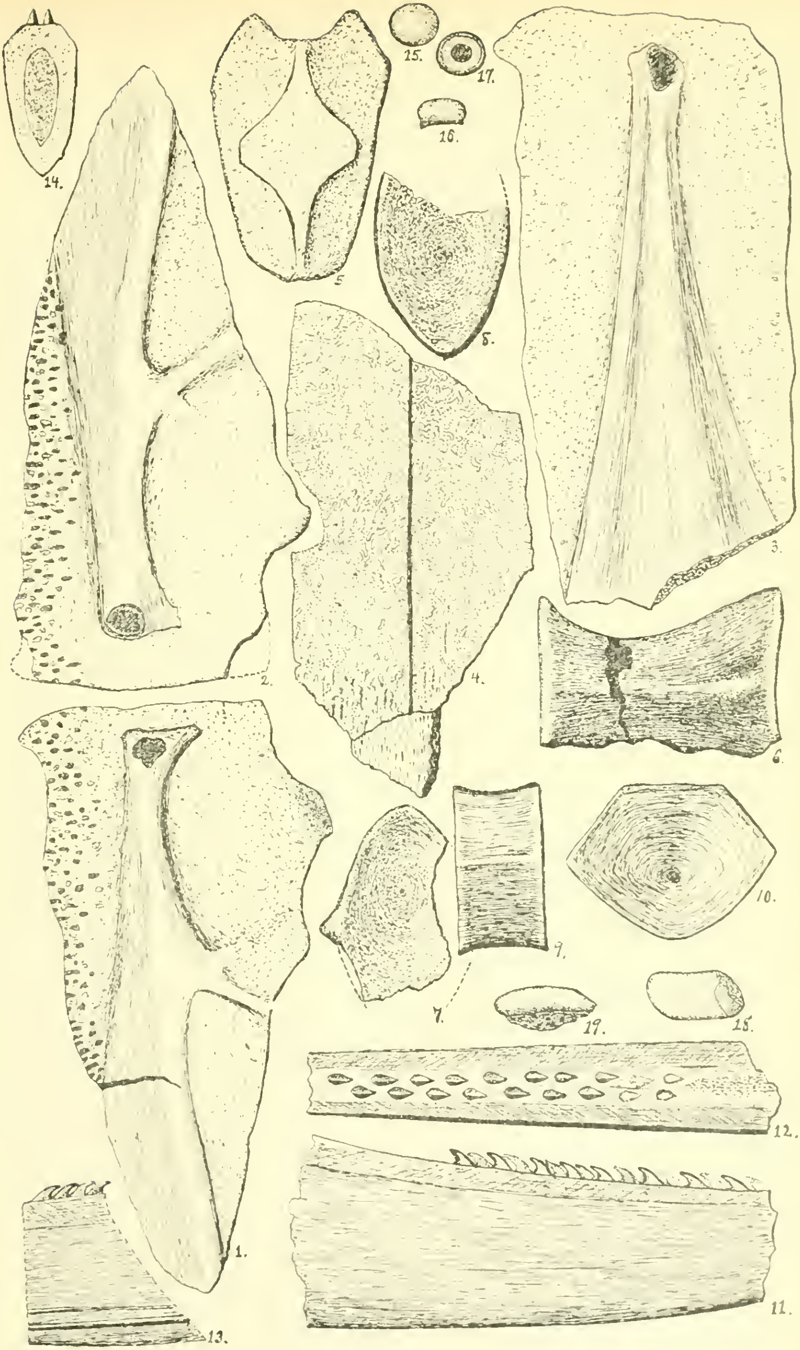

PLATE II.

THE PERMIAN SYSTEM IN KANSAS.

BY F. W. CRAGIN.

North and east of the Arkansas river, Permian rocks occupy a comparatively narrow tract and represent only the lower and middle parts of the system; but south of that river they rapidly widen both their area and the stratigraphic range of their outcrops. They are finely displayed in the southern tier of Kansas counties,* from the eastern border of Cowley to that of Meade, where they present a section that is excelled nowhere east of the Rocky mountains save possibly in northern Texas.

The work of the earlier writers on the Permian of Kansas, Swallow and Hawn, Meek and Hayden, Newberry, Mudge, etc., has been reviewed quite recently by Prof. Prosser, in his "Classification of the Upper Paleozoic Rocks of Central Kansas,"† but (in so far as concerns Kansas) both those early authors and Prof. Prosser have confined their attention to the lower and middle parts of the system, as exposed in central and northern Kansas.‡ Meek and Hayden, Marcou and Geinitz, the earliest students of the Permian ("Dyas") of Nebraska, had in that State only lower and middle outcrops of the Permian available for study. In 1854 and '55, Shumard, Hitchcock and Marcou§ treated of rocks in the Canadian Red river district that belong to the herein-described Cimarron series Marcou referring them to Permian ("Dyas") and Triassic, Shumard and Hitchcock calling them Carboniferous. The highest known terranes of the mid-plains Permian were

*Somewhat less fully in Oklahoma.

†C. S. Prosser, Journal of Geology, Vol. III, Nos. 6 and 7, 1895.

‡Prof. Mudge once visited Harper, Kansas, and there saw the Harper sandstones, but referred them to the Dakota, an identification which was at one time accepted by the present writer and some others.

§In Marcy's Red River Report, 1854; and in Rep. Secy. of War, 1855.

not studied till 1886, when Prof. St. John observed them, describing them the following year in his "Notes on the Geology of Southwestern Kansas,"* and referring them doubtfully to theTriassic.

In the past four decades geologists have repeatedly shown that the passage from the Carboniferous to the Permian system in Kansas is gradual and includes an interval of so-called Permo-Carboniferous rocks which combine the faunæ of both systems. The evidence of continuity and the question of the proper disposal of these intermediate rocks have led to much difference of opinion, some even having gone to the extreme of abandoning the Permian as a system or age, merging it in the Carboniferous, in attempting to avoid the difficulty of the situation.† The Permian in America is, however, a great and widely distributed system, difficult of diagnosis though it often be, from paucity of paleontological data. It is finely developed in Texas, where it has great thickness and has been found‡ to have occasional fossilferous horizons to within less than 300 feet of its summit.

The Permian of the Kansas-Oklahoma basin undoubtedly has many similarities to that of Texas, but it is probably in only one or two of the terranes of the upper Permian, especially in the Medicine Lodge gypsum, that

*Fifth Biennial Report of the Kansas State Board of Agriculture.

†The upper part of what is commonly called the Devonian system in America has so many of the characteristics of the Carboniferous that so high an authority as the late Dr. Newberry treated it as part of the latter system. But to place the entire Devonian in the Carboniferous, or *vice versa*, on account of its transitional relation and the difficulty of assigning the intermediate rocks to either system, would seem of questionable wisdom. The mutilation of the standard geological column by the transfer of the Permian to a lower than systemic rank, seems to be equally unnecessary and to serve no useful purpose, while the retention of the Permian as a system and age in the majority of the leading geological text-books and its degradation in others, is a confusion of terms of which untechnical readers and students of elementary geology have just cause for complaint.

‡By the labors of Mr. W. F. Cummins. See especially his "Notes on the Geology of Northwest Texas" in the Fourth Annual Report of the State Geological Survey of Texas.

stratigraphic continuity or even parallelism of physico-geographic conditions can be traced between them. It therefore seems necessary to treat the Permian north and south of the Ouachita mountain system as belonging to two distinct basins, and profitless to attempt divisional correlation between them.

The following schedule represents the writer's provisional section and

CLASSIFICATION OF THE ROCKS OF THE PERMIAN SYSTEM IN KANSAS.

II. The Cimarron Series.

DIVISIONS.	FORMATIONS.
Kiger.	Big Basin sandstone.*
	Hackberry shales.
	Day Creek dolomite.
	Red Bluff sandstones.
	Dog Creek shales.
Salt Fork.	Cave Creek gypsums.
	Flower-pot shales.
	Cedar Hills sandstones.
	Salt Plain measures.
	Harper sandstones.

I. The Big Blue Series.

DIVISIONS.	FORMATIONS.
Sumner.	Wellington shales.
	Geuda salt-measures.
Flint Hills.	Chase limestones. (Prosser.)
	Neosho shales. (Prosser.)

In round numbers, the thickness of the Permian rocks of Kansas is estimated at 2,200 feet, which may be roughly apportioned among the leading subdivisions as follows, it being borne in mind that there is great geographical variation in the thickness of most of the formations above the Chase and below the Medicine Lodge: In the Big Blue series, 900 to 1,100 feet; in

*The Day Creek and the Big Basin are the only formations of the Kansas Permian that seem to be absolutely *simple* terranes, or to consist each of a single bed.

the Cimarron series, 1,100 to 1,250; in the Flint Hills division, 400, based on 130 for the Neosho and 265 for the Chase, as given by Prosser; in the Sumner division, 550 to 800, of which the Geuda measures occupy 300 to 400 and the Wellington shales, 250 to 450; in the Salt Fork division, 900 to 1,000; and in the Kiger division, about 250.

For rocks near the base of the Permian as here recognized, Prof. Broadhead records a southwesterly dip of over 26 feet to the mile in the vicinity of the Elk-Cowley county line;* and Prof. Wooster gives 20 feet per mile as the westerly dip in the vicinity of the Greenwood-Butler county line.† If the limestones pierced in the first few hundred feet below the rock-salt at Caldwell and Anthony belong, as supposed, to the Flint Hills division, it would seem that the westward element of dip in the lower part of the Kansas Permian continues with little change at least so far west as Anthony, where the summit of these limestones passes below sea-level. The summit of the limestones of this division, on the Walnut creek bluffs, east of Arkansas City, is a little more than 1,100 feet above sea-level; that of the infra-salt-measure limestones at Anthony is 37 feet below sea-level. Calling the difference of elevation of these points 1,140 feet and their distance apart 56 miles, the west element of dip of the summit-limestone of the Flint Hills division for this distance averages about 20 feet per mile, agreeing remarkably with the westing of dip observed by Broadhead and Wooster in basal rocks of this division further eastward.

While the dip of the *lower* Permian rocks of southern Kansas is south of west, that of the *upper* Permian, as

*G. C. Broadhead, in Transac. St. Louis Acad. Sci., Vol. IV, Part 3, p. 488.

†L. C. Wooster, in American Geologist, July, 1890. In his article, "The Permo-Carboniferous of Greenwood and Butler Counties, Kansas," he refers to the dip as "west" in his diagrammatic section, and as "west by south" in his text. It is inferred from this that the dip determined by him was in a direction nearly west, but a little south of a true west line.

indicated further westward in southern Kansas and northern Oklahoma by the Medicine Lodge gypsum, is chiefly south with an easterly element. Thus the dip of the upper Permian is nearly transverse to that of the lower. Whether unconformity, or a quaquaversal flexure, or disparity of sedimentation is the main cause of this discordant relation, remains to be proven. But it is believed that if it be unconformity, it is a succession of minor uncomformities rather than a single large one, a supposition to which the occurrence of conglomerates at several horizons in the middle Permian possibly lends weight; while disparity of sedimentation must apparently be taken into account in solving the problem.

Along the western edge of the area of their outcrop, the Permian Rocks of Kansas are unconformably succeeded by Cretaceous and Neocene sediments: or specifically by the Cheyenne sandstone,* the Kiowa shales,* and the Mentor beds* of the older Cretaceous (perhaps in part by Dakota sandstone of the later Cretaceous also), and by Loup Fork and later fresh-water sediments of the Neocene.

THE BIG BLUE SERIES.

The Kansas Permian presents itself in two series, the lower of which is known to belong to the Permian by its fossils and the upper of which is apparently connected by a bond of stratigraphic continuity with the demonstrated Permian of Texas, as above indicated. The lower series includes the strictly so-called Permo-Carboni-

*On these formations, see "A Study of the Belvidere Beds," in American Geologist, Vol. XVI, pp. 357-385; "The Mentor Beds," in same volume, pp. 162-165; "Descriptions of Invertebrate Fossils from the Comanche series in Kansas, Texas, and Indian Territory," in COLORADO COLLEGE STUDIES, Vol. 5; and earlier papers by the writer in Nos. 9 and 11 of the Bulletin of the Washburn College Laboratory of Natural History, Vols. 6 and 7 of the American Geologist, etc. Also "Outlying Areas of the Comanche Series in Kansas, Oklahoma and New Mexico," by R. T. Hill, in American Journal of Science, September 1895.

ferous, together with several hundred feet of the unmixed Permian, having its base a few feet above the Cottonwood limestone of Prosser (Fusulina limestone of Swallow) and its upper limit at the summit of the Wellington shales. It may be called the *Big Blue series*, from the Big Blue river, which in northern Kansas crosses the somewhat narrowed northern extension of its area of outcrop, cutting both of its divisions. Its rocks include variously colored, in part gypseous and saline shales, limestones many of which are either siliceous or marly, rock-salt, gypsum, and occasional beds of conglomerate. The shales are drab, yellow, greenish, chocolate, maroon, red, white, gray, blue, and dark slate-colored; but other than red in the greater part. The series contains the extensive rock-salt deposits of central and southern Kansas that have become of so great commercial importance within the past few years, and whose products have been manufactured at Hutchinson, Kingman, Kanopolis, Lyons, Anthony, and other towns of that region.

In its lower portion, the Big Blue series is characterized by both Carboniferous and Permian fossils; in higher horizons, by Permian fossils only; and in its upper portion, is devoid of organic remains, so far as at present known.

It embraces two divisions; the lower, or Flint Hills; and the upper, or Sumner.

THE FLINT HILLS DIVISION.

This division of the Big Blue series takes its name from the great monoclinal ridge, called the Flint hills, that extends from the northern part of the Osage Nation northward along the eastern border of Cowley and Butler counties, Kansas, its rocks forming an important part of the ridge and the highlands that constitute in Chase, Morris, Riley and other counties, its dissected northern and northeastern extension. Of these "Permian mount-

ains" as the Flint hills were called by Prof. Broadhead in his geological studies of eastern Kansas,* the rocks of the Flint Hills division occupy the summits, a narrow upper zone of the steep eastern slope, and all of the gentler western slope, extending westward along the south line of the State to the Arkansas river. In the brow of the bluffs of the latter river and its affluent, Walnut creek, at Arkansas City, certain limestones charged with *Athyris subtilita* Hall, *Derbya crassa* M. and H.?† *Productus semireticulatus* Martin, *Septopora biserialis* Swallow, *Schizodus wheeleri* Swallow (?), spines of *Archaeocidaris*, fragments of small crinoid-stems, and other fossils characteristic of the division, being the highest horizons that present a largely brachiopod fauna of Carboniferous affinities, approximately mark the summit of the division. Stratigraphically, the Flint Hills division includes the true Permo-Carboniferous rocks of Kansas, or more definitely, the Neosho and Chase formations of Prof. Prosser's recent paper, "The Classification of the Upper Palaeozoic rocks of Central Kansas."‡

The Neosho formation consists chiefly of shales, with some limestones which are for the most part of no great thickness, and frequently marly.

The Chase consists also partly of shales, but is more conspicuous for its massive limestones, which include three flint-bearing limestones, or so-called flints, that have been named, in ascending order, the *Wreford*, (Hay), the *Florence* (Prosser), and the *Marion* (Prosser).

Prof. Prosser, the most recent authority on the Carboniferous and Permian paleontology of Kansas, lists from

*The Carboniferous Rocks of Southeast Kansas. Am. Jour. Sci., Vol. XXI, pp. 55-57: 1881, and, Carboniferous Rocks of Eastern Kansas. Tran. St. Louis Acad., Sci., Vol. IV, Part 3, pp 481-492: 1884.

†*Derbya multistriata* M. and H.?

‡Charles S. Prosser, Journal of Geology, Vol. III, No. 6, pp. 682-705, and No. 7, pp. 764-800. See latter No., pp. 764-786 and pp. 797-800. October November, 1895.

the Neosho and Chase formations the following fossils, which doubtless constitute the greater part of the

FAUNA OF THE FLINT HILLS DIVISION.

**Chætetes carbonarius* Worthen.
†*Chætetes* sp.
**Zaphrentis* sp.
†*Crinoid stems.
†*Archæocidaris* plates and spines.
†Echinoid spine.
**Spirorbis* sp.
†*Spirorbis orbiculostoma* Swallow.
†*Spirorbis* cf. *permianus* King.
†*Polypora submarginata* Meek.
†**Septopora biserialis* Swall.
†*Fenestella shumardi* Prout.
†**Rhombopora lepidodendroides* Mk.
†**Athyris subtilita* Hall.
†**Chonetes granulifera* Owen.
†**Productus semireticulatus* Martin.
†Same, var. *calhounianus* Swall.
†*Productus costatus* Sowerby.
†**Productus nebrascensis* Owen.
†**Derbya crassa* M. and H.
†*Derbya multistriata* M. and H.
†**Meekella striato-costata* Cox.
†*Meekella* (?) *shumardiana* Swall.
†*Enteletes hemiplicatus* Hall.
†*Spirifera plano-convexa* Shumard.
†**Pinna peracuta* Shum.
†*Aviculopecten carboniferus* Stevens
†*Aviculopecten maccoyi* M. and H.
†**Aviculopecten occidentalis* Shum.
†**Pseudomonotis hawni* M. and H.
*Same, var. *ovata* M. and H.
**Pleurophorus oblongus* Mk.
**Pleurophorus subcostatus* M. and W.
†*Pleurophorus subcuneatus* M. and H.
†**Myalina perattenuata* M. and H.
†**Myalina kansasensis* Shum.
**Myalina swallovi* McChesney.
†*Myalina recurvirostris* M. and W. (?)
†*Edmondia calhouni* M. and H.
**Edmondia* cf. *nebrascensis* Geinitz
†**Allorisma subcuneata* M. and H.
**Schizodus* cf. *curtiforme* Walcott.
†*Schizodus* cf. *wheeleri* Swall.
†*Macrodon sangamonensis* Worth.
†*Bakevellia parva* M. and H.
†*Chænomya minnehaha* Swall.
†*Sedgwickia altirostrata* M. and H.
†Structure like *Stylolites*.
**Discina* sp.
**Aclis robusta*. Stevens.
†**Aclis swalloviana* Geinitz.
**Bellerophon* cf. *sublævis* Hall.
**Bellerophon* cf. *montfortianus* N. and P.
**Macrochilina angulifera* White(?)
†(?) *Glauconome* sp.
†(?) *Orthonema* sp.
†*Straparollus subquadratus* M. and W.
†*Straparollus subrugosus* M. and W.
†*Straparollus* cf. *pernodosus* M. and W.
†Cf. *Loxonema geinitziana* King.
†*Nautilus eccentricus* M. and H.
†*Phillipsia sangamonensis* M. and W.

In this list the asterisk denotes occurrence in the Neosho formation, and the dagger denotes occurrence in the Chase.

For details of the stratigraphy and paleontology of the

division, reference should be made to the account of the Neosho and Chase formations in Professor Prosser's paper above cited.

Reptilian foot-prints are said to have been found on flagstone of this division (in the Chase formation) at Winfield.*

THE SUMNER DIVISION.

Succeeding the Flint Hills division is the *Sumner*. The rocks of this division are largely shales. The limestones are thinner, less frequent, and more impure than those of the Flint Hills division, and as they pass more and more deeply below the surface to the westward, they give place gradually and at length almost wholly to argillaceous shales and rock-salt. The division includes many local beds of gypsum and some of dolomite.

The records of the prospector's drill show for the Sumner division a thickness of about 600 feet at Caldwell and of about 800 at Anthony.

In this division, just before the disappearance of all fossils from the Paleozoic rocks of Kansas, a strictly Permian fauna makes its appearance.

THE GEUDA SALT MEASURES.

In the vicinity of Arkansas City, the southwesterly-dipping, brachiopod-charged limestones and shales of the Chase formation disappear beneath a salt-bearing formation of great commercial importance.

The Permian rocks of Kansas and Oklahoma include *two* extensive rock-salt-bearing formations, the *lower* and the *upper* salt-measures, belonging, one in the Big Blue,

*The proprietor of one of the quarries at Winfield once informed the writer that, in quarrying flagstone for paving, his workmen removed a large slab covered with reptilian footprints. The slab was cut into pavement-blocks and shipped, contrary to orders, and so lost sight of, it having been the owner's intention to preserve it in the interest of science.

and the other in the Cimarron series. Elsewhere in this article, the upper formation is described as the *Salt Plain measures;* the lower formation, which is the basis of the great salt industry of Kansas, is here called the *Geuda salt-measures*, from the town of Geuda Springs, which is located in the area of its outcrop.

At a quarry to which the writer was conducted by the favor of Prof. A. W. Jones of Salina, and which is near the right bank of the Smoky Hill river two or three miles S. S. E. of Salina, is an exposure of perhaps about 30 feet of carbonaceous shales and more or less shaly limestones having rather a pronounced westerly dip and including a fossiliferous horizon in which one lamina of rather hard limestone bears numerous indifferently preserved examples of *Myalina permiana* and smaller undetermined bivalves. This ledge is apparently but little below the summit of the Geuda and overlies a bed of gypsum, which (*fide* A. W. J.) outcrops in the bed of the river near by and is also pierced by a shallow shaft in the vicinity of the quarry. Prof. Jones states that the most southerly appearance of these limestones and shales on the Smoky is about four and a half miles south of Salina. He also states that the gypsum at this quarry is supposed to be the same as that which is worked at Gypsum City; but it seems to the writer, from a consideration of the dip of that gypsum between Hope and Gypsum City, that the gypsum south of Salina, which may be called the *Greeley gypsum* from its occurrence in Greeley township, is higher by not less than a hundred feet than the *Hope gypsum*, as that at Hope and Gypsum City may be called.

The lower limit of the Geuda in north-central Kansas is drawn at the summit of the Marion concretionary limestone of Prof. Prosser, which the latter seems with good reason to regard as the "first cherty limestone" of Prof. Swallow.

The Geuda formation is very complex in its com-

position. It consists primarily of clay-shales, but these are of many colors and sorts, frequently passing into saline, gypseous, and calcareous varieties, or even giving place to limestone, gypsum, or rock-salt. The composition of the formation in and near its area of outcrop is quite different from that revealed by the drill in deep-lying parts remote from that area.

Its area of outcrop, doubtless somewhat narrowed by the leaching out of rock-salt and the settling of superincumbent strata, is in part occupied by a belt of more or less gypseous and saline shales interspersed with beds of massive gypsum and impure limestone, and which has as additional manifestations of its mineralized character, occasional salt-marshes, saline springs, bitter waters, and wells which (where not dug in the superficial deposits of the Neocene) are not infrequently brackish or saline. Near the south line of Kansas, this belt occupies a breadth of some 12 to 18 miles in adjoining portions of Sumner and Cowley counties, the greater portion of which is in Sumner, and with variable breadth extends thence nearly northward through the eastern parts of Sedgwick, Harvey, McPherson and Saline counties, and western parts of Marion and Dickinson, to the region between Salina and Hope, from which region it continues in a nearly northeasterly course through portions of Clay, Riley, Washington and Marshall counties. The so-called "Wellington marble," described by the writer in 1885,* and which comes from the eastern part of Sumner county, and the gypsum used in the manufacture of plaster of Paris, stucco, and cement in Dickinson and Marshall counties, as well as many unused beds of gypsum, belong to this belt. There are, however, in the belt at least two distinct horizons of gypsum, and it is the lower, or Hope, gypsum, that has been hitherto most used in the manufacture of plaster of Paris. The salt-

*Bulletin of the Washburn College Laboratory of Natural History. No. 3. page 87.

springs and salt-marshes at and north of Geuda Springs in Sumner county, and southwest of Arkansas City in Cowley county, are also within this belt. Various geographic names in this belt have been taken from its geological peculiarities. Thus there are several streams called Gypsum creek, a Gypsum township, and even a Gypsum City; the two Slate creeks of Sumner county take their name from slate-colored shales which belong, in the case of the larger stream partly and in the case of the smaller one wholly, to the Geuda measures; Bitter creek, in adjoining parts of Sumner county, Kansas, and K county, Oklahoma, and intermediately the Bitter Creek postoffice, Geuda Springs, and its now abandoned neighbor, Salt City, are all named or, in the case of Geuda Springs, best known from peculiarities of these measures. At Geuda Springs, are several mineral springs differing in the analyses of their several waters, but all belonging to the saline class. These and the salt-marsh of the same locality and the beds of gypsum and limestone not far away, present a fair epitome of the characteristics of the Geuda outcrop.

The shales of the Geuda outcrop are blue, gray, slate-colored, drab, buff, red and various shades of bluish and brownish red. In the deepest known parts of the Geuda the three first-named colors prevail.

The limestones of the Geuda are in part unfossilferous, but those in the lower and sometimes those in the upper part of the formation contain a fauna from which brachiopods and essentially Carboniferous fossils are all but wholly excluded, and which, indeed, consists almost entirely of Lamellibranchia of recognized Permian affinities. The writer has personally identified only a few of the fossils of these limestones, and some of the determinations recorded by the earlier writers on this part of the Kansas section have need of verification or require revision of synonymy. We therefore present here the list of fossils recently recorded from these limestones by

Prof. Prosser,* as embracing a substantial part of the

FAUNA OF THE GEUDA MEASURES.

Septopora biserialis Swallow.	*Bakevellia parva* M. &H.
Derbya multistriata M. & H.	*Schizodus curtus* M. & W.
Pseudomonotis hawni M. & H.	*Schizodus ovatus* M & H.
Same, var. *ovata* M. & H.	*Nuculana bellistriata* Stevens, var. *attenuata* Meek.
Pseudomonotis cf. *variabilis* Swall.	*Nucula* cf. *beyrichi* Schauroth.
Myalina permiana Swall.	*Nucula* cf. *parva* McChesney.
Yoldia subscitula M. & H.	*Dentalium meekianum* Geinitz.
Aviculopecten occidentalis Shumard.	*Macrochilina* cf. *angulifera* White.
Pleurophorus subcostatus M. & W.	Cf. *Aclis swalloviana* Geinitz.
Pleurophorus subcuneatus M. & H.	*Nautilus eccentricus* M. & H.
Edmondia calhouni M. & H.	

Observations recorded by Messrs. Swallow and Hawn and Meek and Hayden in northern Kansas, and others made by the writer in southern Kansas, indicate that *Athyris subtilita* and several other Carboniferous species of fossils may occasionally range up into the basal part of the Geuda: but the fossils thus referred to should perhaps be regarded as more or less varietal representatives of their types.

Several years ago, Mr. E. F. Osborne informed the writer of an early discovery of reptilian footprints near Salina in rocks which must be referred to this division. About four miles east of Salina, at a point about half a mile south of the Smoky Hill river and west of north of Iron mound, is an abandoned quarry. The upper stratum is a ten-inch ledge of so-called "bastard limestone" that was used by the settlers in the earlier days of Salina's history for walling wells, etc.† It is upon this stratum that the footprints were found. It is possible that the reptilian footprints discovered many years ago by Prof. Mudge on rocks of the bluffs of the Republican river in northern Kansas should also be referred to this formation.

Fragments of a charcoal-like sort of fossil wood that occur in the salt mines at Kingman, in rock-salt-bearing shale

*Loc. cit., pages 787 and 788.

†Wells on the old Osborne place and on the Snyder place were walled with this stone, and some of the stone probably remains in foundations of old buildings in the vicinity.

above the main beds of rock-salt, are the only vestiges of plant-life known by the writer to have been found in the Geuda measures.

In the limestones of the Geuda, geodes of celestite are frequent.

Traced by the prospector's drill-records away from its outcrop to its deepest known extensions, the Geuda gradually changes its composition, the variegated shales and the gypsum and limestone gradually disappearing and their place being taken at first partly and at length wholly by more decidedly saliferous shales of almost constantly blue-gray to slate-color and massive beds of more or less nearly pure rock-salt, which constitute the "salt-measures" of current parlance. It is not positively known that these salt-measures everywhere come, in their entire thickness, within limits that correspond with those of the Geuda outcrop; but their dip and position with reference to higher and lower formations are such as to indicate their substantial equivalency to part or (as in the case of Anthony, at least) to practically all of that outcrop. The gradual disappearance of limestones and gypsum with depth, implies a progressive change of physico-geographic conditions in the region between the outcrop and the deep parts of the Geuda, showing that the leaching out of the salt in and near the outcrop—to the probability of which Prof. Hay has called attention in his "Geology of Kansas Salt" and elsewhere—is not the sole (though doubtless a partial) cause of the comparative absence of salt from the Geuda outcrop, and that this absence is partly due to local differences in the physical conditions under which the Geuda sediments were originally laid down.

The salt is present in every grade of occurrence from that of minute particles impregnating the shale, through that of rock-salt and shale intermingled in about equal proportion, to that of massive beds in which clay appears only as sparsely scattered flecks. As may be inferred from what has been already said, the proportion of salt in the

shale and the number and thickness of the rock-salt beds increase with distance from the outcrop.

In strata of shale associated with some of the principal beds of rock-salt, occur iron-red pseudomorphs of halite after selenite.

In the deep-lying part of the formation, remote from the outcrop, the greatest known thickness of these salt-measures is 420 feet, which was obtained in a boring at Anthony. The thickness of the outcrops probably varies from 300 to 400 feet.

The dip of the Geuda measures in southern Kansas is southward and westward; in northern Kansas, it appears from various observations to be northward and westward. From these data it may be inferred that the salt has the form of an anticlinal whose summit-line dips nearly westward. It seems probable that Arlington is located somewhere nearly over the crest of this anticlinal, since the salt-beds decline in either direction (northward and southward) from their position beneath that town. The northward descent is, however, small as compared with that to the southward. Thus,, at Arlington, the summit of the salt-beds is reached at an elevation of approximately 910 feet above sea-level, whence it descends (gradually, as shown by the Sterling and Lyons drillings) to about 815 feet above sea-level at Ellsworth and to only about 383 feet above sea-level at Anthony.

The name here given to this formation is one that has been applied to it in the writer's manuscripts for many months. In a reccent article,* Professor Prosser has called nearly the same formation "the Marion formation;" but aside from the fact that he includes in such formation a zone of "variously colored shales and marls" which is above the zone of gypsum-horizons, and therefore belongs to the lower Wellington, the name, Marion, as a stratigraphic term, is preoccupied, having been previously es-

*Loc. cit., p. 786.

tablished by Professor Prosser in the same article for another quite different stratigraphic value, the *Marion flint and concretionary limestone*, a member of the Chase formation.*

THE WELLINGTON SHALES.

It has been established by numerous borings in the region underlaid by rock-salt in Kansas that there is above the salt-measures a body of clay-shales with a thickness of 250 to nearly 450 feet, and having for the most part the same and darker and lighter shades of the bluish-gray color that prevails in the clays of the rock-salt deposits themselves. This fact, though here stated from the writer's early and independent observations, was recognized by the late Prof. Robert Hay, who called this zone "the gray shales" and "the gray beds,"† apparently in contrast with the "red-beds" of the Cimarron series. It is for this zone of gray beds, which must be distinguished stratigraphically from the underlying and similar gray beds of clay-shale in the Geuda salt-measures, and which is not everywhere gray through its entire thickness, that the formational name, *Wellington shales*, is here proposed.

These shales are thicker in southern than in northern Kansas, attaining their greatest known development beneath the town of Caldwell, where the drill has shown them to have a thickness of 445 feet. They contain satin-spar veins and infrequent and limited saline impregnations, but no rock-salt. Within the area of their outcrop and as reached by borings near it, as respectively at Wellington and Caldwell, the Wellington formation often includes beds of impure limestone and calcareous shales and occasional beds of gypsum and dolomite.

Beneath Ellsworth, the Wellington is 255 feet thick

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*Loc. cit., p. 772.

†Geology of Kansas Salt. Seventh Biennial Report of the Kansas State Board of Agriculture. Part II, pp. 83 to 96. 1891. (See page 87.)

and has a 20-foot zone of red shale 60 feet below its summit. Of the 395 feet of Wellington pierced by the drill at Anthony, 365 feet is the typical blue shale, the remaining 30 feet being a nearly basal zone of red shale.*

At Wellington itself, from which, as a locality within the area of its outcrop, the formation is named, and in whose vicinity are exposed some of the shales that have suggested the term "gray beds," and, together with similar shales of the Geuda, have given provincial but not very accurate name to the Big Slate creek of Sumner county, a considerable part of the more than two hundred feet of shale passed through by the drill before reaching the rock-salt, is alternately red and greenish (or bluish) gray.† This red and gray mottled and banded character appears also in the lower Wellington shale that outcrops in central and western parts of McPherson county and at intervals in the foot of the bluffs of Spring creek from Salina to a point in the southwest vicinity of Bavaria, occupying in the latter situation the interval between the Mentor beds and the zone of shale, gypsum, and Myalina-bearing shaly limestone that forms the summit of the Geuda along the Smoky Hill river south of Salina. But while the red colors so prevalent in the rocks of the Cimarron series, invade the gray in certain quarters of the Wellington, they affect but a small proportion of the whole, and the Wellington is, notwithstanding these and its calcareous inclusions, essentially a thick body of blue-gray and slate-colored shales.‡

The massive ledge of hard, cellular, gray dolomite on the Little Arkansas river at the eastern border of Rice

*Only 25 feet of the blue shale intervenes between this red zone and the summit of the rock-salt.

†This statement of the drill-record at Wellington is made on the authority of Mr. E. W. Davis, driller.

‡At Pratt, salt has been found in drilling without first passing through any large body of blue-gray shales such as must be pierced to reach the salt at Hutchinson, Kingman, Anthony, etc.: but, as elsewhere indicated, the salt that was there so reached does not belong to the Geuda. (On a succeeding page, see account of the Salt Plain measures.)

county, west of south from Windom, is provisionally referred to the Wellington formation.

THE CIMARRON SERIES.

With the Wellington formation, ends the Big Blue, lower, or limestone-bearing, series of the Permian. Succeeding it without break, but possibly with a gradually introduced angular unconformity, are the Harper sandstones and higher prevailingly red formations that comprise the remainder of the Kansas Permian, and constitute the *Cimarron series*, which, for Kansas, is nearly the same as the "red beds."*

So far as known, the series is destitute of any trace of organic remains.

THE SALT FORK DIVISION.

The *Salt Fork division* is so named because all of its formations are found within the drainage-basin of the Salt fork. It includes the Harper, Salt Plain, Cedar Hills, Flower-pot, and Cave Creek formations, and has a maximum thickness of about 1,000 feet.

THE HARPER SANDSTONES.

These constitute the lowest and thickest formation of the Cimarron series. They comprise several hundred feet of more or less mottled, but prevailingly dull-red, or brownish-red, argillaceous and arenaceous shales and sandstones, above the Wellington shales and below the Salt Plain measures. The word, sandstones, as applied to this formation, is intended to imply, not that its rocks consist mainly of sandstone throughout their thickness, but that the frequent low ledges of rock which accentuate the formation are of sandstone. Much of the latter is of the sort quarried at Harper—a reddish-brown or roan-colored sandstone, sometimes mottled and streaked, soft enough to be easily quarried and dressed, but becoming harder by seas-

*Some authors, however, may have included the limestone-bearing Wellington and possibly even the rock-salt-bearing Geuda in their use of the term "red-beds," as applied to Kansas rocks

oning, and constituting an excellent dimension-stone. As this stone first became well known as a building-material from its use at Harper, and as the outcrops of the formation which includes it occupy a large portion of Harper county, the name, *Harper*, seems doubly appropriate for the formation. The term, Harper beds, may be found more convenient, in some cases, in referring to the formation, than that of Harper sandstones.

As one travels westward from Wellington, the red shales and sandstones of the Harper outcrop are first met with near Milan. They occur thence westward to the vicinity of Sharon; but in their western extent are seen only in the lower parts of the country. They occur on middle regions of the two Ninnescahs and of the Chikaskia river. On Bluff creek the sandstone has been largely quarried and employed for business buildings at Anthony, where the Bennett House and the Anthony Roller Mills, both three-story buildings, are constructed of it, and where, as at Harper, it has given excellent satisfaction. Similar stone, most if not all of which belongs to the Harper formation, is used from quarries in the vicinity of Kiowa, Hazelton, Attica, Milan, Spivey, Arlington, and other towns of this region.

No observations have yet been made as to the dip of the Harper sandstones. The prospector's drill has shown that the Wellington shales and the rock-salt-bearing horizons of the Geuda, in a considerable part of southern Kansas, descend and thicken both southward and from their outcrop westward. While the same can not be asserted of the Harper, certain similar facts are known of the latter also. Thus, we find that the *base* of the Harper descends to the southward and to the westward within certain known limits. The base of its outcrop at Caldwell is a rough hundred feet lower than it is some fifteen miles further north, in the eastern neighborhood of Milan; and at Anthony, some twenty-five miles west of the Caldwell-

Milan base-line, the base, as reached by the drill, is apparently some 300 feet lower than at Caldwell. Taking the latter town as the base and Sharon or Attica as the approximate summit of the Harper beds, and making no allowance for dip, the difference of elevation of these places would give about 350 feet as the thickness of the Harper, a figure that would be reduced to about 250 feet, if the base as outcropping near Milan be taken as bench-mark. But that dip or accession of sediments westward, or both, must be reckoned with, is indicated by the fact that at Anthony, which is at least 100 feet below the summit of the formation, (and where the Sumner division has at least its full average thickness), the prospector's drill descended about 550 feet before reaching the apparent summit of the Wellington, making the thickness of the Harper, as thus measured, about 650 feet.

Carbonate of copper, including both azurite and malachite, occurs in the Harper formation, chiefly as stain in calcareous shales in the basal part of the formation* (as at Caldwell) and more rarely in sandstone concretions at higher horizons (as west of Harper); but the occurrences are limited and promise nothing of value from a mining point of view.

Some of the earthy brownish-red and gray shales of the Harper formation, occurring a short distance east of Kingman, form the basis of the "Cherokee Brown Mineral" and "Silver Gray" manufactured by the Kingman Paint Company, and which has had considerable demand in the paint-trade of Topeka, Kansas City and other markets.

THE SALT PLAIN MEASURES.

Occupying an interval between the Harper and the Cedar Hills sandstones, in southern Kansas and northern Oklahoma, is a zone of red shales (? with some sand-

*Really in transitional beds which might perhaps be reckoned equally well as constituting the summit of the Wellington.

stones) in which saline impregnations are common, giving rise to salt springs, salt creeks, salt or brackish wells and other saline waters within and near the area of their outcrop, and resulting in a topography characterized by various closely allied features termed salt-plains, salt-marshes, salt-draws, salt-bars, salt-licks, salines, etc., in which the chloride of sodium is often practically pure, but sometimes associated with other salts.

These salt-impregnated shales constitute what may be regarded as upper, or secondary salt-measures. Comparatively little use has been made of these measures as a salt supply hitherto, owing to the undeveloped condition of this part of the country and to the vast supplies of salt found to the eastward in the Geuda measures. But much use has been been made of the salt of the Great Salt plain, within distances to which it can conveniently be hauled in wagons, as in dry seasons it forms on the "plain" a crust of pure, coarsely crystallized salt several inches in thickness, which is easily removed and available, as taken, for "stock-salt" or, with simple crushing, for all of the domestic uses to which salt is put. This salt-crust is composed in part of pyramidal and intersecting turkey-foot groups of cubical crystals and, as viewed from the brow of the bluffs which overlook it, appears like a field of glistening snow. In the early days of the settlement of western Kansas, the salt was hauled over the old Nescatunga, or Kinsley, trail to Kinsley and other points on the A. T. & S. F. railway. Salt derived from the Salt Plain measures is now manufactured at one point, if not more, in Oklahoma, in which commonwealth it is probable that these upper measures will ultimately become of considerable commercial importance. Rock-salt is alleged to crop out in a ravine near the Salt plain;* but no examination as to the accuracy of this report has been made.

*On the north side of the Cimarron, in a canyon a few miles below the entrance of Buffalo creek.

As local saline impregnations occur to some extent in most of the terranes of the Kansas-Oklahoma Permian, it is possible that it may not always be easy to distinguish the limits of the Salt Plain measures; but the salt-zone which is here especially designated as the *Salt Plain measures* is of considerable extent and is that from which is derived the salt of the Salt plain of the Cimarron river (called often the Great Salt plain), which is the saline usually referred to in the literature of this region when the unqualified expression, "the Salt plain," is employed.

The stratigraphic position of this salt-zone may be seen in Kansas on the east slope of the Cedar hills of Harper county and on the south side of the Salt fork, a few miles below Ætna, where, in each case, beginning immediately below the bright red Cedar Hills sandstones, are the saliferous clay-shales which are the source of the salt that at the one locality gives character and name to the Little Salt creek that traverses adjoining parts of Barber and Harper counties, and at the other has given rise to one of the peculiar flats, or base-levels, so frequent in salt-shale topography, and which are evidently produced through the agency of the salt itself. Little Salt creek, though having the highest source of its salt in a horizon of saline impregnation immediately below the base of the Cedar Hills sandstones, apparently cuts other saline horizons at somewhat lower levels, and its lower portion may intergraduate with and be only arbitrarily separable from the upper of the Harper sandstones.

At Pratt, salt deposits were encountered a few years since, in drilling, without first passing through any thick body of gray shales like the Wellington; and the relation of these to not far distant outcrops of the Cimarron series and to sea-level, indicates that they belong to the upper, or Salt Plain, measures.

The following is a condensed record of the boring at Pratt:

	Thickness.	Depth.
Neocene marl, sand and gravel	89	89
Red sandstone and clay, with veins of water at intervals. (A vein of salt water was cased out at 150 feet)	537	626
Shales, almost wholly red, more or less saline	89	715
Rock-salt, with small admixture of shale	30	745
Salty red and blue shale (mostly red)	13	758
Rock-salt, with small admixture of shale	22	780
Shaly salt gradually becoming red and blue shale	20	800
Red shale with some blue spots	50	850
Rock-salt, with small admixture of shale	20	870
Dark red shale, bluo mottled	134	1004

This section affords the only data here available with which to indicate the thickness of the Salt Plain measures, giving for the vertical range of the rock-salt 155 feet, which, with a few feet added for transitional sediments above and below, probably approximates the thickness of these measures as developed at and near their outcrops in Kansas.

On another page of this article it is shown that if a line be drawn between Ashland and a point about six miles southwest of Medicine Lodge, the Medicine Lodge gypsum dips in both directions (more or less nearly southward and more or less nearly northward) from that line. A similar relation to about the same line apparently obtains in the Salt Plain measures, as these certainly dip from about this line far southward into Oklahoma, and they have an apparent dip of about 8 feet per mile from the same line to the latitude of Pratt.

The saline springs on the upper part of Little Mule creek, in Barber county, Kansas; the great salt spring at the head of Salt creek in Blaine county, Oklahoma, from the brine of which several tons of table-salt are now made daily, and which is said to furnish brine enough for the manufacture of 160 tons a day; and (with less confidence) the Salt plain of the Salt fork in Woods county, Oklahoma (sometimes known as the Little Salt plain) are provisionally referred to the Salt Plain measures.

THE CEDAR HILLS SANDSTONES.

The Salt Plain measures are succeeded by a zone of rocks in which unevenly hard, in part massive concretionary, fine-grained, bright-red sandstones, having some resemblance to those of the Red Bluff terrane of the Kiger division, constitute the leading feature. This may be seen a few miles northwest of Hazelton, Kansas, below the Neocene sands which there form the summit of the Cedar hills. From this occurrence, the terrane takes its name. It is finely displayed in the canyon-cut basal incline of the Gypsum hills, southwest of Medicine Lodge, and in the same southeast of Ætna. The bright-red sandstone in the low bluff north of Sharon and that outcropping on the south fork of the Ninnescah river west of Kingman are provisionally referred to the Cedar Hills formation.

This formation has nowhere been measured. From memory, it is roughly guessed at 50 to 75 feet as seen in the basal incline of the Gypsum hills of the Medicine Lodge river and the Salt fork.

THE FLOWER-POT SHALES.

Next in order above the Cedar Hills sandstones, but entirely eroded from the summit of the Cedar hills, while seen in full thickness a little farther west in the escarpments of the Gypsum hills, southwest of Medicine Lodge, and taking their name from the well-known Flower-pot mound which has been carved out of them by erosion at the point of the divide between East Cedar and West Cedar creeks, are the *Flower-pot shales.* These, for the most part, are highly gypsiferous clays.

Flower-pot mound has been named by the residents of Barber county in allusion to the fact that its top is plumed with several small cedars which, outlined against the sky at a short distance, present a fancied resemblance to plants growing from a flower-pot. It is necessary to consider only the top of the mound to see the flower-pot, since other-

wise the pot were inverted. But to the geologist, the flower-pot is the mound itself; for the slopes of the latter, like most exposures of these clays elsewhere, are strewn with mineral blossoms of divers colors and constitute a very respectable geological bouquet. Light-red, dark-red, bluish-red, pink, greenish-white, bluish-white and gray, mixed in confusion with red in the ascendancy, give much of the outcrop of this terrane a variegated aspect; or as viewed at some distance, a hue approaching the late fashionable color known as "crushed strawberry." The surface is often strewn with fragments of white, pink, red or water-clear satin-spar flecked with green or red clay, and is sometimes also set off with sparkling crystals of selenite. For such a member of the Salt Fork division, a geological posey-bed in appearance, the designation, "Flower-pot," seems very natural, and it is doubtful if a more appropriate name of local geographic origin could be found.

As seen in canyon walls or other vertical exposures, the satin-spar forms a network with irregular rhomboidal meshes. It lies, in fact, in a trestle-work of warped plates traversing the clay in all directions, but chiefly in oblique positions tending toward horizontal. The clay is thus inclosed, sometimes between tortuous subhorizontal and subparallel seams, sometimes in spacious sublenzitoid compartments subject to partition in various directions by intersecting veins. The seams vary from mere paper-seams to plates several inches in thickness.

A noticeable and picturesque feature of the Flower-pot clays is the manner in which their outcrops are carved by the elements. They are, in fact, a theater of rapid erosion, and many weird spectacles present themselves in their relief-forms. In localities where their protective covering of Medicine Lodge gypsum has been removed by erosion, as for example, near the head of Little Mule creek and in the district between Eldred and Ætna, they are frequently cut into rather steeply sloped faces having that peculiar pat-

tern of sculpture that is best designated as *cone-and-gully* erosion, consisting of alternate cones (more strictly semi-cones) and rain-gullies. The cones are sometimes arranged in a close and remarkably uniform palisade-like series on the face of a rampart-like or amphitheater-like bluff, and in such instances are calculated to arrest the attention even of those most indifferent to natural phenomena. Such a palisade of cones may conveniently be called a *conarium*. When viewed at a moderate distance, it recalls the arrangement of points on a backgammon-board. Occasionally the adjacent conaria of two parallel ravines meet, producing a sharp serriform spur running out upon a base-level of erosion. An example of the latter sort (doubtless short-lived in its destiny) is seen near the road from the old Eldred postoffice to Ætna. On the whole, the outcrop of the Flower-pot clays, with its conaria and occasional pinnacles and buttresses, presents a type of erosion similar in many respects to that of the northern Tertiary "Bad Lands." Its mineral-surcharged character renders its occasional smoother tracts little less barren than the ruggeder portions, so that the Flower-pot lands are generally waste-lands.

From the eastern escarpment of the Gypsum hills northwestward in the bluffs of the Medicine Lodge river and its tributaries, the Flower-pot formation may be seen in diminishing exposures, as it gradually descends below the river-valley. It disappears under the latter a few miles below Belvidere. It appears in the divide between East Cedar and Little Mule creeks; on the upper branches of the latter; and on the Salt fork drainage, from the Eldred district on the north and the eastern promontory of the Cimarron-Salt fork divide on the south, northwestward to a point above the mouth of Cave creek. It extends up Big Mule creek at least seven or eight miles from its mouth. It is well displayed in both bluffs of the Cimarron river at the Great Salt plain, and thence down that stream to an unknown distance beyond the bridge of the Panhandle

branch of the Santa Fe railway. It appears again, if remembered correctly, in the lower part of the bluffs that border the Beaver on its south side at the locality where the Panhandle line crosses. It is not seen from that line on the Canadian, though the sections given by Shumard in Marcy's Red River Report make it clear that lower down on the Canadian, and on both forks of Red river as well, a body of similar clays lies beneath the gypsum.

The thickness of the Flower-pot shales on the Salt fork, southeast of Ætna, is in the neighborhood of 150 feet.

THE CAVE CREEK FORMATION.

Above the Flower-pot marls is an important gypsum-bearing formation, consisting usually of either a single stratum of massive gypsum or two such strata separated by an interval of red clay-shale. It may be called the *Cave Creek gypsums*, or *formation*, because well displayed in its fuller development on Cave creek, in Comanche county, Kansas. The formation appears with a similar tripartite character on the north branch of Red river, as indicated by Dr. George G. Shumard on Plate V of Marcy's Red River Report. The lower gypsum horizon (below named and described as the Medicine Lodge) is the heavier and persists throughout the present known extent of the formation; while the upper, or *Shimer* (so named after the township through which Cave creek flows), is less constantly developed as a distinct bed of massive gypsum, not appearing at all on the valley of the Medicine Lodge river, so far as observed.

At the only locality at which it has been measured, viz., on Cave creek at the Comanche cave,* the formation has a thickness of not less than 50 feet, of which the Medicine Lodge gypsum occupies a thickness of 25 to 30 feet, the Shimer gypsum about a third as much, and the interval

*Named and described below, in the account of the Medicine Lodge gypsum.

of red clay, the *Jenkins clay* (named after the former Jenkins postoffice, near Cave creek), 7 to 10 feet.

THE MEDICINE LODGE GYPSUM.—While the Shimer gypsum and the Jenkins clay require merely brief notice here, the former resembling the Medicine Lodge gypsum and the latter the commoner gypsiferous red clay-shales of the Salt Fork division, the Medicine Lodge gypsum calls for a special description, on account of its stratigraphic importance and its more than ordinarily interesting general character.

If, on the road from Harper to Medicine Lodge, the traveller finds himself looking westward across the valley of the Medicine Lodge river on one of those enchanting days for which southern Kansas yields the palm to no other locality, the autumn air being tinged with just enough of haze to purple the remoter vistas of the ruddy landscape,

"The splendor falls on castle walls"

which rear themselves seemingly as low mountains or buttressed escarpments of a table-land crowning the further incline of the valley and bounding a considerable part of the western horizon.

These are the Gypsum hills. They are a northern extension of those on the Red and Canadian rivers, observed by Marcy in his Red river expedition of 1852 and earlier reconnaissance, and illustrated in the report of that expedition in 1854.

The earliest geological study of the Gypsum hills of Kansas was made in 1884 by the writer, who gave an informal description of them before the '84 meeting of the Kansas Academy of Science at Lawrence, publishing a sketch of their physical geology a few months later in the Bulletin of the Washburn College Laboratory of Natural History.*

*No. 3: published about May 1, 1885.

As seen at their eastern border about six miles southwest of Medicine Lodge, the foundations of the Gypsum hills are laid in Cedar Hills sandstone, their walls are reared in the variegated sediments of the Flower-pot shales, and they have, as coping, a massive bed of gypsum to which the stratigraphic name, *Medicine Lodge gypsum*, is here given, in double allusion to its overlooking the valley of the Medicine Lodge river for many miles and to its prominent position opposite the picturesque county-seat of Medicine Lodge, in which was established the first mill to make large commercial use of this gypsum.

For a considerable distance along the Medicine Lodge river, the Salt fork, Big Mule creek, and the Cimarron river, the outcrop of the Medicine Lodge gypsum is prominent and practically continuous. It is usually best displayed on the south side, owing to the fact that here, as generally on the Plains, easterly-flowing streams, while planing down to base-level, are shifting to the south, their valleys consequently having a short and steep slope with bold bluffs and deep canyons as prevailing topographic features on the south side, and a long, more or less alluvium-laden slope with less rugged relief on the north. The Medicine Lodge gypsum, however, is itself conducive to rugged surface-features, and even on the north side of these streams the exposures of the gypsum are sometimes conspicuous. The best of these north-side exposures are seen on southward-running branches, such as Mulberry creek of the Medicine Lodge river drainage east of Sun City, and Cave creek of the Salt fork drainage west of Evansville.

The Medicine Lodge gypsum is seen on the Medicine Lodge river from the eastern border of the Gypsum hills to a point about four miles southeast of Belvidere, where it disappears beneath the Dog Creek and Red Bluff formations of the Kiger division in the floor of the river-valley. On the Salt fork, it extends from the eastern extremity of the high bluffs capped by it a few miles southeast of Ætna,

westward beyond Cave creek, and to a limit not observed by the writer, but stated by Prof. St. John to be at "Cottonwood creek". On the Cimarron river it forms the brow of the bluffs along the south side of the river at the bridge of the Panhandle branch of the A. T. & S. F. railway and extends thence to the southeastern part of Clark county, Kansas. At Ashland, in the latter county, clay-charged gypsum, probably representing this or the Shimer horizon, was pierced in a well at a depth of about 125 feet by Dr. W. J. Workman. The outcrop of the Medicine Lodge gypsum on the Cimarron river has not been explored by the writer below West creek in the western edge of Woods county, Oklahoma: but according to Mr. H. C. Chapman, Editor of the *Okeene Eagle,* it gradually recedes from the river south of the Glass mountains, passing the head of Salt creek in Blaine county, where it is tunneled into a remarkable park-like system of natural bridges, and thence extends southeastward to Darlington (a few miles from El Reno), on the North Canadian. On the latter stream the greater portion of the outcrop of this gypsum is doubtless east of the crossing of the Panhandle line, but a few miles southeast of Beaver City there is a bed of gypsum which may belong to either of the Cave Creek horizons or to a higher one. A section of this bed on lower Clear creek presents a lenticular outline, and the weathered rocks at the foot of the bluff contain salmon-colored nodules in a whitish ground-mass, like plums in a pudding, a feature which is only a phase of the mottled or semi-crystalline character seen in the Medicine Lodge gypsum on the Cimarron and elsewhere.

The principal stratum of gypsum described and illustrated in their Red River Report by Capt. Marcy and Dr. Shumard as occurring on the Canadian and on the forks of the Red river, can scarcely be other than the Medicine Lodge gypsum.

North of the river of its name, the Medicine Lodge

horizon has not been identified. Gypsum is said to occur in Stafford county, Kansas; but this may refer partly or wholly to the so-called "native lime" of the fresh-water Tertiary, which also passes under the misnomer of "gypsum" in western Kansas.

The full thickness of the Medicine Lodge gypsum is not always shown at the immediate outcrop, owing to the solvent effect of meteoric water upon it. On the Medicine Lodge river it is usually between 12 and 25 feet thick. On the Cimarron and Salt fork it is considerably thicker. On Cave creek, a small tributary of the latter stream near Evansville, it has a thickness of 25 to 30 feet.

The prevailing dip of the Medicine Lodge gypsum in northern Oklahoma and an adjacent strip of Kansas is nearly south, apparently a little east of south; but a small area at the north seems to dip in a northerly direction. This attitude of the stratum may readily be seen by a comparison of its elevations at a few leading points. The elevations of the summit of the gypsum referred to sea-level at the points here selected are more or less nearly as follows: (A) at Ashland, as indicated in Dr. Workman's well,* 1840 feet: (B) at point of disappearance of the gypsum in floor of the Medicine Lodge river valley, about four miles southeast of Belvidere, 1744 feet: (M) at brow of the wall of the mesa-like hills between the Medicine Lodge river and East Cedar creek, southwest of Medicine Lodge, 1800 feet: (H) at Heman station, Oklahoma, near the Cimarron river bridge of the Panhandle branch of the A. T. & S. F. railway (roughly) 1500 feet: (G) at disappearance of gypsum below valley of Big Mule creek near the former postoffice of Gallagher, a point not far from the intersection of the lines AM and BH (and which may here be con-

*The gypsum that was encountered at this depth in Dr. Workman's well may possibly represent the Shimer horizon; but as no other zone of gypsum was mentioned as having been met with in the deeper part of this well, it is inferred that it was the Medicine Lodge, and that the Shimer bed was not there developed.

sidered as coinciding with it), 1812 feet. From these elevations we find, in the direction from A to M, a dip of 40 feet in 61 miles, or only about 2/3 of a foot to the mile; from G to H, one of 312 feet in 48 miles, or 6.5 feet per mile; from A to H, one of 340 feet in 67 miles, or about 5 feet per mile; from M to H, one of 300 feet in 50 miles, or 6 feet per mile; from G to B, one of 68 feet in 14.5 miles, or about 4.7 feet per mile; from A to B, one of 96 feet in 46 miles, or 2.1 feet per mile: and from M to B, one of 56 feet in 22.3 miles, or 2.5 feet per mile, the distances, like the elevations, being subject to some correction, but sufficiently exact for our present purpose.

In minor parts, the Medicine Lodge gypsum is nearly pure white: in others it is suffused with leaden-gray or dusky-brownish shades; most commonly it is greyish-white, mottled with feebly defined dark spots. The latter are generally the expression of a tendency that existed in the gypsum, under the original conditions of precipitation, to form crystals, as is shown by the occurrence of the spots in every gradation from ill-defined spot-like segregations to well-formed crystals of selenite. Some of the crystals are of the common rhomboidal patterns, others are of the stellar type. Even the perfect crystals present the appearance of dark spots, as transparent inclusions in an opaque white matrix (comparable with cavities in such a matrix) would naturally appear. Distinct crystals are far more abundant in the gypsum on the Salt fork and Cimarron river than on the Medicine Lodge.

The uneven color and more or less saccharoidal texture of the Medicine Lodge gypsum give to its freshly exposed surfaces, as seen in the quarries of Barber county, an aspect not unlike that of marble, and the resemblance to marble is further increased by the fact that the gypsum takes a fine polish. Hence originated the erroneous names, "Sun City marble," "Kansas onyx," etc., that have sometimes been applied to this gypsum.

With a strength due to the character of its consolidation and to its massiveness, the Medicine Lodge gypsum combines the weakness due to its solubility. Owing to that strength, it is among its fellow-terranes a *piece de resistance*, and is, like the Loup Fork sandstone, responsible for some of the highest bluffs and deepest canyons seen in the central portion of the Plains. Owing to that weakness, it is essentially a *cave formation*. Indeed the gypsum in some localities is fairly honeycombed with earth-filled or empty galleries and spaces.

Its caves are formed in two ways, and may accordingly be classified as *rift-caves* and *arch-caves*.

The gypsum is parted into large blocks by vertical master-joints which often persist for long distances, and which become the conduits of meteoric waters. Enlarged at first by solution and later perhaps in part by mechanical erosion, the joint-fissure becomes a narrow and at length somewhat wider gallery, or *rift-cave*. It may remain open or, if it come to be traversed by an intermittent or variable current carrying sediment, it may become wholly or partly filled with the latter, and so be either a potential cave or an actual one with earthen floor, as the case may be.

In forming the *arch-caves*, water descending through joints or other crevices in the gypsum, is arrested at the summit of the Flower-pot shales and finds its way as a vein along the base of the gypsum, excavating the lower part of the latter and sometimes also a portion of the underlying gypsiferous clay by a corrasion in which solution plays the leading part. In the case of the clay, its fine particles, set free by the solution of the associated gypsum, may be carried off by even a feeble current, the readiness with which these and other red clay sediments of the Cimarron series are held in mechanical suspension in water being frequently attested on the Cimarron outcrop by the slow settling of these sediments in rain-pools.

The rift-cave is doubly typified in what may be called

Sarcophagus cave, at the Natural bridge on Bear creek, south of Sun City. This cave has been opened along two master-joints which intersect at an angle of about 70 degrees, and it accordingly has two entrances, one about five rods south, the other immediately north of the Natural bridge, and neither far above the bed of the creek. In cross-section, and as well seen at the north entrance, the cave has nearly the form of an upright sarcophagus, widening gradually upward to a shoulder-region of maximum breadth, contracting suddenly above the shoulder to a crested head-part, and narrowing to a simple fissure both above and below. The present open portion of the cave extends only from the two entrances to the intersection of the joint-planes, distances of about 55 and 180 feet from the north and south entrances respectively; but both galleries formerly extended in to an unknown distance beyond this point, the continuation being now filled with compact cave-earth. It seems probable that the entire cave was once so filled and that the portion now open was subsequently re-excavated by the water of Bear creek, a portion of which is diverted from the main channel and traverses the cave during ordinary freshets. The north gallery has also an outward extension of its roof for 11 feet in the overhanging wall at its entrance, the lower part of the gypsum having been here cut away and the cave thus much shortened by the creek's erosion. The breadth of the "Sarcophagus," across shoulders, in this gallery is 4 feet and 8 inches at the entrance, gradually increasing in the inner half to 7 feet near the junction of the galleries. The height of this gallery is over 9 feet above the narrow earth-floor at its entrance and gradually diminishes inward. The south gallery has a nearly uniform breadth of 4 feet and 6 inches across shoulders and, containing more earth than the north gallery, has a broader floor and a height only about equal to its breadth.

A fine example of an arch-cave is seen on Cave creek,

a north-side tributary of the Salt fork, not far from Evansville. It is supposed to be the largest of the gypsum-caves of Kansas, and may be called the *Comanche cave*, from being located in Comanche county.* The east and main entrance is picturesquely located a short distance back from the creek in a deep right-hand ravine through which flows a perennial brooklet of limpid but gypsum-tainted water, issuing from the cave itself; it is a broadly-arched portal about 14 feet in height in a wall of gypsum. Near the latter, and a little south of the entrance, a straight and remarkably tall-trunked tree stands sentinel. The cave covers a little less than 150 yards of the course of the brooklet, which, from a small gypsum-walled canyon, enters it through a west portal somewhat like the east one, but smaller. The cave is slightly sinuous and consists of three rooms separated by two low arches. The east room is 14 feet high, 20 to 25 wide, and 123 long. This is followed by an arch, which for 13 feet has a height of only about 6 feet. The middle room is 12 feet high, 30 wide, and 50 long, and is lighted in the south side of its roof through a 6 x 12-foot shaft-like opening that broadens into a funnel in the high ground above the cave.† Through a second arch, this middle room connects with the west room, which is lower and much longer than the others and is flooded with the waters of the brooklet, here expanded into a long pool upon which a small boat has sometimes been used by visitors. The floor of the cave is more or less strewn with blocks of gypsum that have fallen from the roof. A bloom of snowy gypsum covers some parts of the walls and roof, and brown cauliflower-like masses and concretionary layers of clay-impregnated gypsum are forming on the floor in shallow pools along the course of the streamlet.

*The best railroad-point from which to visit the Comanche cave is Coldwater, a convenient approach being via Nescatunga and the John Duckworth place (formerly the post-village of Duckworth).

†Till recently, a cottonwood tree flourished on the slope of this funnel.

A modification of the arch-cave is seen where a vein of water reaches an outcrop of the gypsum and of the upper part of the Flower-pot shales in some ravine and, trickling down over the outcrop of the gypseous clays, causes the latter to soften and fall away in more or less vertical sections, excavating a clay-walled room with a gypsum roof. This may be regarded as a special case of headwater erosion, where the excavating done by the streamlet follows the latter back into a yielding terrane beneath one that is more resistant, and in so doing changes its work from that of trenching to that of undermining. A good example of a cave of this sort is seen in what may be called the *Green room*, a cave in the west bluff of Bear creek, some distance below the Natural bridge and in the immediate vicinity of an older, partly fallen cave formerly much visited by picnicking parties from Sun City. It consists of a single spacious room partly walled up in front by a ridge of talus from which it is necessary to descend into it as into a cellar, and is called "the Green room" in allusion to the fact that its roof is largely incrusted with a beautiful pale-green layer of stalactitic gypsum studded with capitate protuberances like miniature cauliflowers. The latter, formed by the evaporation of dripping water, are more stipitate and have a lighter and more open structure than the large cauliflower-topped masses forming in the pools of the Comanche cave. The color of this roof-crust is only superficial, however, being due to conditions that favor the culture of an undetermined alga, supposed to be one of the *Cyanophyceæ*. The roof of the Green room is traversed by a channel whose sinuous course marks the former continuation of the vein that has produced the cave and is still continuing it inward.

An interesting topographic feature of the Medicine Lodge gypsum is its natural bridges. These are merely remnants of caves that have for the most part fallen in. Sometimes, as near Havard creek, in Barber county, Kan-

sas, the same channel may be alternately canyon and tunnel, or bridge.

The best known of these bridges in Kansas is the Natural bridge of Bear creek, south of Sun City. This spans the canyon of the creek, here about 55 feet from wall to wall. The height of the bridge above the bed of the creek is at the highest point 47 feet, at lowest 31, and at middle 38. The width of the bridge at middle is 35 feet. The upper surface of the bridge declines toward the down-stream side, but not so much that a wagon drawn by a steady team could not be driven across it. The thickness of the arch is therefore greater on the up-stream side, where it measures 26 feet, than on the down-stream. The relief of the vicinity seems to indicate that at a geologically recent time Bear creek here flowed to the east of its present course and that its waters, becoming partially diverted by an incipient cave, enlarged the latter and finally were wholly stolen by it, the cave at length collapsing save in the portion now constituting the Natural bridge. Nor is it impossible that this piracy may be repeated in a minor way by Sarcophagus cave, which at times already draws a portion of the Bear creek waters to the west of its proper course.

Reference has been made to a district especially characterized by natural bridges of the Medicine Lodge gypsum in Blaine county, Oklahoma. For his knowledge of this, the writer is indebted to Mr. Chapman. The district is at the head of Salt creek, a few miles southwest of Okeene and north of Watonga. The bridges are numerous and in some instances are said to be sufficiently large to permit the passage of a load of hay beneath them. The scenery in this district is so remarkable that Mr. Chapman states that he is preparing a bill which he hopes to have passed by Congress, establishing a "Natural Bridge park" containing about a hundred natural bridges.

While following down the Cimarron valley to the Great Salt Plain some years ago, the writer discovered

an interesting deposit of selenite in the Medicine Lodge gypsum. It was a mass of several tons' weight forming the roof of one of the small caves that open upon the river valley a few miles west of the "plain," where the gypsum-ledge occupies a position but little above the level of the "bottom." The mass consisted of an interlocking and partly interpenetrating group of huge crystals. It was found that the latter, though readily cut, could only with great difficulty be removed entire, owing to their toughness and their interlocking relation. Chisels were driven into the mass with difficulty. Prying with iron bars was little more effectual and attended with similar results, the bending of the crystals causing them to cleave into slabs and sections or producing an intermolecular fracture that rendered them opaque. By the destruction of perhaps an equal amount, some 800 pounds' weight was obtained by two assistants and myself as the result of several hours of hard work, and was sent to the museum of Washburn College. The largest crystal (now split into halves) measures a little over three feet in length, is two feet wide and a foot thick, and in part clear enough to read through. It is obliquely penetrated by a comparatively small crystal at one extremity. It is the largest crystal of selenite that the writer has ever seen; but still larger masses are said to have been observed in the Glass mountains.

Some local use has been made of the Medicine Lodge gypsum almost since the founding of the towns of Medicine Lodge and Sun City; but within the last few years two mills* have been built for the manufacture of plaster from it on a commercial scale, and this is doubtless but the beginning of a vast industry that will ultimately be built up in this great gypsum-belt in southern Kansas, Oklahoma and

*That of Best Brothers at Medicine Lodge, making Keene's cement as one of its specialties; and the Standard Cement Company, whose headquarters are at St. Joseph, Mo., and whose mill is on the north side of the Medicine Lodge river, in the west part of Barber county, shipping from Croft.

Texas, a belt which is far greater than the discontinuous one of the Geuda, and is, indeed, one of the greatest gypsum deposits in the world.

THE KIGER DIVISION.

The upper division of the Cimarron series is the *Kiger division*, so named from Kiger creek in Clark county, Kansas, a stream that traverses all of the terranes of this division except the lowest. On the central plains north of the Ouachita mountains, this division includes all of the rocks of the so-called "red-beds" that lie above the Medicine Lodge gypsum. In southern Kansas, it includes the following successive members, beginning with the lowest: the Dog Creek shales, the Red Bluff sandstones, the Day Creek dolomite, the Hackberry shales, and the Big Basin sandstone.

The lower part of this division (including the Dog Creek and Red Bluff terranes as exposed on the Medicine Lodge river drainage) was reconnoitred by the writer in 1884, '85 and '86, but the first knowledge of it as a whole was obtained by Prof. Orestes St. John in his reconnaissance of 1886, and set forth in 1887 in his "Notes on the Geology of Southwestern Kansas."*

In addition to other facts given under the heads of the several formations, relative to the westward extension of the Kiger, it may here be noted that an outcrop of this division, but of undetermined terrane, occurs on the Beaver, six miles west of Beaver City.

THE DOG CREEK SHALES.

The lowest member, or *Dog Creek* terrane, of the Kiger consists of some thirty feet, or locally of a less or greater thickness, of dull-red argillaceous shales, with laminæ of gypsum in the basal part and one or two ledges of unevenly lithified dolomite in the upper. The color of

*Fifth Biennial Report of the Kansas State Board of Agriculture. pp. 132 to 152.

these shales resembles that which prevails in most of the terranes of the Salt Fork division below, more than that of the Kiger terranes above the Dog Creek. The dolomite varies from light-gray to dark-gray, and clay-impregnated portions may partake of the red color of the including shales. In lithological character, it varies from solid stone which serves a fair purpose as a building-stone for the rougher uses, to that which is so contaminated with clay as to be soft and worthless. It is often cellular or cancellated. A dark and cellular variety occurs at the top of a remnantal mound of the Dog Creek at the highest point of the range of lofty tables that forms the eastern front-line of the Gypsum hills southwest of Medicine Lodge, this mound being the most easterly outlier of the Dog Creek formation in Kansas.

The thickness and stratigraphic relations of the Dog Creek formation are well displayed south of Lake City on Dog creek, from which the formation is named, and on Little Bear creek, and thence westward in Barber and Comanche counties and the southeastern part of Clark county, on various branches of the Medicine Lodge river, Salt fork, and Cimarron river. In Oklahoma, it is seen on the bluffs of the latter river in the immediate vicinity of the Great Salt plain.

THE RED BLUFF BEDS.

While the rocks of the Kiger bear certain general resemblances to those of the Salt Fork division, they yet present in the main a different aspect. This prevailing difference is especially due to the thick body of bright-red rocks that constitutes at once the second member of the Kiger division and the major part of the Kiger sedimentation, viz., the *Red Bluff beds*.

This formation consists of some 175 or 200 feet of light-red sandstones and shales. Its thickness is not fully shown in the valley of the Medicine Lodge river, having there been reduced by pre-Cheyenne and later erosion, but is

exhibited in the valley of the Cimarron river in Clark county, Kansas, and in the slope north of the Great Salt plain. Viewed as a whole, it is very irregularly stratified, the component beds, while consisting of nearly parallel laminæ, being in some cases considerably inclined, in others curved, and this oblique and irregular bedding, being on a much larger scale than that of ordinary cross-bedding, at first glance gives the impression of dips, anticlines and synclines that have been produced by lateral pressure, the dips being, however, in various directions, as north, east, etc., etc. It is certain that these older formations of the Plains must have been subjected to even more of the dynamic strain due to oscillatory movements of the earth's crust than the much-fractured Cretaceous rocks of western Kansas, and it is also probable that minor inflections and accidents of the strata have been wrought by the leaching and undermining agencies of solution in ages past, as they are seen in operation to-day producing the numerous basins of western Kansas; but it seems to the writer that neither leaching, which has been suggested by the late Prof. Hay as the cause of similar irregular bedding in lower beds at Caldwell,* nor lateral pressure, nor both of these, should be held wholly responsible for the phenomena, but that these are partly due to the conditions under which the sediments were originally laid down.

The Red Bluff beds exhibit the most intense coloration of any of the rocks of the Cimarron series, being approached in this respect only by the Cedar Hills sandstones. When the outcrops are wet with recent rains, their vividness of color is still greater, and the contrasts of their almost vermillion redness with the other colors of the landscape is most striking. Spots and streaks of bluish or greenish-gray sometimes occur in the red of these rocks, but not to nearly the same extent as in the Salt Fork division.

*Geology of Kansas Salt, p. 5.

The sandstones of the Red Bluff are generally too friable for building-stone ; but in some instances selected portions have proved hard enough for such use and fairly durable.

A marked characteristic of most of these sandstones is their unusually fine texture. When pulverized, or as seen in soils that have been derived from them, they sometimes seem like brick-dust. So light are some of their soils that, walking over them, one may sink shoe-deep, as if walking on the mellow ground of a well cultivated field.

The sandstones are also porous and, especially where overlaid not far away by Neocene sands, are often a source of water. Their springs are rarely strong, being usually seepage-springs, but their spring-waters are in some instances nearly as sweet and soft as those coming directly out of the Neocene sands themselves. Examples of permanent springs of this sort are seen in Red Bluff sandstone at the head of a north-side canyon of North Elk creek, on the Medicine-Elk divide about five miles west of Sun City. Wells dug in Red Bluff sandstone and which at first are failures, or yield only a scanty supply of water, sometimes become valuable wells after the lapse of a few years. The water of such wells is liable to be more or less saline or gypsum-tainted, but is frequently fresh enough to be palatable and available for ordinary uses.

The shales of the Red Bluff are rarely without some admixture of fine arenaceous matter.

The Red Bluff beds, once uncovered, yield rapidly to subaerial erosion and their outcrops generally show a rugged, canyon-cut relief which, in connection with their bright-red color and their frequent setting-off with dark-green cedars, makes some of their landscapes exceedingly picturesque. The sandstones are frequently trimmed off by stream-erosion in a long, straight, vertical wall that resembles the face of a quarry. These and less regularly cut exposures, where rendered conspicuous, have given rise

to many such local names as "the Red bank," "the Red bluff," etc. Thus, for example, we have the so-called "Red bank" on North Elk creek in the northeastern part of Comanche county, southeast of Stokes hill; and on Bluff creek, above Protection, is the "Red bluff"* which gave name to the former postoffice of Red Bluff, after which in turn the *Red Bluff beds* are named.

On the Medicine Lodge river, the Red Bluff beds constitute the highest surviving formation of the Cimarron series and occur as far west as Belvidere. At the latter place, they are unconformably overlaid by the Cheyenne sandstone, only the lower part, about 100 feet, of their thickness being represented. On the Salt fork drainage, they are seen in ravines of the upland slope a few miles south of Deerhead, underlying sandy soils of probably Loup Fork Tertiary origin. Thence they extend up the valley of Big Mule creek to within a few miles of Wilmore, and that of the Salt fork itself to the vicinity of Avilla. On the Cimarron river, the location of the eastern border of the Red Bluff outcrop is unknown to the writer, but it is at least considerably southeast of the Panhandle line of the A. T. & S. F. railway, since the formation is beautifully exposed along that line on the slope south of the river. Westward on the Cimarron drainage, exposures of this formation extend to the lower part of Crooked creek in Meade county, ascending the creek-valley to Odee. They are also seen in the lower bluffs of Tainter's creek (also known as Cottonwood canyon), a beautiful brook heading in the Neocene sands south of the Cimarron, and entering the river a little west of the mouth of Crooked creek. The Red Bluff beds are seen again at intervals on the Beaver in Oklahoma, but in Beaver county, they are largely mantled with fresh-water Neocene sediments. They are probably well developed on

*This bluff is illustrated in fig. 18 of Professor Hay's Geological Reconnaissance in Southwestern Kansas. Bulletin No. 57 of the United States Geological Survey.

the Canadian river and southward, but their extension there has not been seen by the writer, unless certain limited exposures seen low down in the valley of that stream beneath a mantling of Neocene near Canadian, Texas, are part of them.

THE DAY CREEK DOLOMITE.

Upon the latest of the Red Bluff beds rests a persistent stratum of dolomite, varying from less than a foot to five feet or more in thickness. This is the same as the "gray, cherty, sometimes gypsiferous limestone" noticed by Professor St. John* as occuring in Clark county at the head of Day creek. It is a true dolomite, containing with the carbonate of lime an equal or even greater percentage of carbonate of magnesia, as indicated by a qualitative analysis kindly made for the writer by Prof. William Strieby of Colorado College. Though not of great thickness, it is an important member of the upper Permian of southern Kansas and northern Oklahoma owing to its persistence, which makes it a convenient horizon of reference. It may therefore be considered a formation by itself and, to distinguish it from other and less important dolomites of the Cimarron series, be called the Day Creek dolomite, after the above-named locality of its occurrence.

The stone is nearly white in fresh fracture, weathers gray, and often has a streaked and gnarly grain crudely resembling that of fossil wood. It is more or less cellular and, in places, cancellated. Irregular nodules of limonite are here and there imbedded in it. Its cherty hardness and fracture are not due to the presence of silica, as one is tempted to infer, but are characters belonging to it as a dolomite. It is a durable building-stone, as shown by the old buildings and corral-walls of the Fares ranch on West Bear creek, which are built of it; but it is somewhat difficult to trim to desired shapes owing to its erratic fracture,

*Notes on the Geology of Southwestern Kansas, l. c., page 141.

and Mr. Fares informed the writer that when fires were made in a fire-place that was built of it, the stone began to "pop" and crack in pieces, showing its unfitness for use where it would be subjected to much heat. From the skirt of Mount Prospect and the region of the junction of Hackberry and Bluff creeks, the exposures of the Day Creek dolomite extend almost uninterruptedly westward, past East and West Bear creeks, including the vicinity of the Fares place, to Little Sand creek, west of which they are less continuous. The formation appears, however, in the ravine that heads just west of Little basin, and it seems probable that the ledges recorded (l. c., page 142) by Professor St. John "on Gypsum creek a few miles above Cash City, on the borders of Clark and Meade counties," as well as a less characteristic ledge which the writer recently observed on Crooked creek near the present location of Odee postoffice, should be referred to the Day Creek formation. A number of years ago, the writer observed a stratum of dolomite capping the so-called Centennial mound* on the old trail from Kinsley to the Salt plain, in what is now Woodward county, Oklahoma. Some of the field-notes of that reconnaissance have been lost; but if it be remembered correctly, this Centennial mound dolomite was correlated with that which is here called the Day Creek.

At one locality in Clark county, a point on the Little Sand creek drainage passed by the road from the Fares ranch to "St. Jacob's well," the Day Creek stratum presents a peculiar variation. It there becomes a homogeneous, semi-translucent white rock of remarkably pure aspect, unlike any other rock with which the writer is acquainted, but bearing more or less resemblance to fine-grained marble, or to onyx or chalcedony. In honor of Mr. Henry Fares, formerly of the Fares ranch, to whom the writer is indebted for most enthusiastic and valuable assistance in

*Formerly called Sentinel mound?

several of his earlier geological reconnaissances of Clark county and the formerly so-called Public Lands, it is proposed to call this interesting lithologic occurrence, or rock-variety, *Faresite*.

THE HACKBERRY SHALES.

In Clark county, Kansas, the Day Creek dolomite is overlaid by 15 to 20 feet of crumbling, chiefly maroon-colored shales, including some moderately hard laminæ that in weathering check into small cakes and dice-like chips. They are well shown in the region of the junction of Hackberry and Bluff creeks, from the former of which they derive the name of *Hackberry shales*. They follow thence the irregular westerly course of the Day Creek outcrop to the western part of the county. The most westerly occurrence of the Hackberry formation that has been satisfactorily identified by the writer is in the southeastern wall of Big basin; but the writer has observed what he is inclined to consider as remnants of it on lower Crooked creek, at and above Odee, while the dolomite of Gypsum Creek on the Clark-Meade county border, if referable to the Day Creek terrane, involves the not unlikely occurrence of the Hackberry shales with it.

THE BIG BASIN SANDSTONE.

The western, northern and northeastern parts of the bluffy walls of Big basin are formed largely by the calcareous sandstone of the Loup Fork; but on the easterly to southeasterly quarter the rim of the basin is chiefly of Kiger sediments, in which the Hackberry shales appear with a coping of rather massive, blocky, red and grayish-white sandstone. The latter, from this locality of its typical occurrence, may appropriately be known as the *Big Basin sandstone*. It is also seen on Kiger creek, at the Fares ranch on West Bear creek, and elsewhere in Clark county. Its maximum thickness probably does not anywhere exceed 12 feet. While particolored, it has less of the "poikilitic" char-

acter than is seen in most of the particolored rocks of the Cimarron series, the two colors being arranged in two (locally three) broad bands, of which one is almost uniformly red, and one almost uniformly grayish-white with occasional flecks of red.

At one locality on the eastern rim of the Big basin, where it is overlaid by, and not abruptly separated from an incoherent sandstone of the lower Cretaceous,* the Big Basin sandstone is clearly also the highest surviving terrane of the Cimarron series, and therefore of the Permian, if all of the Cimarron series be really of Permian age as here assumed. Moreover, the Big Basin sandstone is the highest terrane of the Cimarron series whose occurrence in any part of Kansas or Oklahoma can here be positively asserted. Were it also the highest terrane of that series ever deposited in this region, it would need be accredited as record of the final shallowing of the great "Dead sea" of the Plains, marking, as nearly as any terrane could, the close of the Paleozoic era in this region. But the writer has an impression (not positive enough to be called a recollection) that he once observed a narrow remnant of red Cimarron shale above the Big Basin sandstone at some point in Clark county; and if this impression be correct, it confirms what might reasonably be inferred on other grounds, namely, that terranes higher than the Big Basin sandstone originally formed a part of the Cimarron series in this region, and that these, with possibly one or two minor exceptions, do not outcrop in Kansas, having been partly removed by erosion in the time-interval indicated by the great post-Cimarron unconformity and partly preserved

*The soft, gray, ferruginous-stained sandstone that here overlies the Big Basin sandstone is a remnant of the Belvidere beds. In the immediate vicinity it also underlies a decomposed remnant of the Kiowa shales, and may be either a western recurrence of the Cheyenne or a sandstone member of the lower part of the Kiowa itself. Its relation to a part of the Kiowa shales is fairly well shown in the west wall of Little Basin, a little way to the eastward.

concealed beneath a blanket of Cretaceous and later deposits.

It might naturally be considered by some that the transitional character of the horizon of passage from the Big Basin sandstone to the Cretaceous sandstone reinforced the earlier generally accepted view that the "red-beds" were Jura-Trias, or at least partly so; but the bond of continuity which has already been referred to as apparently existing between the Cimarron series of Kansas and the paleontologically proven Permian of northern Texas outweights any argument of that sort, and indicates rather that the upper and here lighter-colored zone of the Big Basin sandstone was softened by the invading waters of the Belviderean sea, and its sediments partially and then wholly rearranged as the (for this point) initial deposits of the latter, only gradually becoming supplanted by sediments conveyed from other sources.

SUPPLEMENTARY NOTE.—The *Sumner division* is named after Sumner county, which includes nearly the entire breadth of the area of its outcrop in southern Kansas. The *Cimarron series* is named from the Cimarron river, in whose basin the rocks of both of its divisions are so extensively displayed.

ON THE STRATIGRAPHY OF THE PLATTE SERIES, OR UPPER CRETACEOUS OF THE PLAINS.

BY F. W. CRAGIN.

THE PLATTE SERIES.

The rocks of the North American Interior Cretaceous belong to two great series. The lower of these, consisting chiefly of limestones, with subordinate terranes of shale, marl, and sandstone, and having its fullest development in Texas and Mexico, has been named the *Comanche series*. The upper series, into which shales and sandstones more largely enter, but which has important limestone formations also, may most appropriately be called the *Platte series*, after the Platte river, which, in Colorado and Nebraska, cuts all of the divisions of the series and all of the formations recognized as belonging to it in the typical area of its occurrence, viz., that segment of the North American Interior plateau which extends from the Rocky Mountains eastward, and constitutes the higher portion of the Plains.

The Platte series includes the following divisions: Dakota, Benton, Niobrara, Fort Pierre, Fox Hills, and Laramie.

Following are brief preliminary notices of some of the formations that compose that portion of the Platte series which is included within the limits of Kansas.

THE RUSSELL FORMATION.

The lower formation of the Benton in Kansas. Named from Russell, Kansas, around which it outcrops in ravines.

Consisting of alternating limestones and shales. The shales carbonaceous (bluish) to calcareous (light gray). The limestones including at least two courses that are used as dimension-stone, the more important of which (commonly, 6 to 12 inches thick) is the *Downs limestone* (named from Downs, Kansas, near which are quarries of it,) and which might also be appropriately called the *Fence-post limestone,* as it is extensively used for posts of wire fences. Buildings and posts from the Downs limestone having a characteristic and rather neat, striped appearance due to a median yellow, brown or reddish rusty band in the rock. Outcrops from valley of Crooked creek in southern Gray county to that of Republican river in western part of Republic county, thence crossing Republican-Little Blue divide and passing into Nebraska. Formation includes the *Globigerina bulloides* "Lincoln marble." Characteristic fossils: *Trinacromerum bentonianum, Inoceramus labiatus, I. fragilis* (small phase), *Prionocyclus woolgari* (small phase), etc.

THE VICTORIA FORMATION,

Or *Victoria clays.* The upper formation of the Benton in Kansas. Named from Victoria, Kansas, which is on the outcrop, the clays being also cut largely by Victoria creek. Highly carbonaceous, almost black, sticky clay-shale, with a zone of frequently large "cannon-ball" septaria in the lower part, the *Cannon-ball zone*, supposed to be the same as that of Cannon-ball crossing on the Missouri river, illustrated in Hayden's reports.

Outcrops extend from Walnut creek, Ness county, Kansas, (from north branch of Pawnee river in Hodgeman?) to Whiterock creek and Republican river in northwestern Republic county, and thence through Nebraska, etc. Fossils: immense *Prionocyclus woolgari, Scaphites warreni, Scaphites vermiformis, Inoceramus labiatus, Plesiochelys lovii, etc.*

THE OSBORNE LIMESTONE.

Named from occurence at Osborne and in Osborne county, Kansas. Crowning the Victoria shale and consituting the lower formation of the Niobrara. Limestone in rather thick courses. "Devil's Gap," in southeastern Rooks county, is in this limestone. Courses thicker than in Russell formation: some thin ones, especially at summit. Limestone harder and better for building than Smoky Hill chalk, but softer than and inferior to the Downs limestone. Formation forms bluffs in forks of Walnut creek in Ness county; of Solomon river in Osborne county; occurs at Ft. Hays, forming upper member of the "Ft. Hays" (lower Niobrara and Benton) of Prof. B. F. Mudge; and forms bluffs on Whiterock creek, from above Lovewell to near Whiterock, passing thence into Nebraska through Whiterock-Republic divide. Fossils: *Inoceramus deformis*, *I. brownii*, *Radiolites austinensis*, etc.

THE SMOKY HILL CHALK.

Upper formation of the Niobrara. Named from the Smoky Hill river, on which it is magnificently developed. Chalky and marly limestones and chalk. Bluish and marly in the lower or *Trego zone*, (named from its conspicuousness in Trego county, Kansas,) and yellow (to white, red, etc.) and more chalky in the *Norton zone* (named from occurrence at Norton and in Norton county, Kansas). A horizon of jasper, the *Graham jasper* (named from its occurrence in Graham county, Kansas) near the upper limit. Lenses of hard, so-called "Bell-rock" in base or transition to Osborne limestone. On Smoky Hill river from western Ellis county to vicinity of Ft. Wallace: thence chiefly northward and northeastward. Includes "Castle Rock" in Gove county, and many similar monumental forms. Fossils: *Inoceramus* (*Haploscapha*) *grandis*, *Uintacrinus socialis*, the huge fish, *Portheus molossus*, many large reptiles, *Cimoliosaurus snovii* and other enaliosaurs, with mosasaurs,

pterosaurs, turtles, etc.; toothed birds, etc.: a rich and unique fauna of unusual importance and interest, described by Cope, Marsh, Williston, etc.

THE LISBON SHALES.

Named from Lisbon, Kansas, near which they outcrop. Dark-bluish and brownish shales. Seen above the Smoky Hill chalk in Logan and Wallace counties, Kansas, and Elbert county, Colorado. Contain concretions of yellow phosphate of iron. (Fide Prof. G. H. Failyer.) Supposed to be lower Ft. Pierre. Fossils: *Inoceramus barabini*, *Baculites*, limpets, etc.

THE ARICKAREE SHALES.

Named from Arickaree river, on which they occur in Cheyenne county, Kansas, and an adjoining part of Colorado. Light-colored, olive, yellowish and brownish-gray shales. Referred to the lower part of the Fox Hills division. Fossils: *Pteria fibrosa*, *Scaphites nicollettti*, etc.

PRELIMINARY NOTICE OF THREE LATE NEOCENE TERRANES OF KANSAS.

BY F. W. CRAGIN.

At several localities in Kansas, typically on Bluff creek, in Clark county, in the immediate vicinity of the old Vanhem postoffice, occurs a succession of three terranes: (1) the lowest, consisting of gravels and sands laid down in deep and broad valleys; (2) the middle, consisting of a wide-spread horizon of white to brownish (rarely greenish) volcanic ash; and (3) the highest, consisting of yellowish-brown lacustrine or slack-water marls, containing variously shaped concretions of carbonate and silicate of lime (the former called "native plaster").

THE MEADE GRAVELS.

For the ancient (supposed late Pliocene) gravels, the name *Meade gravels* is proposed, after Meade Center, where they constitute the artesian-water-holding formation. They contain abundant remains of horses, llamas, elephants, turtles, etc., and rarer remains of Megalonyx and *Felidæ*, the species of which will be listed elsewhere, but which include *Elephas imperator* (?), *Megalonyx leidyi*, *Equus complicatus*. *Equus curvidens*, *Auchenia huerfanensis*, etc., indicating the fauna of the Equus beds. These gravels are mostly unconsolidated, but frequently contain hard ledges in Meade county and elsewhere. They are generally 10 to 30 or 40 feet in thickness and frequently grade into the Pearlette.

THE PEARLETTE ASH.

The volcanic ash may be called the *Pearlette ash,* from the old postoffice of Pearlette, in Meade county, where the writer studied an ash-bed of this terrane in 1884. The Pearlette rarely contains fossils. These do not differ from those of the underlying Meade formation. The ash attains a thickness of 13 feet in a bed southwest of Meade Center, considerably less in Clark and the many other counties of Kansas (especially western Kansas), Nebraska, etc., in which the ash has been found. It occurs west at least to Huerfano Park, Colorado (Hills), and east to Sioux City, Nebraska (Todd), and Galena, Kansas (Williston). It frequently passes into the Kingsdown by imperceptible gradations.

THE KINGSDOWN MARLS.

For the marls, the name *Kingsdown marls* is proposed, after the station of that name west of Bucklin on the Rock Island railway, between which and the upper part of Bluff creek, Clark county, they are finely exposed in deep ravines. They are very rarely fossilferous. *Elephas* was found by the writer in them near Vanhem. They are typically developed in Meade county also. They are apparently not less than 100 feet in thickness in Clark county, and more than twice that thickness at certain localities on the divides further westward.

All three of the terranes here described are supposed to be formations of the *Tule division* of Cummins (Equus beds of Cope), and to represent late Pliocene time. They are conformable with each other, and unconformably overlaid with local beds of marl, sand, diatomaceous earth, etc., of supposed Quaternary age.

LITERATURE FOR CHILDREN.*

Prof. E. S. Parsons.

What children are meant by our subject? Though the honor of speaking in this presence has been given me by you who are kindergarten teachers, I have not interpreted your request as limiting me to the discussion of literature suited to children of the kindergarten age. If I had so interpreted it, I should have been compelled at once to decline the invitation, for I feel myself wholly unqualified to speak upon a subject requiring technical knowledge and experience of a sort which I do not possess. I shall try to unfold the thought of literature for children of all ages, those who are just able to understand what is read to them, and every grade up to that which has just crossed the invisible line beyond which is youth. Five and sixteen are good theoretical limits.

What is literature? A second question more important than the first. We ought to resent the loose use of the term, for it has a definite meaning, and one which ought to be kept sacred. All writing is not literature. Intellect cannot make literature; so writing that seeks simply to inform, to instruct, is not literature. Into writing which can be called by this high name, intellect will enter, but it must be intellect, aerated by imagination, to use Lowell's favorite phrase. The wind of the spirit must blow through the essay or the poem if it is to win the right to be called literature. And this quality of imagination must have given to the writing an emotional and an artistic value, the two being closely linked together, before it can take its place in the list of the immortals. Imagination and feeling, then, are the final tests of literature: imagination, which apprehends the deeper significance of life and embodies it in forms of beauty, and feeling, which is the response of the soul to the vision.

*An address delivered before the twenty-second annual session of the Colorado Teachers' Association.

Judged by these high standards, there is not much which can be called literature. Our question to-day is, to what part of this body of writing shall children be introduced? This involves the larger question, are children capable of appreciating literature at all? Shall we give literature to children?

Children will read—that is, most children will. This fact may be taken for granted. The child who will not read under right conditions is an anomaly. The mind in childhood is as hungry as the body, and those of us who are parents know that our boys, as some one has put it, are hollow to their boots. The mental digestion of the child is as strong as his appetite, and he reads and digests with marvelous rapidity. Harriet Martineau once told a story that well illustrates the point. "I have seen a schoolboy of ten," she says, "lay himself down, back uppermost, with a quarto edition of 'Thalaba' before him, on the first day of the Easter holidays, and turn over the leaves, notwithstanding his inconvenient position, as fast as if he was looking for something, till in a few hours it was done, and he was off with it to the public library, bringing back 'The Curse of Kehama.' Thus he went on with all of Southey's poems and some others through his short holidays, scarcely moving through all those days except to run to the library." And Miss Martineau adds: "He came out of the process so changed that none of his family could help being struck by it. The expression of his eye, the cast of his countenance, his use of words, and his very gait were changed. In ten days he had advanced ten years in intelligence; and I have always thought that this was the turning point of his life. His parents wisely and kindly let him alone, aware that school would presently put an end to all excess in this new indulgence."

Children will read, and read voraciously. Publishers are more and more recognizing this fact. The law of demand and supply is finding new proof in the rapid accumulation of juvenile literature. Every publisher's list contains large numbers of new books for the young. Library tables swarm with magazines and weekly papers—good, bad and indifferent—designed to supply this voracious appetite. Children will read, and the publishers are giving them reading.

But is this printed matter, which is every year pouring out in a stream of huge volume from the presses of the world, what we want our children to read? Is it enough to turn them loose in a library of such books and papers? If so, then there is no need of my saying anything more. Nothing more can be said except to suggest a classification of the books by which children may be taught to gobble them down systematically, and so more exhaustively—taught to scrape the platter clean, as it were, at the literary banquet. Children have strong digestions, but such a process, I fear, would justify the words of the "Fable for Critics:"

> "Reading new books is like eating new bread,
> One can bear it at first, but by gradual steps he
> Is brought to death's door of a mental dyspepsy."

Is such a process safe for our children? Is it desirable?

There are great authorities who have given no uncertain answer to these questions. Charlotte Yonge has said: "We have little liking for books for boys," and she goes on to say in conclusion of a somewhat extended treatment of the subject: "Our conclusion as to children's literature is a somewhat Irish one, for it is, use it as little as possible, and then only what is substantially clean and good. Bring children as soon as possible to stretch up to books above them, provided those books are noble and good." Charles Dudley Warner speaks with still greater emphasis: "As a general thing, I do not believe in books written for children. * * * * I am not sure but it would be a gain if all so-called children's books were destroyed and the children depended altogether on what we call adult literature. I know of a family of young children who read, or had read to them, a translation of the 'Iliad.' They were perfectly captivated by it, and they got more out of it, even though not able to read it themselves, than they would have got from a whole library of the stuff children now commonly read."

You remember Emerson's dictum, the first of his three rules of reading: "Never read any book that is not a year old," implying that if you wait a year before reading a book, in a vast majority of cases you will not read it at all.

Nowhere does this rule apply so well as in the case of children's books. If we do not accept in its entirety the position which the first two writers quoted above have taken, we can at least accept it with Emerson's qualification. If at the end of a year the book has proved it has a right to live, then let the child read it. Only most of us, after having thoroughly studied the subject, would desire a still longer period of probation.

If the child should not read so-called children's books, what should he read? Miss Yonge and Charles Dudley Warner have already answered the question. Let them read adult literature—the books their elders read. This statement needs at least one limitation. Let the children read, not what their elders read, but what their elders ought to read, for the grown-up members of the family, as often as the children, read the latest and the trashiest books. In fact, what the children read is largely determined by what their elders read. Trash on the library table means trash in the nursery.

But the question rises at once in many of your minds, is the child capable of understanding adult literature? Is there not the danger of shooting above the heads of the children in seeking to limit them to such books?

It would not do before such an assemblage as this to base our answer to such a question on anything less than child study. This is now the fashionable subject of research, and one of our good fashions it is, too, though it has its absurdities, like most other fashions. What are the characteristics of the child mind which has this voracious literary appetite? Theories have no weight in this age unless they are theories that have developed out of facts. "Children are indeed treated and written about sometimes," said a very wise writer on our general theme more than thirty years ago, "Children are indeed treated and written about as though they were little fools, and any baby talk or twaddle were good enough for them; but we are in the main inclined to believe that they are great fools who make this mistake, and so sadly libel God's handiwork." "The great human mind is in the little child as in the gray-headed sage," but the intellect—what we

mean in our formal use of the word—is not the faculty most early developed. We who are parents often find our children startlingly good reasoners, but that is not the aspect of the mind which stands out in them most prominently. Any one who knows children knows that the affections and the imagination are what chiefly characterize the child nature. "Give a child a little love and you will get a great deal in return," some one has said. Give a child a glimpse into the imaginative world, and he will build castles and people them, fight battles and win them, create a new world and live in it, far from the madding crowd of life's cares and pain.

"The supreme endowment of human nature is the gift of imagination," and it is given to the child at the beginning of life. Oftentimes the imagination atrophies for want of use. But it need not have this fate. The poet, the seer, is he who does not let it die, but who, through all his life, looks out upon nature and men with the eyes of a child, seeing not merely what is at the surface, but what is underneath, not merely the hard, apparent reality, but the blessed and more real ideality.

Many persons, hard and fast realists, grow angry at the suggestion that there is any value in the fancies of childhood and in the power which calls them into being. They would, if they dared, criticise the Creator for mingling this ingredient in the human composition, and they seek to eliminate it by denying the child mind its natural food of fairy story and myth, by strapping it about with the intellectual bands of the alphabet and the spelling book and the prosaic primer and arithmetic, much as the Chinese mother of high caste binds and dwarfs the foot of her little baby girl. To the annointed eye such realists have the grotesque—the pitiful—hobble of the fine Chinese lady of mature years. But the satisfaction of the imagination and the affections in early life does not mean the crippling and dwarfing of the intellectual powers. They will unfold soon enough. They are simply latent. And when they begin to develop, and after they have grown to maturity, nothing will aid them so much in doing their work as "the witch, imagination."

The affections and the imagination, then, are the powers which dominate the child nature. Let us revert to what we said about literature: "Imagination and feeling are the final tests of literature." Imagination and emotion, the chief traits of childhood; imagination and emotion, the chief tests of literature—the two were evidently made for one another.

It is singularly interesting that three of the greatest classics of childhood were not written for children at all. "Pilgrim's Progress" was a new type of sermon written by the tinker preacher in his prison cell at Bedford; "Robinson Crusoe" was a pseudo-history from the pen of one of the first great English realists; "Gulliver's Travels" was a political satire by the greatest of English satirists. The same thing is true of the stories of the Bible, of the "Arabian Nights," of the folk lore which strikes a sympathetic chord at once in the child's nature. The truth is that the childlike is the eternal. Goethe said at the close of his "Faust" that the Eternal Womanly is to lead us on. Has not our age, by reason of its more intimate knowledge of human life, come to the profounder truth that it is the Eternal Childlike which is to lead the world to its final perfection? At this season the Christ from His manger cradle sways the world, and the heart of child is always the best standard of religion, character, art, literature. Except ye be converted and become as little children ye cannot enter the kingdom of painting or of books any more than the kingdom of God and His Christ.

Child study, then, reveals the fact that the child nature is the counterpart of what is best in books, that children can appreciate literature. But do the conclusions of the psychological laboratory find themselves sustained in the practical experience of those who have had to deal with children? You remember that beautiful story from the pen of Laura E. Richards, "Captain January"—one of those books of recent times which at once reveal the fact that they are literature. The red dory was just about to set out for the lighthouse home when the captain was hailed by an old friend with the same title. The conversation drifted straight to the topic always uppermost in the lighthouse keeper's mind, his "little

gal." "'She's learnin!' he added proudly: 'learnin' well! I'll bet there ain't no gal in your school knows more nor that little un does. Won'erful, the way she walks ahead.'

'Get the school readers, hey, and teach her yourself, do you?' queried Captain Nazro.

'No. sir!' replied the old man; 'I don't have no school readers. The child learns out o' the two best books in the world—the Bible and William Shakespeare's book: them's all the books she ever seed—*saw*, I should say.'

'William Shak—' began Captain Nazro, and then he broke off in sheer amazement, and said simply, 'Well, I'm blowed!'"

Then follows the charming story of the little ten-year-old child with "her cloud of pale-gold hair" and her soft, deep, shadowy blue eyes, to whom Samson and Imogen and Ariel were as real as the cold waves that washed her to the rocky shore and to the warmth of the old sailor's heart. "An ideal picture," you tell me. Is it merely ideal, or was it drawn from life? Last May I received a letter from an Omaha lady in which, writing about another subject, she spoke of her little girl, then nine years old: "M—— is very fond of her father's library. * * * * The books she is simply devoted to are the plays of Shakespeare and the Bible." Was Captain January so far wrong in taking the advice of his minister and bringing up the "little gal" on those, the two best books in the world? Harriet Martineau, after relating the story I have quoted, tells of her own childhood-reading: "I devoured all of Shakespeare, sitting on a footstool and reading by firelight. * * * * I made shirts with due diligence, being fond of sewing: but it was with Goldsmith, or Thomson, or Milton, open in my lap, under my work, or hidden by the table." One of our own State teachers, one whom we all delight to honor, told me a few weeks ago that he had read all of Goethe's "Faust" with his little thirteen-year-old girl, to her great enjoyment, and that last summer she read alone all of Chaucer's "Canterbury Tales." Many teachers have found young children delighted with Dante. I have to my own satisfaction discovered that little children, not yet able to read, will be deeply interested in the stories of the Bible read to them from

the Book itself, where they turn to their plays from the reading of a paraphrase of the stories written expressly for children. The truth is. as Miss Burt has stated it in her book, which contains so many wise words: "Age has little to do with the ability of children to receive classic thought." Illustrations of this fact could be given by almost any teacher who has enough of the literary spirit in himself to be competent to teach literature.

It has been wisely said that a taste of a great thought is worth far more than the full comprehension of a small one. No teacher ought to expect, or to desire, the full comprehension on the part of a child of a great work of the imagination. Nothing is so inimical to the development of a genuine literary appreciation as the insincerity which a teacher will invariably foster by such expectations. But in literature it is not true that a bird in the hand is worth two in the bush. A little glimpse into a large and generous world is far better than the full understanding of a mean and paltry one. The child needs something that will expand his nature, something he can grow to, not leave behind. One would hardly think of citing Dr. Johnson as an authority on the education of children, especially when it is remembered that it was he who had the most to do with making our English orthography such a terror to young and old alike, but no one ever said a truer word on the subject than this: "Babies do not want to hear about babies; they like to be told about giants and castles, and of somewhat which can stretch and stimulate their little minds."

Of course, what has just been said does not mean that all pieces of great literature are equally fitted to a child's mind at any period in its life. But it does mean that it is not necessary to go outside the world's great literature for fit mental food for a child's imaginative and emotional nature.

The problem, then, as Mr. Horace E. Scudder has put it, is not one of creation, but one of selection. "The task is to find literature for the child, not to make it. The permanent in literature springs from the necessity of the writer to create, not from the attempt to fit the creation to the needs of the reader."

Literature is a spontaneous product; it cannot be made to order; it makes itself. He who attempts to make it to order fails before he begins. Genius is unconscious, and builds better than it knows; it produces because it must; and its readers, usually a later generation, recognize the full meaning of what has been written. The work of him who is to guide a child's reading is to select out of the works which bear the unmistakable stamp of genius—a stamp which the test of time always reveals—those which are best suited to the child at the particular stage of his development.

But can anything more definite be said? I have no desire to give lists of books suited to children. Such manuals as Miss Burt's "Literary Landmarks" and Hewin's "Books for the Young" will supply such lists for those who need them. But a word or two of more detailed suggestion will not be amiss. Poetry will find its place early in a child's reading, if not at the beginning. It is concentrated emotion and imagination, and much of it is as simple literature as there is in the language. Mr. Scudder is of the opinion that the poetry which children should first read by themselves should be, not Whittier, nor Longfellow, nor Wordsworth, but "Mother Goose." His reasons are well worth repeating: "'Mother Goose' helps the child to make a passage from the known to the unknown. The cat he knows, the boy he knows; but the cat in the well, little Johnny Green, big Johnny Stout, the bell with its swinging, resounding note—all these are in the region of the just not known; and when he reads, half sings, the ditty, his mind is given wings with which to soar a little way. Again, 'Mother Goose' is cheerful, and the task of reading literature is lightened. Further, 'Mother Goose' is full of human associations, and entering literature by these passages, the child is treading steps worn by generations of use. There is no waste. He is becoming familiar with the permanent in literature; he is not conning that which will be left behind with childhood. Rather, he is acquiring a currency which will, in later days, be drawn forth for use in the exchange when 'we that are children have children.'"

After "Mother Goose" Mr. Scudder suggests simple poems of Whittier, Longfellow, Blake, Wordsworth. In prose every teacher, as well as every parent, knows there is no book for young, as well as old, students of literature like the Bible. In the Bible we have imagination and emotion; we have simplicity combined with exquisite purity of thought and style; we are dealing with what has endured the test of time; we are introducing the child to what is of permanent value, to that which alone can unlock much of what is best in English literature. Macaulay's remark that he who aspires to be a critic of English literature must have the Bible at his finger's ends has received another iteration in a recently published utterance of the professor of the English language and literature in Yale University (in a paper upon Nineteenth Century literature) that "it would be worth while to read the Bible carefully and repeatedly if only as a key to modern culture, for to those who are unfamiliar with its teachings and diction much that is best in the English literature of the present century is as a sealed book."

Mrs. Wesley, it will be remembered, the day her children were five years old, set them to learning the alphabet. The second day she put them to reading in the first chapter of Genesis, and made the Bible thereafter their text-book. Such training produced not merely the superb energy of John Wesley, but also the rare poetic power of his equally gifted brother, Charles.

It is almost a truism to quote John Ruskin's experience, but to omit his testimony is to fail to call to the stand the best witness of the influence of the Bible as literature. He tells the story in the early part of "Præterita": "I have next with deeper gratitude to chronicle what I owed to my mother for the resolutely consistent lessons which so exercised me in the Scriptures as to make every word of them familiar to my ear in habitual music,—yet in that familiarity reverenced, as transcending all thought and ordaining all conduct. This she effected, not by her own sayings or personal authority, but simply by compelling me to read the book thoroughly, for myself. As soon as I was able to read with fluency, she began a course of Bible work with me, which never ceased till

I went to Oxford. She read alternate verses with me, watching, at first, every intonation of my voice, and correcting the false ones, till she made me understand the verse, if within my reach, rightly and energetically. It might be beyond me altogether—that she did not care about—but she made sure that as soon as I got hold of it at all, I should get hold of it by the right end. In this way she began with the first verse of Genesis, and went straight through to the last verse of the Apocalypse—hard names, numbers, Levitical law, and all—and began at Genesis the next day. * * * * After our chapters (from two to three a day, according to their length, the first thing after breakfast, and no interruption * * * * allowed) I had to learn a few verses by heart, or repeat, to make sure I had not lost, something of what was already known; and with the chapters thus gradually possessed from the first word to the last, I had to learn the whole body of the fine old Scottish paraphrases, which are good, melodious and forceful verse, and to which, together with the Bible itself, I owe the first cultivation of my ear in sound." And concluding his discussion of the whole matter, he says: "Though I have picked up the elements of a little further knowledge—in mathematics, meteorology and the like, in after life—and owe not a little to the teaching of many people, this maternal installation of my mind in that property of chapters I count very confidently the most precious, and, on the whole, the one *essential* part of all my education." It was this which gave him the place accorded him without any reservation by the latest authoritative historian of nineteenth century literature, that of the greatest of English prose writers.

Such readings in the Bible, and in simple poetry, with a large intermingling of fairy stories and folk lore, will make up the bulk of a child's early reading. What shall follow? Some say, after such a beginning turn a child loose in a well selected library and let him browse. Such an experience is of the utmost value to the child. It leads him into familiarity with books. He tastes many, and is led into fascinating regions before unknown. The delight of the explorer is added to the delight of the reader. Lowell began in such a way the extensive browsings which made him what he called himself,

the last of the great readers. Led by his own inclination. John Fiske had read at seven Rollin, Josephus. Goldsmith's "Greece," Shakespeare, Bunyan and Milton. As some one said of this reading of his, "I dare say he skipped to get the stories, but they were great stories." At eleven he had read Gibbon, Robertson. Prescott, Froissart. This was his reading for pleasure, and was wholly apart from the intense study going on at the same time.

But objections have been and can be brought against this habit of reading. One, which is directed not merely against this, but against much of what has already been said, is that it means the overcrowding of the child's brain. In an article published during the summer in one of our best weeklies, a writer discussed this objection under the question, "Are Our Children Precocious or Infantile?" and she concluded that. while naturally quick, our American children are in reality infantile in their acquirements. Part of the fault must be laid at the door of the necessity we are under of educating children in regiments, but part also must be answered for by the foolish notion that it is a severer tax on the brain "to learn that a certain combination of three letters means cat than that a certain furry creature, having a long tail and catching mice, *is* a cat," that it breaks down the brain power to read Southey's "Life of Nelson," but builds it up to read Oliver Optic's "Soldier Boy" and "Sailor Boy." The opposite is the truth. The book full of unreality and sentimentality without genuine emotion and high imagination is what breaks down the mind, and with it the body, while the good book is a tonic, "as refreshing and strengthening as the sunshine and the sea water."

But a more definite objection is urged to this habit of library browsing. It is said that if the library is at all inclusive of great literature there will be much in it which will pollute the child's imagination. But is this true? Has any one a grain of evidence to support the theory that a child, early taught to love the best literature and drawn to it of its own accord, will be polluted by the coarseness which is here and there a blot upon it? The little girl who last year read through the "Canterbury Tales" expressed to her father her delight,

but said there were some things in them which Chaucer ought to have been ashamed to write. Did it hurt the child to read those things and let her purity condemn them? Harriet Martineau, to whose wisdom I have already appealed, says: "The last thing that parents need fear is that the young reader will be hurt by passages in really good authors, which might raise a blush a few years later. Whatever children do not understand slips through the mind and leaves no trace; and whatever they do understand of matters of passion is to them divested of mischief. Purified editions of noble books are monuments of wasted labor; for it ought to be with adults as it is with children—their purity should be an all-sufficient purifier."

But, believing as I do that, where it is possible, this browsing habit should form a part of every child's education, it is not everywhere, or in every case, possible. It is a rare exception to find a home with such a well-selected library. Most homes have nothing worth the name. The public library in most places does not permit free entrance to the shelves. The book stall, which meant so much to a browser like Dr. Johnson, has not become much domesticated in America. The child cannot be taught to browse, because there is no pasturage.

And more than this, such reading, while valuable as an adjunct, is too desultory to give a full and rounded introduction to literature. It is the opinion of those who have most carefully studied this subject that "between the ages of six and sixteen a large part of the best literature of the world may be read if taken up systematically at school," and the writer who makes this statement adds, "that the man or woman who fails to become acquainted with great literature in some form in that time is little likely to have a taste formed later." Such a wide acquaintance with literature cannot be obtained by browsing; browsing must be accompanied by systematic, developing courses of reading.

At the risk of wearing out your patience, I must say a few words about still another subject. What end shall we set before the child to be gained by reading? No end at all, we can say at once. Reading to the child ought to be an end in itself.

"We get no good by being ungenerous even unto books,
And calculating profits, so much good by so much reading."

But what end shall we set before *ourselves* that the child shall gain from the reading? "Many teachers have felt," as Mr. Scudder has so pithily put it, "that Pegasus ought to be hitched to a tip cart," that reading should teach history, geography, biography, science — in other words, that literature should be made to help the child to obtain information, a purely intellectual achievement. But the writer of literature did not write to give information. He wrote because he had to, because it was the great joy of his life to make the product of his imagination live in the written word. And the spirit in which he wrote the book is the spirit in which it should be read. To teach geography by it is to make it tasteless and insipid; to make it a medium of criticism is to make it dead and joyless. The child should learn to delight in it, to read it only for the delight it can inspire. If the child does not like what is given him to read, let his choice prevail, and let the teacher, or parent, study to find what will delight him. Those who persist in trying to harness up Pegasus to practical things may find it necessary to insert in the newspaper the kind of advertisement which Lowell suggests John Bull will some day have to put into the *London Times:*

> "Lost, strayed or stolen, from the farmyard of the subscriber, the valuable horse Pegasus. Probably has on him part of a new plough harness, as that is also missing. A suitable reward, etc.
>
> "J. BULL."

What has been said at once answers the objection that the use of great literature in the schoolroom will kill it for the scholars for all the future. Certainly it will be so killed if the teacher does not know what literature is, and treats it as he would a dry-as-dust text-book. But the teacher who, instead, leaves criticism alone, explains only enough to stimulate the child's interest, and seeks to lead him out into the perennial delights of the great creations—he will never have to bear the accusation of killing a good book. "The great end of literature is not to inform, but to inspire." Let us thoroughly grasp this truth and we shall never again teach literature as it has so often been taught.

To accomplish what has been set forth will require a wise planning of a child's time. We must remember that a child's

early years are in many respects the most precious of its life. A wise mother told me a few days ago that, hard as it seemed to some of her friends, she had never allowed that most excellent boy's and girl's paper, the *Youth's Companion*, to enter the house. Not that she objected to a single word printed in it, but she felt that it had a too dissipating influence upon her children's reading. Coming week after week, it gave her children no time for the reading of great books—books which have stood the test of time, and have been a fountain of literary inspiration. The wisdom of her choice for her children is illustrated by the fact that a few weeks ago her first born was elected to write the ode for the graduating exercises of his class at Harvard next June. It would be well for us all to ask ourselves the question, does it pay for us, or for our children, to give to the reading of current books so much of the time we might give to what has proved itself to be the world's greatest literature?

The results upon the child of such reading as I have been advocating are too evident to detain us. A child, after his earliest years, talks like those with whom he associates. "Live with wolves and you will learn to howl," runs the proverb. Live with the great masters of the English speech, and if there is any literary instinct in you, you will learn to use clearly, simply and musically the greatest of all languages. But this is by no means the most important result to be gained. This is an age full of unconscious, if not conscious, materialism. Aspiration fades more quickly to-day than ever before from the human soul. Imagination and the higher powers of the soul tend to die in the suffocating atmosphere of mere fact. But in him who has been taught in youth to wander in the green pastures and beside the still waters of our great English literature the soul will not die. For him there will always be open what the poet Spenser so beautifully called "the world's sweet inn," where he can ever find rest and refreshment, and in which there is always to be found a large upper chamber called Peace, whose windows open toward the sun's rising.

WARMING UP.

DR. E. G. LANCASTER.

It is a common and generally observed phenomenon that when one starts for a long walk there comes a period of fatigue, which, with many people, passes away after continued walking. The same phenomenon may be noticed with most people in any kind of mental or physical activity. A student sits down for a long period of study or writing. There may be at first a feeling of fatigue or awkwardness or that he has not yet "found the combination" which, after a time, gives place to a growing facility that increases until exhaustion demands a rest.

The same thing may be noticed in animals. Dogs on the chase, the animals pursued, and especially race-horses show the effect of warming up. It is said of two famous trotters, each of which has reduced the world's record within a few years, that the period of warming up was very characteristic. One was driven two miles at a 2:30 gait, rested, rubbed, taken out for his fast mile but driven a full mile at about the same pace as before, until approaching the wire, when he was driven at full speed and the record was taken thus on the fourth mile. So much warming up was necessary. The other was jogged one mile, rubbed, then taken to the quarter pole and gradually urged to her highest speed and the record was taken on this the second mile.

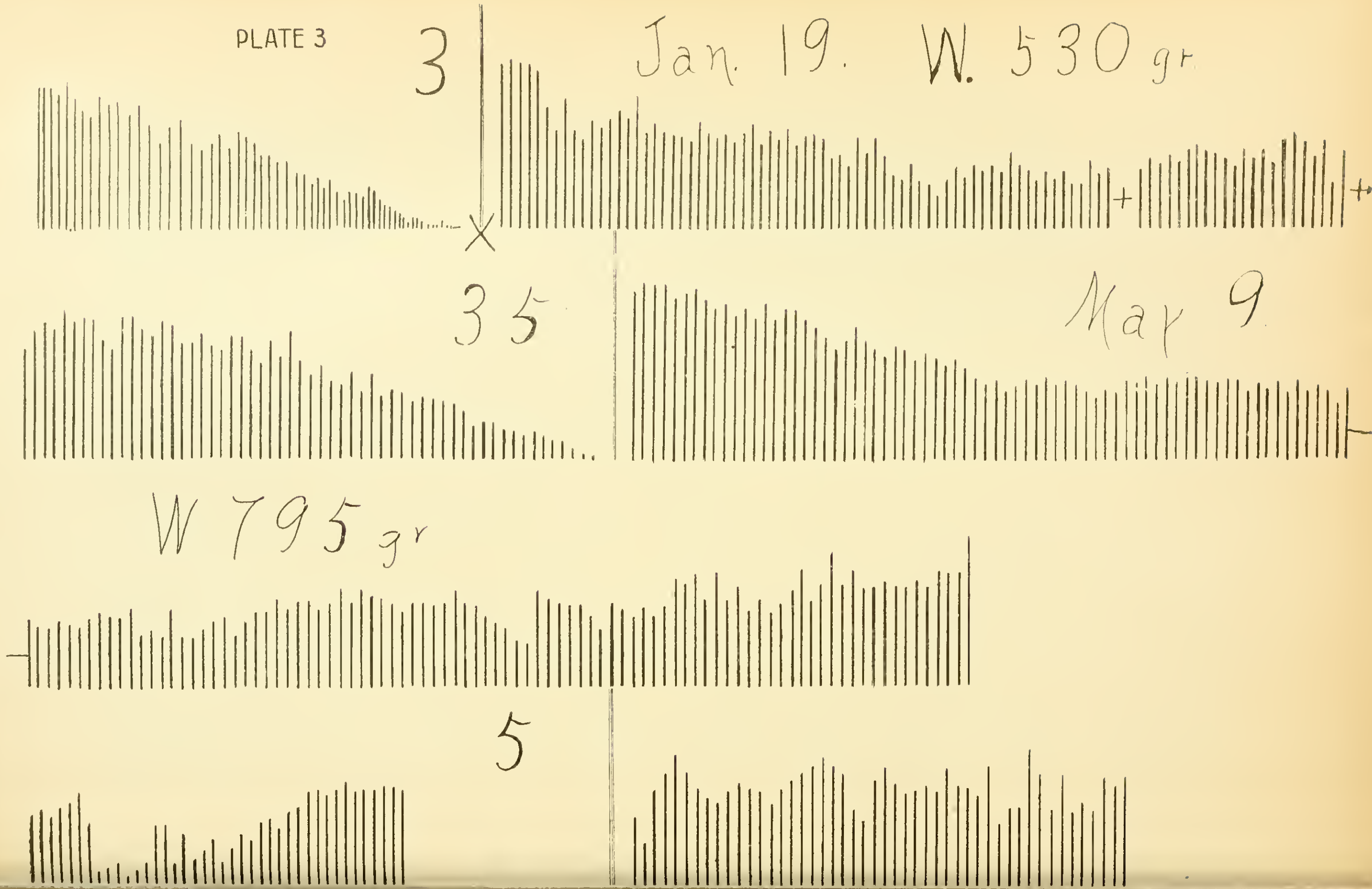
Athletes, especially ball-players, realize the importance of practice just before the games, to be followed by a slight rest. A pitcher would hardly enter the box till he had got his arm in working order by a few minutes' practice. Orators often are dull at first but warm up. It is said that Wendell Phillips was often hissed for his slow, uninteresting speech but rallied to the occasion at such times with his masterly oratory. We need not multiply instances. It can be seen and felt almost anywhere or any time, in one way or another.

It fell to my lot in the psychological laboratory, at Clark University, to determine if possible exactly what happens in this warming up process and whether the cause is located in the brain, muscle, or elsewhere. The results obtained were certainly suggestive. Having in mind the work done by Mosso, and especially the closely related work of Lombard, we began in a similar way with the ergograph, which is a machine for measuring work done by registering the height to which the weight is lifted each time. We used the middle finger of the right hand. The muscles are so closely related in the forearm, and in some persons probably grown together to such an extent that the experiment there was valueless. When one combination of muscle fibers fatigued, another took its place and the finger could be worked with considerable power, though variable, all the time. We then arranged a thimble for the index finger and a hand rest, to which the hand was bound so that it could not be tilted to allow a bending of the finger, thus having but one motion possible—that of separating the index from the other fingers by the use of the *abductor indicis*, which is a y-shaped muscle between the first joints of the thumb and index. Being separate from all others, this muscle can be used for an exact record. The marks in the curve represent the height to which the weight was pulled each time.

Ten or twelve subjects were tried, though most of the results were gained from four or five. The others were used to verify the work. Some of the experiments were so painful that they were not often repeated.

The two curves (see Plate 3), No. 3 taken January 19 and No. 35 taken May 9, show the general effect of warming up on one subject whom we may call G. They show, too, the value of exercising a muscle regularly, since 530 grams at first fatigued the muscle in about 95 seconds while in May 795 grams produced the same effect in 115 seconds. The weight was pulled once every second at the swinging of a pendulum. (These curves have been reduced but the general character remains.)

PLATE 3



In the first curve (No. 3) it is seen that with G. the curve falls quite steadily to the end. With G. and with others it is found that this curve is as characteristic and uniform as their signatures.

After the muscle was completely tired the weight was removed (at line marked X) and a very light one put in its place merely to keep the muscle active, for five minutes. Then the same weight as used at first was replaced and the second curve taken. In both of these cases, all of them in fact that were tried on most of the subjects, there was not room enough on the drum of the kymograph to record the second curve and they were broken off before there was any evidence of fatigue. In one instance G.'s warmed up curve with 600 grams was continued a long time and then additions made until he was pulling 1.075 grams, which he raised to a good height till stopped by the operator. He could not move more than 800 grams before the warming up occurred. With this subject there was a feeling of muscular exaltation that made it a pleasure to lift the weight after the warming up occurred. It was found that this was true after he said that he was tired out and did not feel able to go through the experiment. He noticed the same thing with studying. Others have spoken of sitting down to study with a tired feeling that almost persuaded them to give up the attempt, but beginning to work that feeling disappeared and the best work followed without fatigue for two hours or more. That looks as if the first feeling of fatigue is a ruse of nature to get out of work. It only indicates that the system is still building up or in the anabolic state. One cannot, however, judge by his feeling what his warmed up curve may be. All but one of the subjects showed this warming up phenomenon.

After marking the fact of warming up we attempted to locate this increase of power. The first interesting point noticed was that a general warming up, such as sparring, running or general active exercise in the gymnasium produced the same warmed up condition of this small muscle, which was not used at all or very slightly in the general

warming. The curve is almost a reproduction of the second part of No. 3.

Next electric stimulus was tried. At first the electrodes were placed over marked points on the muscle. The results were unsatisfactory. Since the current in such a case seems to stimulate only a part of the muscle fibres, the slightest moving of the hand or even a change in the position of the body or feet would move the instrument or muscle enough to get a large increase of power at once, which at first seemed like a recovery of power. Then we located the branch of the radial nerve which supplies the motor power to this muscle and applied the electrodes to it. The result was a much larger amount of work done and much more uniform results. The same attempt was made to work the muscle a time, then give it five minutes' warming up and then apply again the same stimulus to the nerve as before. The results were not always the same and varied much in different subjects. In no case, however, did the curve after the warming up show a decided improvement. In a few cases and in the subject G., who showed the most constant improvement in the warmed up curve, there was sometimes a slight improvement. This seemed, however, to be entirely due to peripheral conditions. The flexibility of the skin in the palm of the hand and about the index, the natural stiffness or resistance about the joints that have not been used in a certain way for a long time, might explain the slight rise in the curve after these resisting elements had been lessened or relaxed by use. This looked as if there was nothing more than a slight improvement at most in the muscle itself and the end plates of the nerves. See curve No. 5, Plate 3.

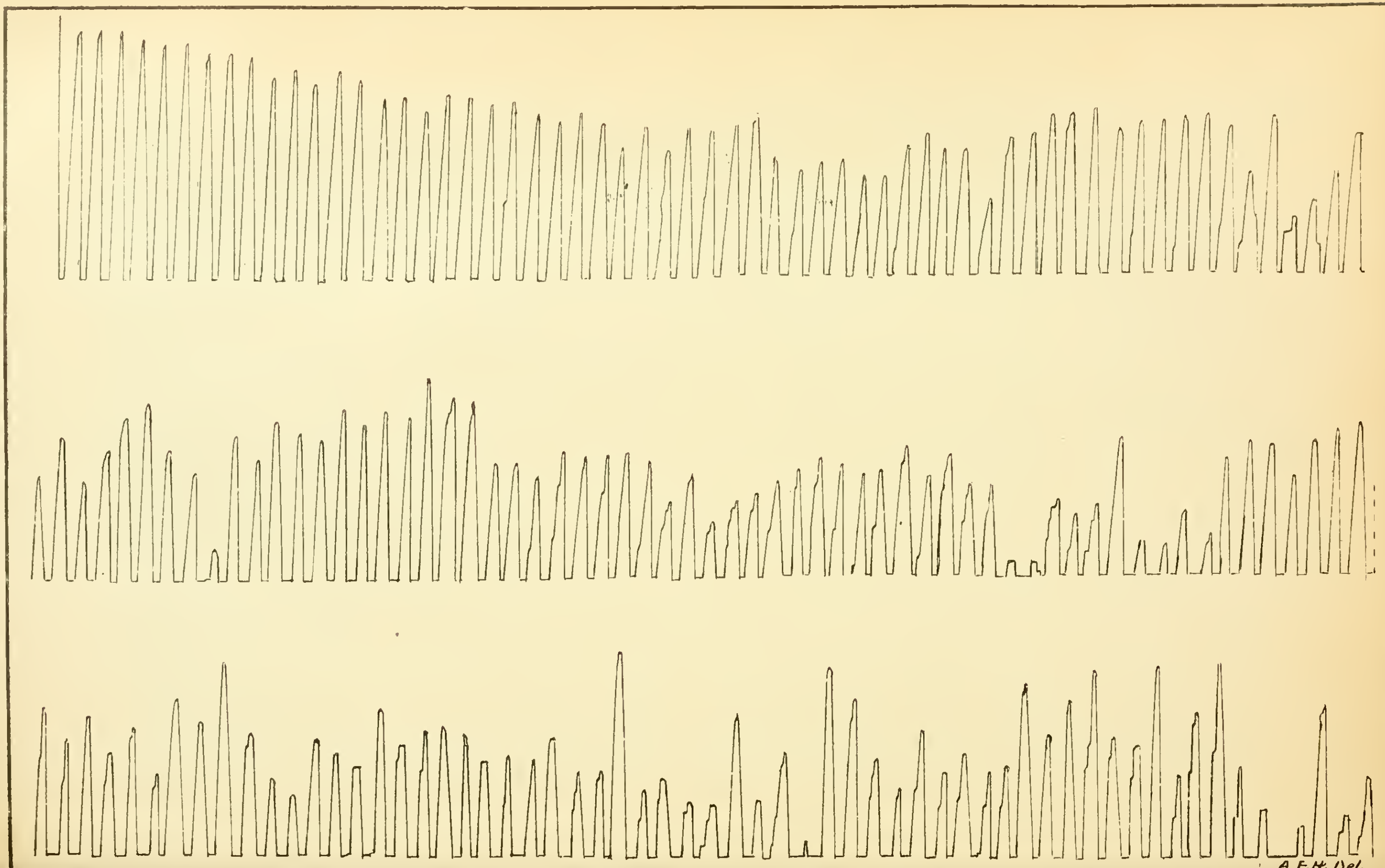
Then there were many instances where the curve after the electric warming up was less than before. In a few cases, the same stimulus, applied to the nerve, could not raise as large a weight after the first curve had been taken and a period of five minutes had been given for rest and warming up. On removing 150 grams the subject raised it to a good height. The 150 were put back after a few successful pulls of the lighter weight and the same condition prevailed as at

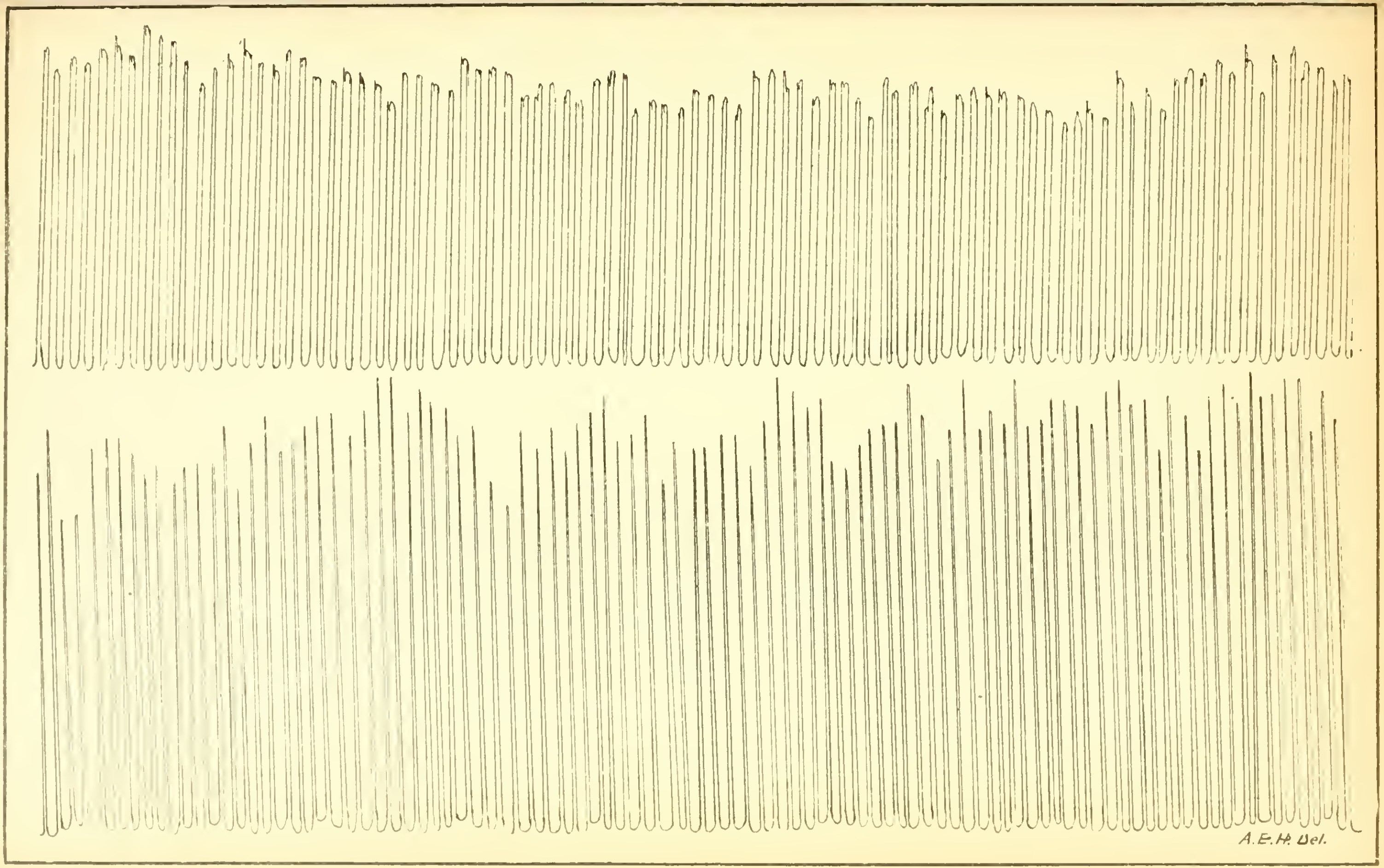
first. The heavier weight, which in the first curve, before the warming up, had been raised as high as the lighter weight after it, could not be raised at all. This was done until we were satisfied that the same stimulus to the nerve produced less work after a time than at first and that the warming up did not improve it.

This is in line with another experiment where the weight was pulled by electric stimulus applied to the nerve every two seconds during a long period of time—15 to 16 minutes. The current was on one second and off one second. That result may be seen on curve No. 13, Plate 1. The weight was 525 grams and the current as strong as the subject could endure. The pain decreased until the sensory nerves were nearly or quite fatigued and failed to report any pain to the brain, for after 10 minutes or so the current was scarcely appreciable. In the curve there may be noticed an almost rhythmical rise and fall. At first this would seem to contradict Lombard's work and give a "recovery" by electric stimulation. It would look as if there was a change caused by fatigue in the muscle or nerve endings. It might be due to a clogging of the contractile substances which worked themselves clear again, or it might be due to an unconscious assistance by voluntary effort. The curve is suggestive but not decisive. There is no warming up but rather a loss of power. Lombard's experiment was repeated here and his results obtained. After voluntary fatigue the electric power was used and then the voluntary, alternating, when it was found that the power had returned though the muscle had been worked all the time. To compare with that another curve is shown, No. 16 (Plate 2), in which the same weight was pulled voluntarily every other second and a distinct gain of power or warming up appears. There was no fatigue but a feeling of muscular exaltation in this *abductor indicis* at the finish. This also shows that the warming up may be gained without fatigue if moderate work is done at first.

An attempt was made next to shut off the effect of increased circulation, thinking that the warming up might be caused by an extra blood supply to the muscle. An Esmarch's

PLATE I. CURVE 13.




A. E. H. del.

bandage was used to wind the arm, beginning with the fingers, so that the blood was all driven back to a point near the shoulder. A lighter weight was then used, 175 grams at first, since the hand was rendered partially numb by the experiment. The effect was not all that we hoped but showed one or two noteworthy results. Since the winding occupied one or two minutes, the hand was rapidly cooling before the curve began. It was interesting, therefore, to note that the first curve with each subject, taken with a bloodless arm, was a *facsimile* of his normal curve taken with the blood in the muscle. This was especially noticeable with one subject whose curve was very characteristic, always having a rally at a certain point near the finish. This rally appeared just the same when the blood was shut off. Knowing one's curve before, the curve with the bloodless arm could be identified easily. After the first curve was taken and the weight could not be moved again, the weight was taken off and three minutes given for exercise of the muscle and warming up. Then the same weight was attached. At first in each case it could not be moved. Then with G. the power came back and the weight was raised, and for a time his characteristic warmed-up curve was recorded, which curve always differed from the curve taken before the warming up. The pain in the arm and hand became so severe that he begged to be released. See curve No. 18, Plate 4. A recovery of power was found with other subjects but not very marked except in one case where a nearly normal power was gained for a few seconds, but the violent pain and numbness rendered the hand useless. Sometimes after the first curve and five minutes' attempt at warming up the hand could not be opened or shut and no motion of the index was possible. After one experiment with G., when the bloodless hand had become useless and no curve was possible after the five-minute rest following the first curve, the bandage was removed and in one minute the curve was taken and the usual warmed up curve was the result. This shows that the warming had occurred but the hand was stiff and cold so that the effect could not be registered.

One fact came out here in conflict with the teaching of physiology in some colleges. The rate of fatigue was just the same, with the circulation entirely shut off, as it was before. Other experiments were made in this line but nothing valuable resulted.

The next experiment seemed to be more decisive. Subject G. had been a subject for an hour or more in another room learning nonsense syllables, where he had shown a distinct warming up toward the end of the hour. His curve was taken very soon after learning syllables and it was found to be his warmed up curve, though he had been quiet physically all the afternoon. The curve showed no flagging at all. It was continued with no fatigue except the characteristic rise and fall of a few centimeters seen on all of his warmed up curves. This clue was followed. The subject was set to adding for twenty minutes, at his most intense mental effort and immediately afterward his curve was taken. It resulted always the same way. The warming up was apparent and his characteristic curve was registered. Learning of syllables was tried again and proved to give a somewhat more complete warming up than the adding. See Plate 4, curve No. 45 for adding effect and No. 50 for memorizing effect.

In later tests a record was also taken of the respiration and pulse. It was found in every case that the respiration did not change much in rate and the changes in height are due somewhat to motions of the body other than respiratory.

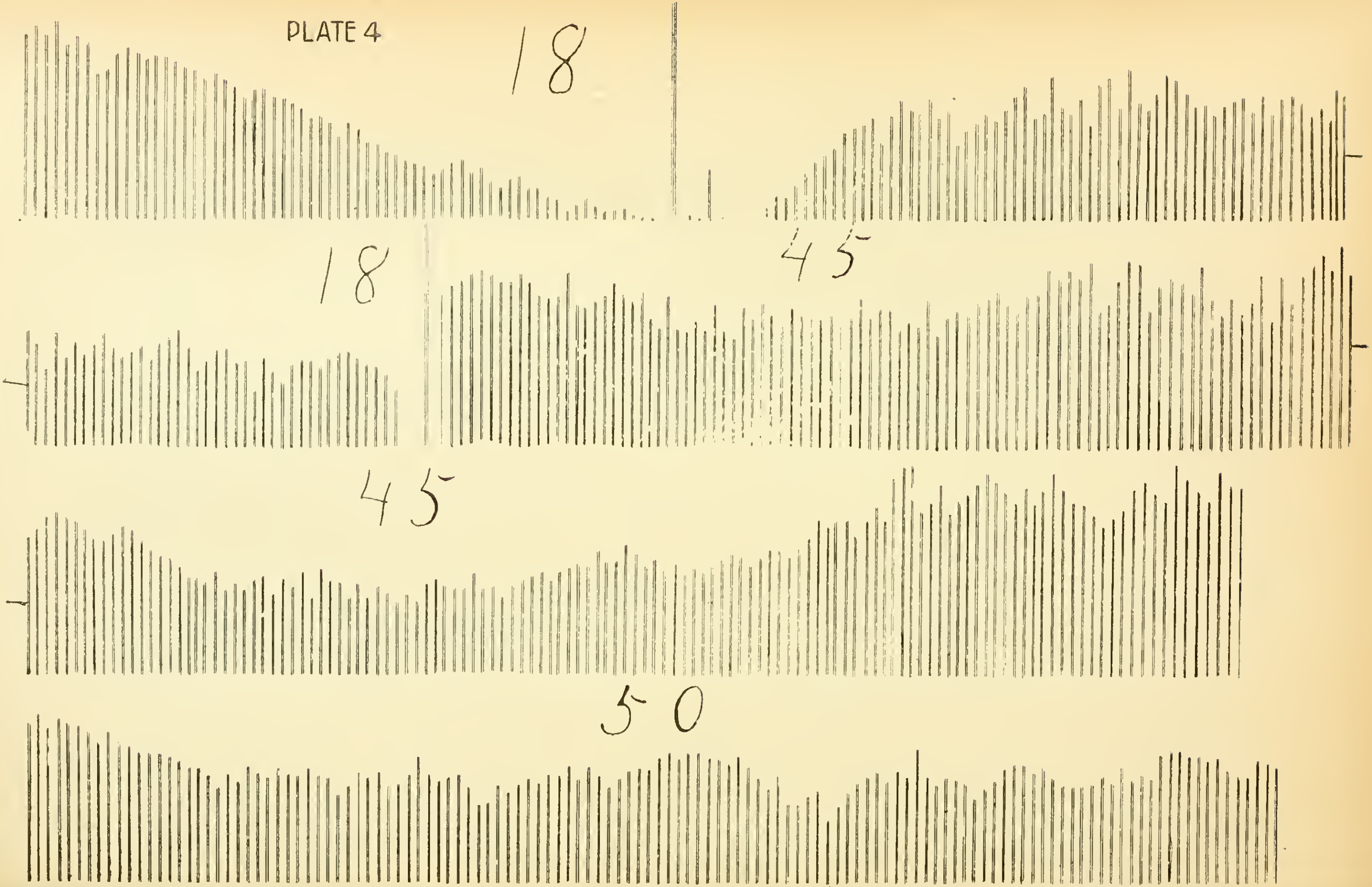
The pulse regularly falls from 4 to 6 beats a minute in this experiment. It would seem thus that increased respiration or pulse has no share in the phenomenon of warming up.

It has been noticeable, too, in other experiments conducted in this laboratory where the subject was adding for forty minutes or learning syllables, or alternately learning and adding, that the pulse falls though there is a gain in the time required to add or learn the syllables.

This experiment shows the following:

1. Warming up is general but not universal. One subject always did his best work first. He shows no warming up in mental work either. He can do his best studying the first half hour and regularly decreases.

PLATE 4



2. Warming up is not gained with electric stimulation, while it is gained, in those who show the phenomenon, by voluntary effort.

3. A bloodless arm fatigues at the same rate as a normal arm, and some recovery or warming up is possible even then.

4. This warming up may be gained from mental activity and the effect of it shown in a muscle. It seems justifiable, then, to locate the warming up in the brain cells.

Now, what happens in the cortex to produce this phenomenon? Four causes are suggested.

1. The use of a center may react on the local arteries and increase the blood supply and nourishment of the nerve cells. This is called hyperæmia.

2. The stimulus may spread and involve a much larger area. The automatic motor impulses which may be seen to accompany any vigorous effort or new motion, may be gathered and sent along these active channels.

3. The nerve cells may discharge more vigorously when excited by products caused by the previous activity. That is, it may be a case of auto-intoxication.

4. The amœboid motion of the nerve cells as explained by Golgi, Ramon y Cajal, Krapotkin and others, best explains this warming up. According to them the dendrites, or tree-like branches of the nerves, which arise from the central end of the nerve cell, tend to ball up or contract, when at rest, in sleep, etc. When some stimulus is given, these expand and make connections with their neighbors, and whole areas are thus harnessed for activity which do not respond ordinarily.

In conclusion a few observations might be suggested. Orators, athletes, students and working men often are aware of this effect and practice mental or physical gymnastics, or both, when about to make an unusual effort. Some orators always excite themselves by recitation of thrilling literature and by physical efforts before meeting an audience. The effect is important on those who are influenced thus. It means, if we take the fourth explanation above, that larger areas of the brain are brought into activity. The question

of "second wind" is closely allied though not the same as warming up. It is manifest that generally when people speak of getting their second wind they mean they are warmed up. This would apply always in all cases except violent exercise, like running, where both phenomena are present. There is no doubt that warming up gives better control of the muscular system in running and other violent exercise. The difficulty in breathing which passes away may be merely an adjustment of the breathing to the increased quantity of blood in the lungs caused by rapid muscular activity.

Second wind, when spoken of in connection with study, speaking, and muscular activity involving only a part or few of the muscles, and in slow movements, is a misnomer. There is no such thing. It is warming up. The subject who shows no warming up in the above experiments, gets his "second wind" in running. His physical system probably can exert more power at first than later, but he adjusts his breathing to the demand.

The importance of warming up is greatly underestimated. If one is properly trained so that he can warm up at will he is able to use a tremendous power, even that of the insane or frightened person, and then relax to rest and recruit the exhausted nerve cells.

Every one has great power, both mental and physical, that he has never used. The insane laborer may become a poet or musician of high order. What he can do at all, is in his power any time, if he were so trained as to use it.

Lombroso, in his book "Genius and Insanity," tells of various things which men of genius do to get their inspiration. A great crisis, national or individual, develops, as in the case of the Polish boy, the latent possibilities in one's character and makes the Lincoln or the Grant out of the country lawyer or the tanner. The person who cannot warm up can never meet a great occasion. The person who warms up greatly and yet controls his activity may do anything within human possibilities. Not only the large motor areas of the brain come to aid the part in central use, but the sensory

areas which coincide with the motor in location in the brain are also in active coöperation. Dr. Richard H. Storrs, who was wont to pound his own upholstered pulpit, in a grand burst of eloquence struck the marble desk where he was preaching and broke two bones in his hand, but was unconscious of the fact till the sermon was over.

Napoleon referred to the same thing when he said that the fate of battles was the result of an instant of latent thought. "The decisive moment appeared, the spark burst forth and one was victorious." The deep desire in the child and adolescent and in grown people to such an extent as was shown last year by the thousands who gathered in all cities to get the news from Carson City, may be a natural hunger, perhaps depraved in adults, for such excitement as will push into function new and larger brain areas.

Education should mean a culture of one's potential energies that they may become actual when occasion demands. The great public schools in England have made men as perhaps no other schools have done in the past century. Why? May not one reason be that they have in peculiar ways taught their students to rise to the occasion and have furnished inspiration which has made possible such men as Wellington and Gladstone. The application of this problem may be wide and important if carried into all educational work. In order to do a great work we must have a highly developed brain, which can be thrown into activity quickly and as a unit of power.

EQUATIONS OF MOTION OF A VISCOUS LIQUID.

PEARL EUGENE DOUDNA, B. A., M. A.

Historical Introduction.

Two and a half centuries B. C., Archimedes (287?–212) wrote a work entitled *De Iis Quae Vehuntur In Humido.* He maintained that every particle of a fluid mass, when in equilibrium, is equally pressed in every direction. The laws and properties of liquids were investigated by this ancient mathematician sufficiently to enable him to devise a hydrostatic means of determining the purity of the precious metals. He made a further practical application of the results of his studies in this direction by the invention of the screw engine, or the Archimedean screw.

The Alexandrian School is accredited with the construction of a few hydraulic machines, such as the siphon and the force pump. However, fluid motion was probably first studied by a Roman, Sextus Julius Frontinus, inspector of the public fountains at Rome in the reigns of Nerva and Trajan, about the close of the first century A. D. In his work entitled *De Aquaeductibus Urbis Romae Commentarius*, is found a description of the great aqueducts at Rome. Frontinus constructed five new aqueducts, making in all fourteen. He also describes the methods used in determining the amount of water discharged from ajutages and the methods of distributing the water of the aqueducts and fountains. He observed that the amount of water discharged depends upon the height of the water in the reservoir above the orifice as well as the area of the orifice.

Fifteen centuries later Castelli (1628) advanced the theory that the velocity of discharge is proportional to the height of water in the reservoir above the orifice. Torricelli, a contemporary of Castelli, observing that a small jet of water rushing from an ajutage rises to a height almost equal to

that of the water in the reservoir, concluded that the velocity of discharge is equal to the velocity which the water would acquire by falling freely through a distance the same as the height of the water above the orifice. As a result of this observation he obtained the correct relation between the velocity of discharge and the head of water (1643).

Mariotte (1620?–1684), the author of a posthumous work, entitled *Traité du Mouvement des Eaux et des Autres Fluides* (1686), made extensive use of the theorem of Torricelli. He seems to have been the first to attempt to reconcile theory and experiment by attributing the retardation to friction. The filaments sliding along the surface of the pipe were supposed to be retarded and other filaments having a greater velocity than those near the surface were retarded by rubbing against the slower ones. The retardation was supposed to be proportional to the distance from the axis of the pipe.

Guglielmini, a contemporary of Mariotte, devoted himself to the study of the motion of the water in rivers and canals. He assumed that every particle in a vertical section moves with a velocity equal to the velocity of discharge from an orifice at an equal depth below the surface of the water, and explained the discrepancy between theory and fact as due to transverse currents caused by the irregularities in the bed of the stream. Later, however, when Mariotte showed that the same retardation takes place in a glass tube, where it cannot be explained by cross-currents, Guglielmini accepted the explanation of the French philosopher, but also maintained that viscosity had considerable to do in retarding the motion.

In the latter part of the seventeenth century Varignon (1654–1722) gave to the Académie des Sciences de Paris a very natural and plausible explanation of the relation existing between the velocity of discharge and the head of water. Having remarked that when water flows from a cylindrical vase (vessel) through a small orifice in the bottom the water in the vase moves with a very slow and sensibly uniform motion for all the particles, he concluded that there is no acceleration and that the portion of the fluid escaping each instant receives all its movement from the pressure produced by the

weight of the column of fluid having the area of the orifice as a base. Therefore its weight, which is proportional to the area of the orifice multiplied by the height of the fluid in the vessel, must be proportional to the quantity of motion engendered in the particles which escape through the orifice each instant. Continuing this analysis, he finally arrives at the theorem already established by Torricelli by an entirely different method.

The investigation of the flow of water in rivers attracted considerable attention in Italy, probably on account of the extensive landscape gardening. Besides Guglielmini, who was inspector of rivers in Milanese, Marquis Polini deserves mention in this connection. In 1695 he wrote *De Motu Aquae Mixto*, and in 1718 another work concerning the flow of water through orifices and short tubes.

Newton (1642–1727) investigated the effects of friction and viscosity in diminishing the velocity of running water. In book 2, § 9, of the *Principia*, he offers the following hypothesis: The resistance which arises from the viscosity of a fluid, other things being equal, is proportional to the velocity with which the particles separate from each other. This may be said to be the fundamental principle underlying all that part of hydrodynamics which deals with viscous fluids. The vena contracta as well as the oscillation of waves seems to have been considered first by Newton.

Daniel Bernoulli published his *Hydrodynamica, Sive De Viribus et Motibus Fluidorum Commentarii*, in 1738, in which he bases his theory upon the suppositions that the surface of a fluid contained in a vessel which is being emptied by an orifice remains always horizontal and that the horizontal strata always remain contiguous to each other, and that the particles descend vertically with a velocity inversely proportional to the horizontal section of the reservoir. His principle was not acceptable to his contemporaries, consequently John Bernoulli and Maclaurin each attempted to solve the problem by independent methods but did so without marked success.

Jean-le-Rond D'Alembert (1717–1783), aided by the discoveries of Euler (1707–1783), took the first great step in determining the general equations of motion of a perfect

fluid. "When generalizing James Bernoulli's theory of pendulums he discovered a principle of dynamics so simple and general that it reduced the laws of motion of bodies to that of their equilibrium. He applied this principle to the motion of fluids, and gave a specimen of its application at the end of his *Dynamique* in 1743. It was more fully developed in his *Traité Des Fluides*, which was published in 1744, where he resolves, in the most simple and elegant manner, all problems which relate to the equilibrium and motion of fluids. He makes use of the very same suppositions as Daniel Bernoulli, though his calculus is established in a very different manner. He considers, at every instant, the actual motion of a stratum as composed of a motion which it had in the preceding instant and of a motion which it has lost. The laws of equilibrium between the motions lost furnish him with the equations which represent the motion of the fluid. Although the science of hydrodynamics had then made considerable progress, yet it was chiefly founded on hypothesis. It remained a desideratum to express by equations the motion of a particle of fluid in any assigned direction. These equations were found by D'Alembert from two principles: first, that a rectangular canal, taken in a mass of fluid in equilibrium, is itself in equilibrium; second, and that a portion of fluid, in passing from one place to another, preserves the same volume when the fluid is incompressible, or dilates itself according to a given law when the fluid is elastic. His very ingenious method was published in 1752, in his *Essai Sur la Resistance Des Fluides*. It was brought to perfection in his *Opuscules Mathematiques*, and was adopted by Euler." Philosophers had attempted in vain to determine the laws of fluid motion independent of all hypotheses. However, the method of fluxions proved inadequate to the purpose, and it was only after Euler had contributed to science his calculus of partial differences that the object was reached. D'Alembert first applied the new calculus to the motion of water, and he and Euler both succeeded in obtaining equations of motion for a perfect fluid restricted by no particular hypothesis.

Chevalier Dubnat, a French engineer, published in 1786 a very satisfactory theory of the motion of a fluid, based upon the experiments of himself and others. "He considered that if water were a perfect fluid, and the channels in which it

flows infinitely smooth, its motion would be continually accelerated, like that of bodies descending on inclined planes. But since the motion of rivers is not accelerated, and soon arrives at a state of uniformity, it is evident that the viscosity of the water and the friction of the channel in which it descends must equal the accelerating force. Dubuat, therefore, assumes it as a proposition of fundamental importance, when water flows in any channel or bed, that the accelerating force which obliges it to move is equal to all the resistances which it meets with, whether they arise from the viscosity or from the friction of the bed."

In 1779 Abbé Bossut conducted very extensive investigations to determine the amount of retardation in pipes due to adhesion and friction. He reached the conclusion that with pipes of the same length the discharge is proportional to the diameter, and when the diameters are equal the discharge is inversely proportional to the square root of the length.

The general equations of motion are so complex that little could be done with them. This resulted in the numerous efforts which were made by one class of investigators, such as Dubuat, Bossut and Prony, to develop by experiment simpler formulas for the velocity of running water. However, the more theoretical mathematicians, among whom were Lagrange, Laplace, and Poisson, endeavored to obtain solutions for the equations of D'Alembert, and thus solve the various problems of hydrodynamics. These men, together with other eminent mathematicians of both France and Great Britain, devoted much time to the investigation of waves and tides.

Navier (1785–1836) was probably the first to offer equations of motion for a viscous fluid (1822). He bases his investigation upon the following suppositions: The fluid is composed of ultimate molecules, acting upon each other with forces which, when the fluid is at rest, are functions simply of the distance, but when the molecules recede from or approach each other are modified by this circumstance so that two molecules repel each other less strongly when they are receding and more strongly when they are approaching each other than when they are at rest; and the alteration in the attrac-

tion or repulsion is supposed to be proportional to the velocity with which the molecules, separated by a given distance, are approaching or receding from each other. A second supposition involves the symmetrical arrangement of the particles. Navier dealt only with an incompressible fluid, and arrived at the following equations:

$$X-\frac{1}{\rho}\left(\frac{dp}{dx}-A\left(\frac{du^2}{dx^2}+\frac{du^2}{dy^2}+\frac{du^2}{dz^2}\right)\right)=\frac{\delta u}{\delta t}$$

and the other two can be written by making similar changes in the corresponding equations for a perfect fluid. A is a constant, depending on the nature of the fluid. The other terms are the same as those used in the following pages.

Poisson (1781–1840) derived equations not only for an incompressible fluid, but also for an elastic fluid in which the change in density is small. He treated the subject from the standpoint of an elastic solid, supposing the fluid to be continually beginning to be displaced like an elastic solid, and continually rearranging itself so as to make the pressure equal in all directions, as is the case with a fluid at rest. His equations are written thus:

$$X-\frac{1}{\rho}\left(\frac{dp}{dx}-A\left(\frac{du^2}{dx^2}+\frac{du^2}{dy^2}+\frac{du^2}{dz^2}\right)-B\frac{d}{dx}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right)=\frac{\delta u}{\delta t}$$

with the other two to correspond.

For an incompressible fluid these agree with those obtained by Navier, since in that case the expression for cubical expansion disappears. It is to be noticed that both the foregoing methods involve a consideration of the ultimate molecules of the fluid.

Barré de St. Venant (1797–1886) first succeeded in obtaining the equations independent of any consideration of the ultimate moleeules. He attempted to connect the oblique pressures in different directions about a point with the differential coefficents $\frac{du}{dx}\cdot\frac{du}{dy}$ etc., which express the relative motion of the fluid particles in the immediate neighborhood of the point, by assuming the tangential force on a plane passing through the point to be in the direction of the principal sliding along that plane. He then employs theorems

by Cauchy, connecting the pressures in differet directions in any fluid, to show that the tangential force in any direction along a plane is proportional to the sliding along the same plane resolved in the same direction. He published this theory in 1843.

In 1845 G.G. Stokes derived the same equations as Poisson, but found that $A=3B$. He based his investigation upon three principal hypotheses: "First, that the difference between the pressures on a plane in a given direction passing through a given point P of a fluid in motion and the pressure which would exist in all directions about P if the fluid in its neighborhood were in a state of relative equilibrium depends only on the relative motion of the fluid immediately about P; that the relative motion due to any motion of rotation may be eliminated without affecting the differences of the pressures above mentioned." (Stokes.) He then arrives at the conclusion that the stresses due to viscosity are functions of the rates of strain. After speculating as to the molecular constitution of the fluid, he arrives at the hypothesis that these are linear functions. This is his second hypothesis. For gases he introduces a third: "When a gas is expanding equally in all directions, the stresses P, Q and R are the same as if the fluid were frictionless." * * * * In his report to the British Association in 1846 "On Recent Researches in Hydrodynamics," Stokes claims that the prineipal feature of his investigation consists in eliminating from the relative motion of a fluid about any particular point the relative motion which corresponds to a certain motion of rotation and examining the nature of the relative motion which remains; and that the method employed does not necessarily require the consideration of the ultimate molecules.

The second assumption of Stokes is not altogether satisfactory, since it rests upon the supposition that the velocity is small.

In 1861 O. E. Meyer derived the ordinary equations if $B=O$, which, according to other investigators prior to this time, are true only for an incompressible fluid.

He begins his investigation after some preliminary considerations with the following statement: "The internal friction of a fluid takes place between the different strata of the same fluid, and is proportional to the differential coefficient of the velocity along the normal to the plane separating the strata." But this hypothesis explains only six of his nine initial expressions. For instance, if we take the three initial expressions for the retarding forces parallel to the x axis, viz.,

$$-n\frac{du}{dx}\,dy\,dz,$$

$$-n\frac{du}{dy}\,dz\,dx,$$

$$-n\frac{du}{dz}\,dx\,dy,$$

the second and third readily follow from the hypothesis, but the first has no definite meaning, unless we extend his hypothesis, as W. M. Hicks has interpreted it in the British Association Report for 1881, so that it reads: "The friction on a small plane in a given direction in the plane is proportional to the rate of variation perpendicular to the plane of the component of the velocity in the given direction, whilst there is a normal part proportional to the rate of variation perpendicular to the plane of the component perpendicular to this plane."

The subject has been investigated by Stefan (1862), and later by Maxwell, Levy, Klutz, and Butcher, without adding materially to what had been done already.

However, the work of Maxwell resulted in the determination of the constant called the coefficient of viscosity. Following Maxwell, numerous experiments have been made for the purpose of determining the value of the coefficient of viscosity. Helmholtz, Piotrowski, Maxwell, Meyer, and Poissenille may be named as having made the most elaborate series of experiments.

Lamb, in his *Treatise on Fluids* (1879), derives the equations of motion for a viscous fluid by a method based on those of St. Venant and Stokes. Basset, in 1888, published

A Treatise on Hydrodynamics, in which he uses Stokes' method. Both authors accept the second hypothesis of Stokes, so that we have no assurance that their equations hold, except in the case of slow motion. Both fully realize this, and restricted the application of their results accordingly. Of all methods so far, that of Stokes has received the most general approval.

In the following paper we have derived the equations for the motion of a perfect fluid in order that we might extend the same methods as far as possible in obtaining the equations for a viscous liquid (the discussion is limited to an incompressible fluid, or liquid). The second section is devoted to the consideration of a viscous liquid. The discussion is based upon the definition of the coefficient of viscosity as agreed upon by experimental physicists. Then by comparing the motion of a viscous liquid with that of a perfect liquid we are enabled to give a definite meaning to all nine of the initial expressions obtained by Meyer. To obtain the equations referred to cylindrical and spherical coördinates we have not used the method of transformation, but have derived them by analysis similar to that used in determining the equations when referred to rectangular coördinates. By this method a definite meaning attaches to each term in the resulting equations. This part of the subject is given in Sections 3 and 4, and will appear in a later issue of the *Colorado College Studies*. The essential features of Sections 1 and 2 were completed before April 1, 1895; of Sections 3 and 4 before June 1, 1896. The notes at the end of Section 4 were added during the spring of 1897.

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DERIVATION OF EQUATIONS.

SECTION I.—*Equations of motion of a perfect fluid.*

D'Alembert's Principle: Since D'Alembert's Principle is used so frequently in what follows, it may be well to say by way of explanation of this principle that its use means the equating of

Impressed Force—Pressure—Expressed Force

to the forces lost through friction.

An Ideal Fluid.—With this as with other problems, it is easier to consider first the ideal condition. An ideal fluid is one in which there is no friction; the particles move by one another without causing any mutual retardation; hence without any loss of energy.

The Derivation of the Equations of Motion of a Perfect Fluid.—To derive the equations of motion of a perfect fluid is a comparatively easy matter.

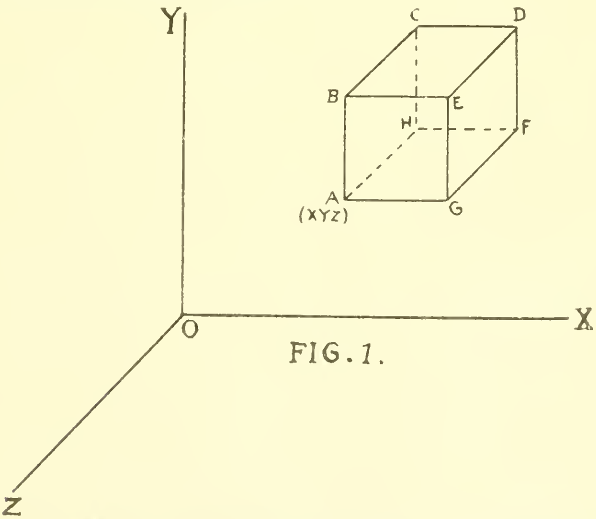

FIG. 1.

Let $A\ D$ (Fig 1) represent an element of such fluid; x, y, z be the coördinates of the point A; $A\ G$, $A\ B$, and $A\ H$ equal to dx, dy, and dz respectively; X, Y, and Z the impressed forces in directions x, y, and z; and u, v, and w the velocities

in the same directions; and finally let p represent the pressure at the point (x, y, z) and m the mass of the element.

From the conditions of the problem

$$u=f\ (x, y, z, t)$$

and it follows immediately that

$$\frac{\delta u}{\delta t}=\frac{du}{dt}+u\frac{du}{dx}+v\frac{du}{dy}+w\frac{du}{dz}=f_x.$$

This last is an expression for the effective or expressed force in the direction of the x axis. In a similar notation $\frac{\delta v}{\delta t}$, or f_y and $\frac{\delta w}{\delta t}$, or f_z represent the effective forces in the direction of the y and z axes respectively.

The total impressed forces are mX, mY, and mZ; the total effective forces mf_x, mf_y, mf_z; the pressures on the faces AC. AE, and AF are $p\ dy\ dz$, $p\ dy\ dx$, and $p\ dx\ dz$ respectively; on the faces opposite these the pressures are $p\ dy\ dz+\frac{dp}{dx}\ dx\ dy\ dz$, $p\ dy\ dx+\frac{dp}{dz}\ dz\ dy\ dx$, and $p\ dx\ dz+\frac{dp}{dy}\ dy\ dx\ dz$.

The excesses of pressure on the opposite faces are:

$$\frac{dp}{dx}\ dx\ dy\ dz,\ \frac{dp}{dz}\ dx\ dy\ dz,\ \text{and}\ \frac{dp}{dy}\ dx\ dy\ dz.$$

Since the hypothesis excludes the possibility of any loss through friction, we have now considered all the forces involved. Equating separately the forces acting in the direction of each axis by D'Alembert's principle, we obtain the following equations:

$$mX-mf_x-\frac{dp}{dx}\ dx\ dy\ dz=0$$

$$mY-mf_y-\frac{dp}{dy}\ dx\ dy\ dz=0$$

$$mZ-mf_z-\frac{dp}{dz}\ dx\ dy\ dz=0$$

Substituting for m its value $\rho\ dx\ dy\ dz$ in which ρ represents the density of the fluid, and then dividing by $dx\ dy\ dz$, the resulting equations are:

$$X\rho - f_x\rho - \frac{dp}{dx} = 0$$

$$Y\rho - f_y\rho - \frac{dp}{dy} = 0$$

$$Z\rho - f_z\rho - \frac{dp}{dz} = 0,$$

or as usually written:

$$X - \frac{dp}{\rho dx} = \frac{\delta u}{\delta t}$$

$$Y - \frac{dp}{\rho dy} = \frac{\delta v}{\delta t}$$

$$Z - \frac{dp}{\rho dz} = \frac{\delta w}{\delta t}$$

So far we have obtained three relations between the five quantities u, v, w, p, and ρ. When these equations are used in application to an incompressible fluid, ρ is constant; but when gases are under consideration, a fourth equation is supplied by Boyle's Law,

$$p = k\rho.$$

Continuity.—The condition of continuity furnishes a fifth equation. If the fluid be continuous, then the increase within a given space during any given time must equal the excess of inflow over outflow across the boundaries in the given time.

The mass of fluid contained by the element at any moment, is

$$\rho\ dx\ dy\ dz$$

and its increase in the time dt is

$$\frac{d\rho}{dt}\ dt\ dx\ dy\ dz.$$

The flow over the surface $dy\ dz$ in time dt is

$$u\ \rho\ dt\ dy\ dz$$

and the flow over the opposite face is

$$u\ \rho\ dt\ dy\ dz + \frac{d\,(u\rho)}{dx}\ dt\ dy\ dz\ dx,$$

giving an increase in the element of

$$\frac{d\,(u\rho)}{dx}\ dt\ dx\ dy\ dz.$$

Similarily the increments in the directions of the y and z axes are found to be

$$\frac{d\,(v\rho)}{dy}\,dt\,dx\,dy\,dz$$

and

$$\frac{d\,(w\rho)}{dz}\,dt\,dx\,dy\,dz.$$

Equating the whole increment to the partial increments and dividing by $dt\,dx\,dy\,dz$, we obtain

$$\frac{d\rho}{dt}=\frac{d\,(u\rho)}{dx}+\frac{d\,(v\rho)}{dy}+\frac{d\,(w\rho)}{dz}.$$

But for an incompressible fluid, ρ being constant, this becomes

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0.$$

SECTION II.—*The Equations of Motion of a Viscous Fluid.*

Having determined the equations of motion of a perfect fluid when referred to rectangular coördinates, we will now consider a viscous fluid. This consideration will combine those forces already dealt with in the case of a perfect fluid and the forces lost through internal friction.

The problem is to determine the relations existing between these lost forces and the velocities.

Coefficient of Viscosity.—It is necessary at this point to define a quantity called the coefficient of viscosity.

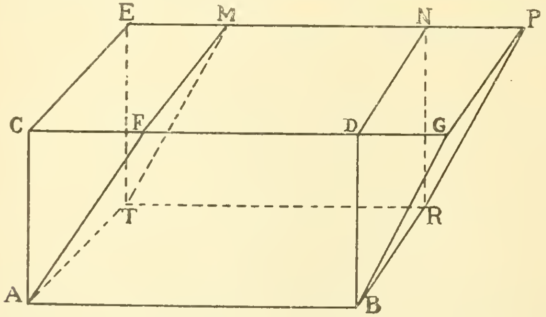

FIG. 4.

If $A\,E\,D\,R$ represents the initial position of an element

subjected to a shearing force exerted parallel to the planes of the strata, *e. g.* plane $A\,T\,R\,B$, such that the element takes the form $A\,M\,G\,R$, the coefficient of viscosity is defined as the ratio of the shearing stress to shear per unit of time. Putting this definition in the form of an equation, we have

$$\frac{\textit{shearing stress}}{\textit{shear per unit of time}}=\mu$$

in which μ is the coefficient of viscosity; or shearing stress $=\mu$ x shear per unit of time. Let u be the velocity in the direction of $A\,B$; and the point A be located by x, y, z; the sides of the element be dx, dy, dz. Then

$$u=f\,(x,\ y,\ z)$$

$$\frac{du}{dy}=f^1\,(x,\ y,\ z)=\text{rate of normal displacement}$$

along y. This force acts upon the surface of the element $dx\ dz$. Hence the shearing force on this surface is

$$F=\mu\frac{du}{dy}dx\ dz.$$

The force on the opposite face is

$$F^1=\mu\frac{du}{dy}\,dx\ dz+\mu\frac{d^2u}{dy^2}\,dx\ dz\ dy.$$

The difference of the forces on the opposite faces of the element considered is the retarding force due to the viscosity of the fluid.

Derivation of the Equations.—Sufficient has now been said to proceed to the derivation of the equations. We shall take up this problem in much the same way that we did the problem of a perfect fluid. But this new element of viscosity must be taken into account. It is evident that our equations, when derived, must be the same as those already obtained for a perfect fluid with terms attached to account for the retardation due to viscosity. For if in our new equations μ be equated to zero the equations of a perfect fluid must remain.

Forces to be Equated.—The forces to be equated are

Impressed force,
Pressure,
Effective force, and
Retarding force, or force lost,

which give, when equated,

impressed force—pressure—effective force=
—retarding force,

or, in the more usual form,

impressed force—pressure+retarding force=
effective force.

It is to be noticed that the sense in which the retarding force is taken is opposite that in which the impressed force is taken.

Rectangular Co-ordinates.—Take an element PB, the co-

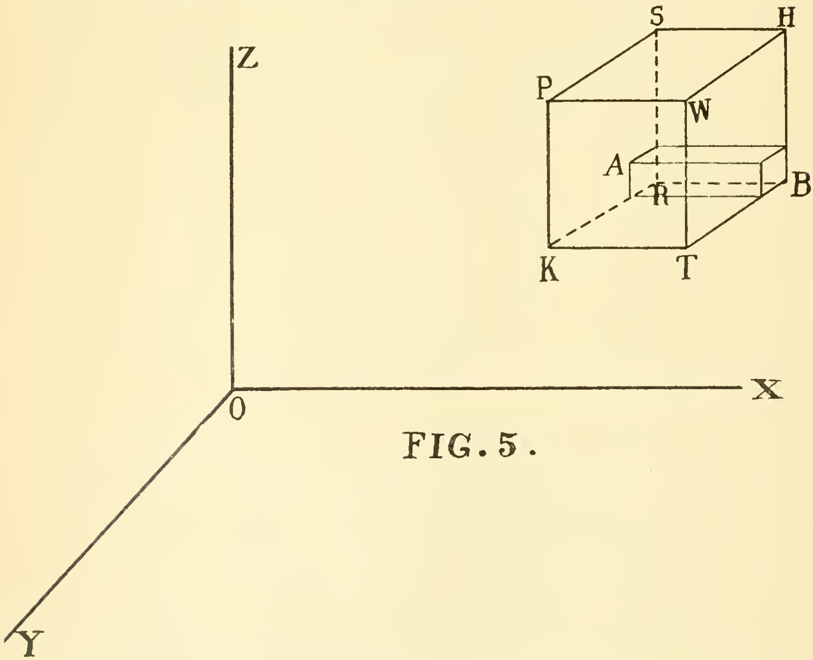

FIG. 5.

ördinates of the point R being x, y, z and the dimensions of the element being dx, dy, dz.

Suppose the velocity uniform in the direction of x. Then any parallelopiped, such as AB, would move as a solid; but the element is composed of a great many such parallelopipeds. If the velocity in the direction of x varies as we pass from R to K, the layers in slipping by each other will produce mutual retardation. The rate of change in u along the normal to its direction is

$$\frac{du}{dy}$$

From the definition of viscosity it follows that the shearing force upon the face SB is

$$\mu\frac{du}{dy}dx\,dz,$$

and that upon the opposite face is

$$\mu\frac{du}{dy}dx\,dz+\mu\frac{d^2u}{dy^2}dx\,dz\,dy,$$

indicating a loss of force within the element equal to

$$\mu\frac{d^2u}{dy^2}dx\,dy\,dz.$$

In a similar manner we find the retardation upon the face $dy\,dx$ to be

$$\mu\frac{du}{dz}dx\,dy,$$

and on the opposite face to be

$$\mu\frac{du}{dz}dx\,dy+\mu\frac{d^2u}{dz^2}dx\,dy\,dz,$$

showing a loss within the element of

$$\mu\frac{d^2u}{dz^2}dx\,dy\,dz.$$

All of the loss within the element is now accounted for, provided the velocity in the direction of x is uniform. It remains to examine the effect of an accelerating force along the axis x. To simplify the problem, suppose the motion two

dimensional. In this case it is necessary to consider only a section, such as AC (Fig. 6).

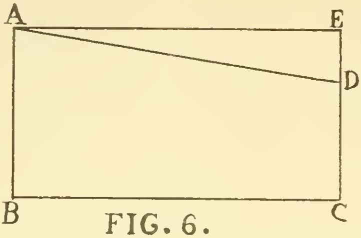

FIG. 6.

Suppose under certain conditions a certain motion exists at B. If the fluid is perfect a certain amount will flow into the parallelogram AC across the line AE, due to the acceleration of the velocity u. ED represents the whole increase of inflow at E over that at A.

$$ED = AB - DC.$$

But $\dfrac{AB}{DC} = \dfrac{u^1}{u}$, u being the velocity across AB, and u^1 that across DC.

By composition $$\frac{AB-DC}{AB} = \frac{u^1-u}{u^1}$$

which represents the distance of inflow at E when a unit length of AB is considered. The time of inflow is dt, hence

$$\frac{AB-DC}{AB\,dt} = \frac{u^1-u}{u^1 dt}$$

gives the rate of displacement in the direction of x.

$$\frac{1}{dt}\left(\frac{u^1-u}{u^1}\right) = \frac{1}{dt}\left(1-\frac{u}{u^1}\right) = \frac{1}{dt}\left(1-\frac{u}{u+du}\right)$$

$$= \frac{1}{dt}\left[1-\left(1-\frac{du}{u}+\frac{du^2}{u^2}-\frac{du^3}{u^3}+\frac{du^4}{u^4}-\text{etc.}\right)\right]$$

$$= \frac{1}{dt}\left(\frac{du}{u}-\frac{du^2}{u^2}+\frac{du^3}{u^3}-\frac{du^4}{u^4}+\text{etc}\right)$$

Since this series converges very rapidly, unless the in-

crease in motion is very great all of the terms except the first may be dropped. This gives

$$\frac{1}{dt}\left(\frac{du}{u}\right)=\frac{du}{dx}.$$

The retarding force on the face $dy\,dz$ is

$$\mu\frac{du}{dx}dy\,dz,$$

and on the opposite face

$$\mu\frac{du}{dx}dy\,dz+\mu\frac{d^2u}{dx^2}dy\,dz\,dx,$$

showing a loss for the element of

$$\mu\frac{d^2u}{dx^2}dy\,dz\,dx$$

when the fluid is viscous.

The retardation in the direction of the x axis must equal the retardation of inflow, for otherwise there would be a discontinuity.

It is necessary to extend this principle to the case where the inflow is across two faces instead of one.

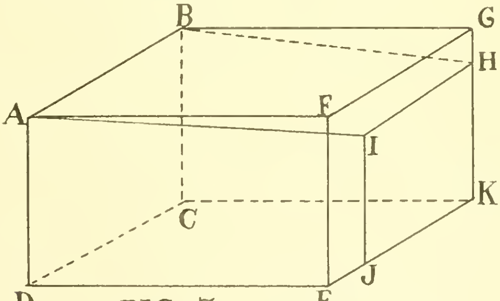

FIG. 7.

$AB\,.\,AD-HI\,.\,IJ=$total areal increase of inflow.

$$\frac{AB\,.\,AD}{HI\,.\,IJ}=\frac{u^1}{u}$$

$$\frac{AB\,.\,AD-HI\,.\,IJ}{AB\,.\,AD\,dt}=\frac{u^1-u}{u^1dt}=\text{rate of inflow.}$$

But $$\frac{u^1-u}{dt\,u^1}=\frac{1}{dt}\left(1-\frac{u}{u+du}\right).$$

Expanding this and neglecting, as before, all terms except the first, we get $\frac{du}{dx}$ for the rate of inflow.

This shows that the lost forces obtained for two dimensional flow hold in the case where the inflow is over two faces. This could have been concluded without the above proof, for the amount flowing in is fixed, and it is immaterial whether the flow is all in the direction of z or partly in the direction of z and partly in the direction of y.

The forces in the direction of the y and z axes can be obtained in the same way.

It is to be noticed that a force element of the second order is neglected if, for example, there is a normal increase in the velocity u, for in that case the inflow will be accelerated.

Equating the forces found and writing the remaining two symmetrical equations, we have

$$X - \frac{dp}{\rho dx} = \frac{\delta u}{\delta t} - \frac{\mu}{\rho}\left(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right)$$

$$Y - \frac{dp}{\rho dy} = \frac{\delta v}{\delta t} - \frac{\mu}{\rho}\left(\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2}\right)$$

$$Z - \frac{dp}{\rho dz} = \frac{\delta w}{\delta t} - \frac{\mu}{\rho}\left(\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + \frac{d^2w}{dz^2}\right)$$

(To be Continued in Volume VIII.)

EQUATIONS OF MOTION OF A PERFECT LIQUID AND A VISCOUS LIQUID, WHEN REFERRED TO CYLINDRICAL AND POLAR CO-ORDINATES.

BY P. E. DOUDNA,* A. M.

(Continued from Vol. VII, p. 18.)

EQUATIONS OF A PERFECT LIQUID.

Cylindrical Co-ordinates: The equations of motion of a perfect fluid when referred to cylindrical coördinates may be derived from those already obtained for rectangular coordinates by transformation. The following, however, is a more satisfactory method in so much as it gives a definite interpretation to each term appearing in the resulting equations. Moreover, the method to be applied in the derivation of the equations when referred to cylindrical coördinates and polar coördinates forms the basis of several important principles involved in the derivation of the equations of motion of a viscous fluid referred to the same systems of coördinates.

The velocities u, v, w, and the impressed forces X, Y, Z have the directions of r, θ, and z, respectively; v is angular velocity.

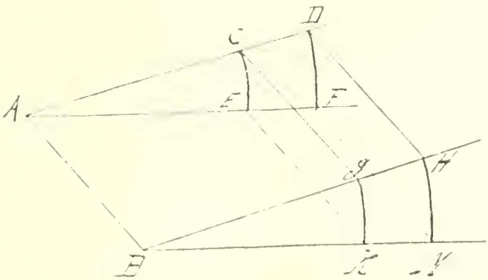

Fig 8

$X\rho\, r\, d\theta\, dz\, dr$ = impressed force in the direction of r.

* Died at Colorado Springs, January 6, 1900.

$\frac{\partial u}{\partial t}\rho r\, d\theta\, dz\, dr$=effective force in same direction due to an acceleration in the velocity u. The velocity v through centrifugal force contributes to the effective force in this direction

$$-v^2\rho\, r^2 d\theta\, dz\, dr.^*$$

The velocity w can have no influence upon the forces in the direction of r.

$$-\frac{dp}{dr}\, rd\theta\, dz\, dr$$

is the difference in pressure on the two faces CK and DM. Equating by D'Alembert's Principle, we have

$$X\rho\, rd\theta\, dz\, dr-\frac{dp}{dr}rd\theta\, dz\, dr-\frac{\partial u}{\partial t}\rho\, rd\theta\, dz\, dr+v^2\rho\, r^2 d\theta\, dz\, dr=0.$$

Dividing by $\rho\, rd\theta\, dz\, dr$ and transposing, this equation becomes

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-v^2 r.$$

Similarly for the forces acting in the direction of θ

$$Y\rho\, rd\theta\, dz\, dr=\text{impressed force,}$$

$$r\frac{\partial v}{\partial t}\rho\, rd\theta\, dz\, dr=\text{effective force}$$

contributed by acceleration in v.

$\frac{dp}{rd\theta}rd\theta\, dz\, dr$=difference in the pressures on the opposite faces. But there is still another resulting force acting in this direction due to the fact that while a particle

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* If there were no motion in the direction of r, a force directed toward the center would be necessary to maintain the configuration of the element. This force would be

$$v^2\rho\, r^2 d\theta\, dz\, dr-\frac{dp}{dr}\, rd\theta\, dz\, dr.$$

Since motion along r is in a straight line, it follows from Newton's second law that, if the velocity in that direction is uniform, the above relation still holds.

If the motion in this direction is accelerated,

$$\frac{\partial u}{\partial t}\rho\, rd\theta\, dz\, dr=X\rho\, rd\theta\, dz\, dr+v^2\rho\, r^2 d\theta\, dz\, dr-\frac{dp}{dr}\, rd\theta\, dz\, dr,$$

$$\text{or }\frac{\partial u}{\partial t}=X+v^2 r-\frac{dp}{rdr}.$$

is being carried in the direction θ it is also being carried in the direction of r by the velocity u; consequently, at the end of the time dt it will have fallen behind by an amount equal to

$$(r+dr)\,d\theta - rd\theta \text{ or } dr\,d\theta.$$

From physics, $s=\frac{1}{2}ft^2$

$$f=\frac{2s}{t^2},$$

where f=acceleration, s the space described, and t the time. Substituting,

$$f=2\frac{dr\,d\theta}{dt^2}=2\,uv\,*$$

which is the force due to this fall, multiplying by $\rho r d\theta\,dr\,dz$ we get for the total of this force

$$2\,uv\,\rho r d\theta\,dr\,dz.$$

The velocity along z as before contributes no component to the forces in this direction. Equating the forces found and simplifying, we get

$$Y-\frac{dp}{\rho r d\theta}=r\frac{\delta v}{\delta t}+2\,uv.$$

Neither u nor v can contribute a component to the forces acting in the direction of the z axis, therefore the equation of these forces will be identical with the corresponding equation for rectangular coördinates.

* If from an elevation on a uniformly rotating body a particle falls towards the axis of rotation it constantly gains angular velocity without the application of any force in the direction of θ. This is true of falling bodies on the surface of the earth. A rotating body, if contracting, would increase its angular velocity. To maintain the motion uniform under such conditions, a force must act in the direction opposite to the direction of motion. This force is

$$2\,uv\rho\,rd\theta\,dz\,dr-\frac{dp}{rd\theta}\,rd\theta\,dr\,dz.$$

If such a force were acting the configuration of the element would be maintained. If the motion were from instead of toward the center an opposite force would be necessary to keep the velocity uniform or to maintain the configuration of the element.

Collecting the results just obtained, we have

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-v^2r,$$

$$Y-\frac{dp}{\rho r d\theta}=r\frac{\partial v}{\partial t}+2\ ur.$$

$$Z-\frac{dp}{\rho dz}=\frac{\partial w}{\partial t}.$$

The equation of continuity is

$$\frac{d(ur)}{dr}+r\frac{dv}{d\theta}+\frac{rdw}{dz}=0.$$

It is unnecessary to derive this last equation, since the method by which it is obtained is identical with that used to obtain the similar equation referred to rectangular co-ordinates.

Spherical Co-ordinates.—Angle WOF (in Fig. 9)$=\theta$ and

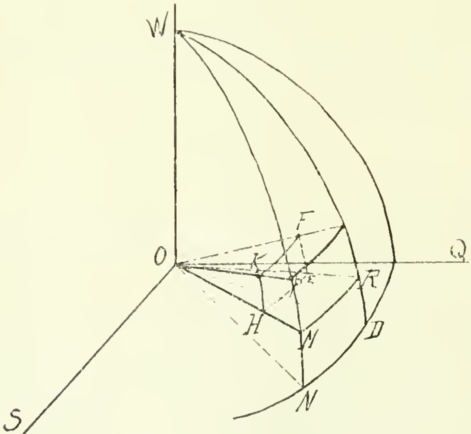


Fig 9.

$QOD=\varphi$. Let u, v and w represent the velocities in the directions r, θ and φ respectively, v and w being angular; X, Y and Z are the symbols for the corresponding impressed forces. The mass of the element FM is

$$\rho\ dr\ rd\theta\ r\sin\theta\, d\varphi=m.$$

Then Xm= impressed force in the direction of r; $\frac{\delta u}{\delta t}m$ = effective force due to the acceleration in u; the centrifugal force $-rv^2m$ (see note, p. 2) results from the velocity v; the centrifugal force contributed by w is $-r\sin\theta\, w^2m$ (see note, p. 2) in the direction of OD, and this gives the component $-r\sin^2\theta\, w^2m$ along the radius vector r; $\frac{dp}{dr}dr\ rd\theta\ r\sin\theta\, d\varphi$= the difference in pressure on the two opposite faces.

Equating and transposing as before

$$X-\frac{dp}{\rho dr}=\frac{\delta u}{\delta t}-rv^2-r\sin^2\theta\, w^2.$$

With reference to the direction of θ, we have

Ym=impressed force,

$r\frac{\delta v}{\delta t}m$=effective force due to the acceleration in angular velocity v.

$\rho\frac{dp}{rd\theta}m$=difference in pressure on the opposite faces.

Since as a particle moves from E to F it is carried by u the distance ER, it falls behind by an amount equal to $dr\ d\theta$, which, when expressed in terms of force, is equivalent to

$2\ uv$ (see note, p. 3).

Multiplying by mass,

$2\ uv\ m$

represents the component of effective force contributed by the combined action of u and v; w cannot exert an effective force such as this, since the particle always moves in the same parallel, but it does exert a centrifugal force in the direction of θ, or rather there is an effective component of the centrifugal force acting in the direction of θ due to w. We already know (see above) that the centrifugal force due to w along OD is

$-r\sin\theta\, w^2$.

Since the tangent to EF at E makes an angle θ with OD, the component of

$-r\sin\theta\, w^2$

along the tangent is

$$-r \sin \theta \cos \theta \, w^2.$$

Multiplying by mass, we have

$$-r \sin \theta \cos \theta \, w^2 m.$$

Equating and simplifying, the resulting equation is

$$Y - \frac{dp}{\rho r d\theta} = r\frac{\partial r}{\partial t} + 2\, uv - r \sin \theta \cos \theta \, w^2.$$

The forces along the φ co-ordinate are

Zm=impressed force,

$\sin \theta \, r \frac{\partial w}{\partial t} m$=effective force due to the accelera-

tion of angular velocity w,

$\frac{dp}{\rho r \sin \theta d\varphi} m$=difference in pressure on the op-

posite faces.

Since by the combined action of u and w a particle falls behind

$$dr \sin \theta d\varphi,$$

which, converted into terms of force, gives

$$2 \sin \theta \, wu \text{ (see note, p. 3)},$$

we obtain a total force contributed by the combined action of these two velocities equal to

$$2 \sin \theta \, wu \, m;$$

v also carries the particle through the distance EF, and since the line FK is shorter than EH there will be a resultant effective force due to the combined action of v and w.

$$EH = r \sin \theta \, d\varphi,$$

$$FK = r \sin (\theta - d\theta) \, d\varphi,$$

$$= r d\varphi \, (\sin \theta \cos d\theta - \cos \theta \sin d\theta).$$

Substituting, for $\cos d\theta$, 1 and, for $\sin d\theta$, $d\theta$

$$FK = r d\varphi \sin \theta - r d\varphi \cos \theta \, d\theta,$$

$EH - FK = r d\varphi \cos \theta \, d\theta$, or the distance through which the particle may be said to fall under the combined action of u and v. This is equivalent to a force

$$2\, r \cos \theta \, wv,$$

or for the mass of the element,

$$2\, r \cos \theta \, wv \, m.$$

By equating and simplyfying, we obtain the third equation,

$$Z-\frac{dp}{\rho r\sin\theta d\varphi}=r\sin\theta\frac{\delta w}{\delta t}+2\sin\theta\ uw+2\ r\cos\theta\ vw.$$

Equation of Continuity.—The increase in the element for time dt is

$$\frac{d\rho}{dt}r^2\sin\theta\ d\varphi\ dr\ d\theta;$$

the difference between inflow and outflow, in the direction of r, is

$$\frac{d\,(ur^2\rho)}{dr}\sin\theta\ d\varphi\ dr\ d\theta;$$

in the direction of θ,

$$\frac{d\,(rv\rho\sin\theta)}{rd\theta}r^2 d\varphi\ d\theta\ dr;$$

in the direction φ,

$$\frac{d\,(r\sin\theta\, w\rho)}{r\sin\theta d\varphi}r^2\sin\theta\ d\varphi\ d\theta\ dr.$$

Equating and dividing by $r^2 d\varphi\ d\theta\ dr$,

$$\frac{d\rho}{dt}\sin\theta=\frac{d\,(ur^2\rho)}{r^2dr}\sin\theta+\frac{d\,(rv\rho\sin\theta)}{rd\theta}+\frac{d\,(r\sin\theta\, w\rho)}{r\sin\theta\, d\varphi}\sin\theta;$$

or

$$\frac{d\rho}{dt}\sin\theta=\frac{d\,(ur^2\rho)}{r^2dr}\sin\theta+\frac{d\,(v\rho\sin\theta)}{d\theta}+\frac{d\,(w\rho)}{d\varphi}\sin\theta;$$

or if ρ is constant,

$$\frac{d\,(ur^2)}{r^2dr}\sin\theta+\frac{d\,(v\sin\theta)}{d\theta}+\frac{dw}{d\varphi}\sin\theta=0.$$

Collecting the equations obtained, we have

$$X-\frac{dp}{\rho dr}=\frac{\delta u}{\delta t}-rv^2-r\sin^2\theta\ w^2,$$

$$Y-\frac{dp}{\rho r\ d\theta}=r\frac{\delta v}{\delta t}+2\ uv-r\sin\theta\cos\theta\ w^2.$$

$$Z-\frac{dp}{\rho r\sin\theta d\varphi}=r\sin\theta\frac{\delta w}{\delta t}+2\sin\theta\ uw+2\ r\cos\theta\ vw.$$

$$\frac{d\,(ur^2)}{r^2dr}\sin\theta+\frac{d\,(v\sin\theta)}{d\theta}+\frac{dw}{d\varphi}\sin\theta=0.$$

EQUATIONS OF A VISCOUS LIQUID.

Cylindrical Co-ordinates.—In treating the equations for cylindrical and spherical co-ordinates, it is found that the total

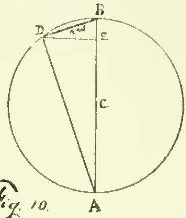


Fig. 10.

rate of displacement, as, for example, on the face CK (Fig. 8) along the normal, is not expressed by $\frac{du}{dr}$. for if the velocity u is constant there must be an inflow equivalent to $\frac{u}{r}$. Consequently, in the following treatment account is taken of this, and then the element considered as uniform in size, *i. e.*, $rd\theta\ dz$ is a constant so long as we are considering the acceleration in u along r. We may therefore consider the total rate of displacement in the direction of r at surface r$d\theta\ dz$ as

$$\frac{du}{dr}+\frac{u}{r}.$$

It has been shown in the derivation of the centrifugal force that the distance $BE=\frac{r^2 r^2}{2r}$, or a velocity $\frac{r^2 v^2}{r}$. (See Fig. 10.) If rv is a variable the rate of change on surface $dr\ dz=zv$. Therefore the total rate of displacement in direction of θ at surface $dr\ dz$ is

$$\frac{du}{rd\theta}-2v.$$

The rate of displacement along z at surface $rd\theta\ dr$ is

$$\frac{du}{dz}.$$

For the direction of θ.

$r\frac{dr}{dr}$ is the linear displacement at surface $rd\theta\ dr$.

But if r is constant, the rate of linear displacement along r at this surface is

$$2r$$

The total retardation at surface $rd\theta\ dz$ is

$$\mu\left(r\frac{dv}{dr}+2r\right)rd\theta\ dz.$$

We have seen before that in order that the form of the element may be maintained there must be a rate of displacement at $rd\theta\ dz$ of

$$\frac{u}{r}$$

This gives a rate of inflow at $dr\ dz$ of

$$\frac{2u}{r},$$

which, added to the rate of inflow due to change of velocity of rr along $rd\theta$, gives a total rate of

$$\frac{dr}{d\theta}+\frac{2u}{r}.$$

In the direction of z the rate is

$$\frac{dr}{dz}$$

The rates for the direction of z are the same as in rectangular co-ordinates.

Collecting these results, we have the following rates of displacement:

$$a.\ \begin{cases}(1)\ \dfrac{du}{dr}+\dfrac{u}{r} & \text{normal to } rd\theta\ dz,\\[2ex] (2)\ \dfrac{du}{rd\theta}-2r & \text{normal to } dr\ dz.\\[2ex] (3)\ \dfrac{du}{dz} & \text{normal to } rd\theta\ dr.\end{cases}$$

$$b.\ \begin{cases}(1)\ r\dfrac{dr}{dr}+2r & \text{normal to } rd\theta\ dz,\\[2ex] (2)\ \dfrac{dr}{d\theta}+\dfrac{2u}{r} & \text{normal to } dr\ dz,\\[2ex] (3)\ r\dfrac{dr}{dz} & \text{normal to } rd\theta\ dr.\end{cases}$$

$$c.\begin{cases}(1)\ \dfrac{dw}{dr}\text{ normal to } rd\theta\, dz,\\[2ex] (2)\ \dfrac{dw}{rd\theta}\text{ normal to } dr\, dz,\\[2ex] (3)\ \dfrac{dw}{dz}\text{ normal to } rd\theta\, dr.\end{cases}$$

All of the above surfaces, except c (1), are to be considered constant for the rates have been reduced to those of a uniform element. In the case of c (1), if w is constant there is no loss within the element. but if w varies along r the total loss is increased by the increasing area of the opposite face. Finding the difference in displacement of opposite sides and multiplying by μ we get,

$$a.\begin{cases}(1)\ \mu\left(\dfrac{d^2u}{dr^2}+\dfrac{du}{rdr}-\dfrac{u}{r^2}\right)rd\theta\, dr\, dz,\\[2ex] (2)\ \mu\left(\dfrac{d^2u}{r^2d\theta^2}-\dfrac{2\,dr}{rd\theta}\right)rd\theta\, dr\, dz,\\[2ex] (3)\ \mu\left(\dfrac{d^2u}{dz^2}\right)rd\theta\, dr\, dz.\end{cases}$$

$$b.\begin{cases}(1)\ \mu\left(r\dfrac{d^2r}{dr^2}+\dfrac{3\,dr}{dr}\right)rd\theta\, dr\, dz,\\[2ex] (2)\ \mu\left(\dfrac{d^2r}{rd\theta^2}+\dfrac{2\,dr}{r^2d\theta}\right)rd\theta\, dr\, dz,\\[2ex] (3)\ \mu\left(r\dfrac{d^2r}{dz^2}\right)rd\theta\, dr\, dz.\end{cases}$$

$$c.\begin{cases}(1)\ \mu\left(\dfrac{d^2w}{dr^2}+\dfrac{dw}{rdr}\right)rd\theta\, dr\, dz,\\[2ex] (2)\ \mu\left(\dfrac{d^2w}{r^2d\theta^2}\right)rd\theta\, dr\, dz,\\[2ex] (3)\ \mu\left(\dfrac{d^2w}{dz^2}\right)rd\theta\, dr\, dz.\end{cases}$$

If these expressions are subtracted from the effective forces in the corresponding equations for a perfect liquid the resulting equations are

$$X-\frac{dp}{\rho dr}=\frac{\delta u}{\delta t}-rv^2-\frac{\mu}{\rho}\left(\frac{d^2u}{dr^2}+\frac{du}{rdr}-\frac{u}{r^2}+\frac{d^2u}{r^2d\theta^2}-\frac{2\,dv}{rd\theta}+\frac{d^2u}{dz^2}\right),$$

$$Y-\frac{dp}{\rho rd\theta}=r\frac{\delta v}{\delta t}+2\,uv-\frac{\mu}{\rho}\left(r\frac{d^2v}{dr^2}+\frac{3\,dv}{dr}+\frac{d^2v}{rd\theta^2}+\frac{2\,du}{r^2d\theta}+r\frac{d^2v}{dz^2}\right),$$

$$Z-\frac{dp}{\rho dz}=\frac{\delta w}{\delta t}-\frac{\mu}{\rho}\left(\frac{d^2w}{dr^2}+\frac{dw}{rdr}+\frac{d^2w}{r^2d\theta^2}+\frac{d^2w}{dz^2}\right).$$

The cause that contributes $-rv^2$ also contributes the rate of displacement $-2v$ normal to $r\sin\theta\, d\varphi\, dr$ if rv is a variable. Similarly there are rates, $-2\sin\theta w$ normal to $rd\theta\, dr$ and $-2\cos\theta w$ normal to $rd\theta\, dr$ corresponding to $-r\sin^2\theta w$ and $-r\sin\theta\cos\theta w^2$ respectively. The first forms a part of the rate of displacement of u along $r\sin\theta d\varphi$ and the second a part of the displacement of v in the same direction. In taking account of the acceleration of u along r and of v along θ, it must be remembered that if everything is constant there is an inflow equivalent to a rate of displacement of $\frac{2v}{r}$ at surface $r^2\sin\theta d\varphi\, d\theta$ in the first case and of $\cot\theta\, v$ at surface $r\sin\theta d\varphi\, dr$ in the second case. To the rate of change in v along r and of v along θ, $2v$ and $\frac{2u}{r}$ respectively must be added as in cylindrical co-ordinates. If w is constant along r, θ and φ, as in the foregoing cases, the rates of displacement at the initial surfaces are $2\sin\theta w$, $2\cos\theta w$, and $\frac{2u}{r}+2\cot\theta\, v$ respectively.

Grouping the rates of displacement, we have

(1) $\frac{du}{dr}+\frac{2u}{r}$ normal to $r^2\sin\theta d\varphi\, d\theta$,

(2) $\frac{du}{rd\theta}-2v$ normal to $r\sin\theta d\varphi\, d\theta$,

(3) $\frac{du}{r\sin\theta d\varphi}-2\sin\theta w$ normal to $rd\theta\, dr$,

(4) $r\frac{dv}{dr}+2v$ normal to $r^2\sin\theta d\varphi\, d\theta$,

(5) $\frac{dv}{d\theta}+\frac{2u}{r}+v\cot\theta$ normal to $r\sin\theta d\varphi\, dr$.

(6) $\frac{dv}{\sin\theta d\varphi}-2\cos\theta w$ normal to $rd\theta\, dr$.

(7) $r\sin\theta\frac{dw}{dr}+2\sin\theta w$ normal to $r^2\sin\theta d\varphi\, d\theta$,

(8) $\sin\theta\frac{dw}{d\theta}+2\cos\theta w$ normal to $r\sin\theta d\varphi\, dr$,

(9) $\frac{dw}{d\varphi}+\frac{2u}{r}+2\cot\theta v$ normal to $rd\theta\, dr$.

Among the expressions above, (2), (4) and (7) are similar to c (1), page 10, in that the losses within the element are affected by the increase of the opposite faces in one dimension. In the case of (2) the changing dimension to be considered is $r\sin\theta d\varphi$; of (4), $r\sin\theta d\varphi$; and of (7), $rd\theta$. Taking this into account, we obtain the following expressions for the losses within the element:

(1) $\frac{\mu}{\rho}\left(\frac{d^2u}{dr^2}+\frac{2\,du}{r}-\frac{2\,u}{r^2}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(2) $\frac{\mu}{\rho}\left(\frac{d^2u}{r^2d\theta^2}+\frac{du\cot\theta}{r^2d\theta}-\frac{2\,dv}{rd\theta}-\frac{2\,v}{r}\cot\theta\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(3) $\frac{\mu}{\rho}\left(\frac{d^2u}{r^2\sin^2\theta\, d\varphi^2}-\frac{2\,dw}{rd\varphi}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(4) $\frac{\mu}{\rho}\left(r\frac{d^2v}{dr^2}+4\frac{dv}{dr}+\frac{2\,v}{r}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(5) $\frac{\mu}{\rho}\left(\frac{d^2v}{rd\theta^2}+\frac{2\,du}{r^2d\theta}+\cot\theta\frac{dv}{rd\theta}-\frac{v}{r\sin^2\theta}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(6) $\frac{\mu}{\rho}\left(\frac{d^2v}{r\sin\theta^2d\varphi^2}-\frac{2\cos\theta dw}{r\sin\theta d\varphi}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(7) $\frac{\mu}{\rho}\left(r\sin\theta\frac{d^2w}{dr^2}+4\sin\theta\frac{dw}{dr}+2\frac{\sin\theta w}{r}\right)r^2\sin\theta d\varphi\, d\theta\, dr$,

(8) $\frac{\mu}{\rho}\left(\frac{\sin\theta d^2w}{rd\theta^2}+\frac{3\cos\theta dw}{rd\theta}-\frac{2\sin\theta w}{r}\right)r^2\sin\theta d\varphi\, d\theta\, dr$.

(9) $\frac{\mu}{\rho}\left(\frac{d^2w}{r\sin\theta d\varphi^2}+\frac{2\,du}{r^2\sin\theta d\varphi}+\frac{2\cot\theta dv}{r\sin\theta d\varphi}\right)r^2\sin\theta d\varphi\, d\theta\, dr$.

Finally we get by annexing these quantities to the corresponding equations for a perfect liquid:

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-v^2r-r\sin^2\theta w^2-\nu\left(\frac{d^2u}{dr^2}+\frac{2\,du}{rdr}-\frac{2u}{r^2}+\frac{d^2u}{r^2d\theta^2}+\right.$$
$$\left.\frac{du}{r^2d\theta}\cot\theta-\frac{2\,dv}{rd\theta}-\frac{2v}{r}\cot\theta+\frac{d^2u}{r^2\sin^2\theta\, d\varsigma^2}-\frac{2\,dw}{rd\varsigma}\right)$$

$$Y-\frac{dp}{\rho rd\theta}=r\frac{\partial v}{\partial t}+2\,uv-r\sin\theta\cos\theta\, w^2-\nu\left(r\frac{d^2v}{dr^2}+\frac{4\,dv}{dr}+\frac{2\,v}{r}+\right.$$
$$\left.\frac{d^2v}{rd\theta^2}+\frac{2\,du}{r^2d\theta}+\frac{dv}{rd\theta}\cot\theta-\frac{v}{r\sin^2\theta}+\frac{d^2v}{r\sin^2\theta d\varsigma^2}-\frac{2\cos\theta dw}{r\sin\theta d\varsigma}\right)$$

$$Z-\frac{dp}{\rho r\sin\theta d\varsigma}=r\sin\theta\frac{\partial w}{\partial t}+2\sin\theta\, uw+2\,r\cos\theta\, vw-$$
$$\nu\left(r\sin\theta\frac{d^2w}{dr^2}+4\sin\theta\frac{dw}{dr}+\frac{2\sin\theta w}{r}+\sin\theta\frac{d^2w}{rd\theta^2}+\right.$$
$$\left.3\cos\theta\frac{dw}{rd\theta}-\frac{2\,w\sin\theta}{r}+\frac{d^2w}{r\sin\theta d\varsigma^2}+\frac{2\,du}{r^2\sin\theta d\varsigma}+\frac{2\cot\theta dv}{r\sin\theta d\varsigma}\right)$$

NOTE I.

For convenient reference the various equations are collected from the different parts of the foregoing paper; and a tabulated analysis of the terms appearing in the equations is annexed. Special attention is given to the terms in the equations referred to cylindrical and polar co-ordinates.

RECTANGULAR CO-ORDINATES.

Perfect Liquid.

$$X-\frac{dp}{\rho dx}=\frac{\partial u}{\partial t}.$$

$$Y-\frac{dp}{\rho dy}=\frac{\partial v}{\partial t}.$$

$$Z-\frac{dp}{\rho dz}=\frac{\partial w}{\partial t}.$$

Viscous Liquid.

$$X-\frac{dp}{\rho dx}=\frac{\partial u}{\partial t}-\frac{\mu}{\rho}\left(\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}\right),$$

$$Y-\frac{dp}{\rho dy}=\frac{\partial v}{\partial t}-\frac{\mu}{\rho}\left(\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}+\frac{d^2v}{dz^2}\right),$$

$$Z-\frac{dp}{\rho dz}=\frac{\partial w}{\partial t}-\frac{\mu}{\rho}\left(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}+\frac{d^2w}{dz^2}\right),$$

CYLINDRICAL CO-ORDINATES.

Perfect Liquid.

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-v^2r,$$

$$Y-\frac{dp}{\rho r d\theta}=r\frac{\partial v}{\partial t}+2\,uv.$$

$$Z-\frac{dp}{\rho dz}=\frac{\partial w}{\partial t}.$$

Viscous Liquid.

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-rv^2-\frac{\mu}{\rho}\left(\frac{d^2u}{dr^2}+\frac{du}{rdr}-\frac{u}{r^2}+\frac{d^2u}{r^2d\theta^2}-\frac{2\,dv}{rd\theta}+\frac{d^2u}{dz^2}\right)$$

$$Y-\frac{dp}{\rho rd\theta}=r\frac{\partial v}{\partial t}+2\,uv-\frac{\mu}{\rho}\left(r\frac{d^2v}{dr^2}+\frac{3\,dv}{dr}+\frac{d^2v}{rd\theta^2}+\frac{2\,du}{r^2d\theta}+\frac{r\,d^2v}{dz^2}\right)$$

$$Z-\frac{dp}{\rho dz}=\frac{\partial w}{\partial t}-\frac{\mu}{\rho}\left(\frac{d^2w}{dr^2}+\frac{dw}{rdr}+\frac{d^2w}{r^2d\theta^2}+\frac{d^2w}{dz^2}\right).$$

POLAR CO-ORDINATES.

Perfect Liquid.

$$X-\frac{dp}{\rho dr}=\frac{\partial u}{\partial t}-rv^2-r\sin^2\theta\,w^2,$$

$$Y-\frac{dp}{\rho r\,d\theta}=r\frac{\partial v}{\partial t}+2\,uv-r\sin\theta\cos\theta\,w^2.$$

$$Z-\frac{dp}{\rho r\sin\theta d\varsigma}=r\sin\theta\frac{\partial w}{\partial t}+2\sin\theta\,uw+2\,r\cos\theta\,vw.$$

Viscous Liquid.

$$X-\frac{dp}{\partial dr}=\frac{\partial u}{\partial t}-v^2r-r\sin^2\theta\,w^2-\frac{\mu}{\rho}\left(\frac{d^2u}{dr^2}+\frac{2\,du}{rdr}-\frac{2u}{r^2}+\frac{d^2u}{r^2d\theta^2}+\right.$$
$$\left.\frac{du}{r^2d\theta}\cot\theta-\frac{2\,dv}{rd\theta}-\frac{2v}{r}\cot\theta+\frac{d^2u}{r^2\sin^2\theta d\varsigma^2}-\frac{2\,dw}{rd\varsigma}\right),$$

$$Y-\frac{dp}{\rho rd\theta}=r\frac{\partial v}{\partial t}+2\,uv-r\sin\theta\cos\theta\,w^2-\frac{\mu}{\rho}\left(r\frac{d^2v}{dr^2}+4\frac{dv}{dr}+\right.$$
$$\frac{2v}{r}+\frac{d^2v}{rd\theta^2}+\frac{2\,du}{r^2d\theta}+\cot\theta\frac{dv}{rd\theta}-\frac{v}{r\sin^2\theta}+\frac{d^2v}{r\sin^2\theta d\varsigma^2}-$$
$$\left.\frac{2\cos\theta\,dw}{r\sin\theta d\varsigma}\right),$$

$$Z-\frac{dp}{\rho r\sin\theta d\varsigma}=r\sin\theta\frac{\partial w}{\partial t}+2\sin\theta\,uw+2\,r\cos\theta vw-\frac{\mu}{\rho}\left(r\sin\right.$$
$$\frac{d^2w}{dr^2}+4\sin\theta\frac{dw}{dr}+\frac{2\sin\theta w}{r}+\sin\theta\frac{d^2w}{rd\theta^2}+3\cos\theta\frac{dw}{rd\theta}-$$
$$\left.\frac{2\,w\sin\theta}{r}+\frac{d^2w}{r\sin\theta\,d\varsigma^2}+\frac{2\,du}{r^2\sin\theta d\varsigma}+\frac{2\cot\theta\,dv}{r\sin\theta d\varsigma}\right).$$

Equations of Continuity for a Liquid.

Rectangular co-ordinates, $\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0.$

Cylindrical co-ordinates, $\frac{d(ur)}{dr}+r\frac{dv}{d\theta}+r\frac{dw}{dz}=0,$

Polar co-ordinates. $\frac{d(ur^2)}{r^2dr}\sin\theta+\frac{d(v\sin\theta)}{d\theta}+\frac{dw}{d\varphi}\sin\theta=0.$

An Analysis of the Equations.

Rectangular Co-ordinates—The terms found in the equations, when referred to rectangular co-ordinates, need but little explanation. The terms $\frac{d^2u}{dx^2}$, $\frac{d^2u}{dy^2}$, and $\frac{d^2u}{dz^2}$, are due to the variation in the velocity of u in the direction of x, y and z respectively. And the terms found in the other equations are due to a similar variation of v and w.

Cylindrical Co-ordinates—Equations for a viscous liquid.

X—axis:

$-rv^2$ results from centrifugal force due to v,

$\frac{d^2u}{dr^2}$ results from $\frac{du}{dr}$.

$\frac{du}{rdr}-\frac{u}{r^2}$ results from displacement $\frac{u}{r}$ (see page 8),

$\frac{d^2u}{r^2d\theta^2}$ results from $\frac{du}{d\theta}$.

$-\frac{2\,dv}{rd\theta}$ results from same cause that produces centrifugal force $-rv^2$ (see page 8),

$\frac{d^2u}{dz^2}$ results from $\frac{du}{dz}$.

Y—axis:

$2\,uv$ results from the combined action of u and v, (see page 3 and note,)

$r\frac{d^2v}{dr^2}+\frac{dv}{dr}$ results from $r\frac{dv}{dr}$.

$\frac{2\,dv}{dr}$ results from linear displacement $2\,v$ along r. (see page 9).

$\frac{d^2v}{rd\theta^2}$ results from $\frac{dv}{d\theta}$,

$\frac{2\,du}{r^2d\theta}$ results from $\frac{u}{r}$ (see page 9).

$\frac{rd^2v}{dz^2}$ results from $\frac{dv}{dz}$.

Z—axis:

The equation for the forces acting along z is exactly like that for rectangular co-ordinates.

Spherical Co-ordinates:

X—axis:

$-v^2r$ results from centrifugal force due to v,

$-r\sin^2\theta w^2$ results from centrifugal force due to w,

$\frac{d^2u}{dr^2}$ results from $\frac{du}{dr}$,

$\frac{2\,du}{rdr}-\frac{2\,u}{r^2}$ results from $\frac{2u}{r}$ inflow (see page 11),

$\frac{d^2u}{r^2d\theta^2}+\cot\theta\,\frac{du}{r^2d\theta}$ results from $\frac{du}{rd\theta}$ (see page 11).

$-\frac{2\,dv}{rd\theta}-\frac{2\,v}{r}\cot\theta$ results from same cause that produces centrifugal force $-rv^2$, (see also 11, 12).

$\frac{d^2u}{r^2\sin^2\theta d\varphi^2}$ results from $\frac{du}{d\varphi}$,

$-\frac{2\,dw}{rd\varphi}$ results from same cause that produces centrifugal force $-r\sin^2\theta\, w^2$.

Y—axis:

$2\,uv$ results from combined action of u and v.

$-r\sin\theta\cos\theta\, w^2$ results from centrifugal force due to w,

$r\frac{d^2v}{dr^2}+2\frac{dv}{dr}$ results from $r\frac{dv}{dr}$ (see pages 11, 12),

$\frac{2\,dv}{dr}+\frac{2\,v}{r}$ results from (see pp. 9, 11. 12).

$\frac{d^2v}{rd\theta^2}$ results from $\frac{dv}{d\theta}$,

$\frac{2du}{r^2 d\theta}$ results from $\frac{u}{r}$ (see page 9).

$\cot\theta \frac{dv}{rd\theta} - \frac{v}{r\sin^2\theta}$ results from $v \cot\theta$ (see page 11).

$\frac{d^2v}{r\sin^2\theta d\varsigma^2}$ results from $\frac{dv}{\sin\theta d\varsigma}$.

$-\frac{2\cos\theta\, dw}{r\sin\theta d\varsigma}$ results from same cause that produces centrifugal force $-r\sin\theta\cos\theta\, w^2$.

Z—axis:

$2\sin\theta\, wu$ results from combined action of u and w. (see page 11.)

$2\, r\cos\theta\, vw$ results from combined action of v and w, (see page 11.)

$r\sin\theta \frac{d^2w}{dr^2} + 2\sin\theta \frac{dw}{dr}$ results from $r\sin\theta \frac{dw}{dr}$.

$2\sin\theta \frac{dw}{dr} + \frac{2\sin\theta w}{r}$ results from $2\sin\theta\, w$,

$\sin\theta \frac{d^2w}{rd\theta^2} + \cos\theta \frac{dw}{rd\theta}$ results from $\sin\theta \frac{dw}{d\theta}$,

$2\cos\theta \frac{dw}{rd\theta} - \frac{2\,w}{r}\sin\theta$ results from $2\, r\cos\theta\, w$.

$\frac{d^2w}{r\sin\theta d\varsigma^2}$ results from $\frac{dw}{d\varsigma}$,

$\frac{2\,du}{r^2\sin\theta d\varsigma}$ results from $2\sin\theta\, wu$.

$\frac{2\cot\theta\, dv}{r\sin\theta\, d\varsigma}$ results from $2\cos\theta\, v$ (see p. 12).

NOTE II.

Advantages of the preceding method of derivation.

The method of deriving the equations of a viscous liquid which has been received most favorably is that of Stokes. Both Lamb and Basset have used his method, the first in part and the second entirely, in their works on Hydrodynamics.

But Stokes' method is based upon two hypotheses. (See introduction.) The second hypothesis has not been received

very favorably because it involves the assumption that the motion is slow.

The method which I have used is based upon the definition of the coefficient of viscosity and involves no hypothesis save that of continuity of motion, which has been long received without question.

The derivation of the equations is much simpler than by former methods, unless an exception is made of the method of O. E. Meyer. The equations of Meyer are based upon an assumption which does not satisfactorily explain parts of the terms. (See introduction.)

The development of the equations of motion by analysis for cylindrical and spherical coördinates enables us to give a definite meaning to each term in the equations, and may aid materially in solving them.

The method definitely determines when the equations hold and when they do not, as is shown in Note III.

The principal feature of this paper is the relation established between the motion of a perfect and a viscous liquid.

NOTE III.

The first approximation made in this paper is found on pages 46 and 47, *Colo. Col. Studies*, Vol. 7, where all terms but the first are neglected in the expression.

$$\frac{1}{dt}\left(\frac{du}{u}-\frac{du^2}{u^2}+\frac{du^3}{u^3}-\frac{du^4}{u^4}+\text{etc.}\ .\ .\right).$$

This approximation is admissable unless the acceleration in the velocity is very great, which is not usually the case.

In the second place, we suppose the rate of inflow over the face in a direction perpendicular to the velocity considered uniform along the normal to the face. It is easy to conceive that the inflow might be accelerated. But if it were the acceleration neglected would be of the second order, and might reasonably be neglected.

If our conclusions are true, the equations hold not only when the motion is slow, but in all cases, except when the motion is rapidly accelerated.

NOTE IV.

On the motion of a fluid in which a circular cylinder of infinite length is slowly rotating.

Whatever motion takes place in the fluid is due to internal friction, and is two dimensional, *i. e.*, in the plane perpendicular to the axis of the cylinder.

Let the very natural supposition be made that the layer of fluid contiguous with the surface of the cylinder moves with the surface, then the concentric layers will have a motion due to friction. There will be no motion parallel to the axis, consequently $w=0$: the only force acting along r is that due to centrifugal force, but since the velocity of rotation is small the products and squares of velocities may without sensible error be neglected, consequently u also is equal to 0. Therefore to completely determine the motion of the fluid we need consider only the second of the equations for cylindrical coordinates.

Since the initial condition was one of rest, the conditions of equilibrium demand that

$$\frac{\delta v}{\delta t}=\frac{\mu}{\rho}\left(r\frac{d^2v}{dr^2}+3\frac{dv}{dr}+\frac{d^2v}{r\,d\theta^2}\right).$$

$\frac{2\,du}{r^2d\theta}$ disappearing, since $u=0$.

$\frac{r\,d^2v}{dz^2}$ disappearing, since this motion is two dimentional.

Since the angular velocity of rotation is small, squares and products of the velocities may be omitted, hence $\frac{\delta v}{\delta t}$ becomes $\frac{dv}{dt}$; after the rotation has continued long enough for the motion of the fluid to become steady, $\frac{dv}{dt}=0$. This last is on the supposition that the velocity of rotation is not accelerated: and since the motion is symmetrical to the axis of the cylinder, $\frac{d^2v}{r\,d\theta^2}$ becomes 0. The final form for the motion, where

angular velocity is constant, is

$$0=r\frac{d^2v}{dr^2}+3\frac{dv}{dr}.$$

Solving $$rv=kr+\frac{c}{r}$$

At an infinite distance rv is insensible, consequently

$$k=0,$$

$$rv=\frac{c}{r},$$

$$av^1=\frac{c}{a},$$

$$c=a^2v^1,$$

$$rv=\frac{a^2v^1}{r}.$$

But if we consider the cylinder as filled with liquid and consider the motion of this inclosed liquid,

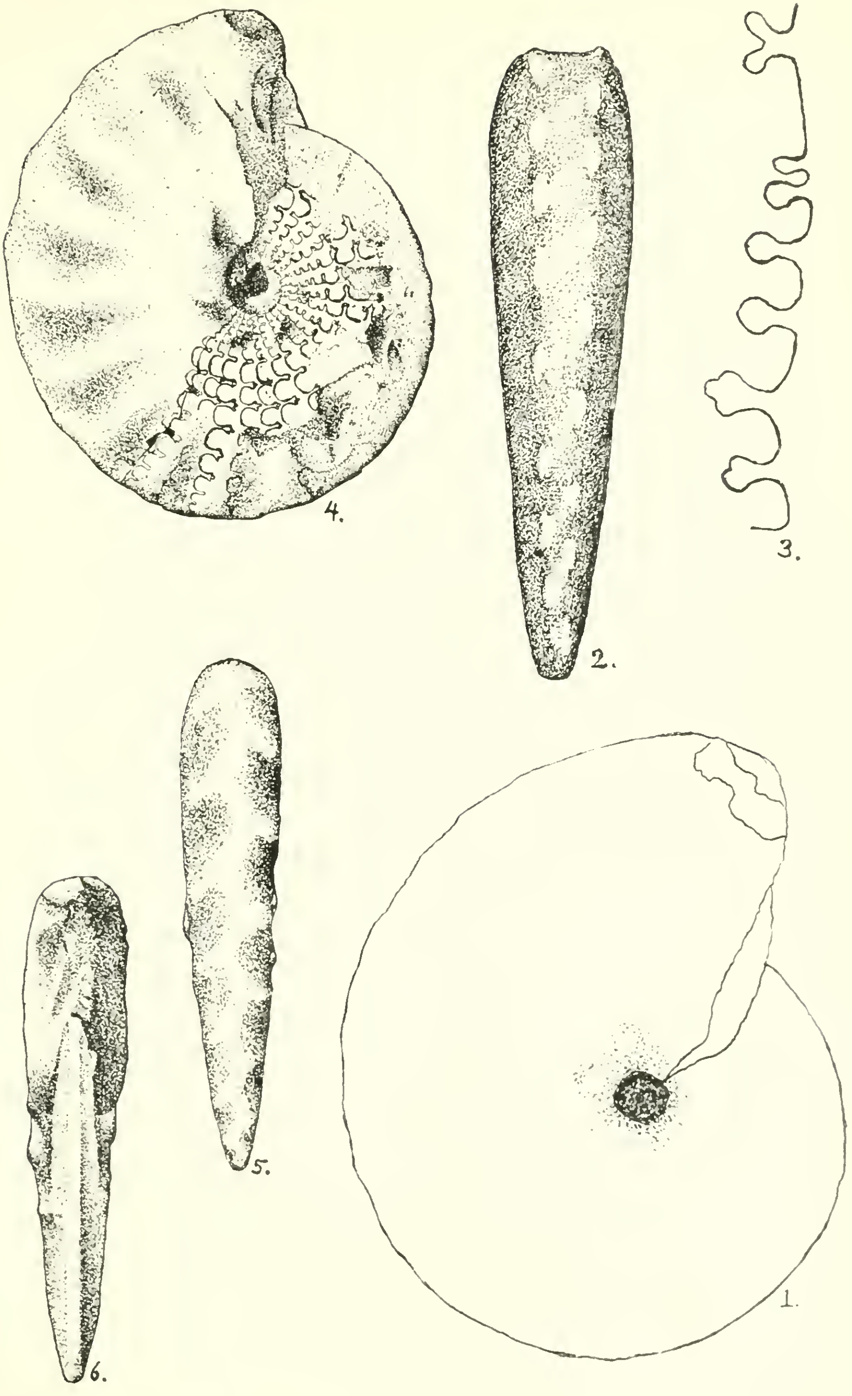
$$c=0.$$

$$av^1=ka.$$

$$k=v^1,$$

$$rv=rv^1,$$

where v^1 is angular velocity of cylinder. Hence the inclosed fluid moves as a solid body.—(See BASSET, p. 282, Vol. 2.)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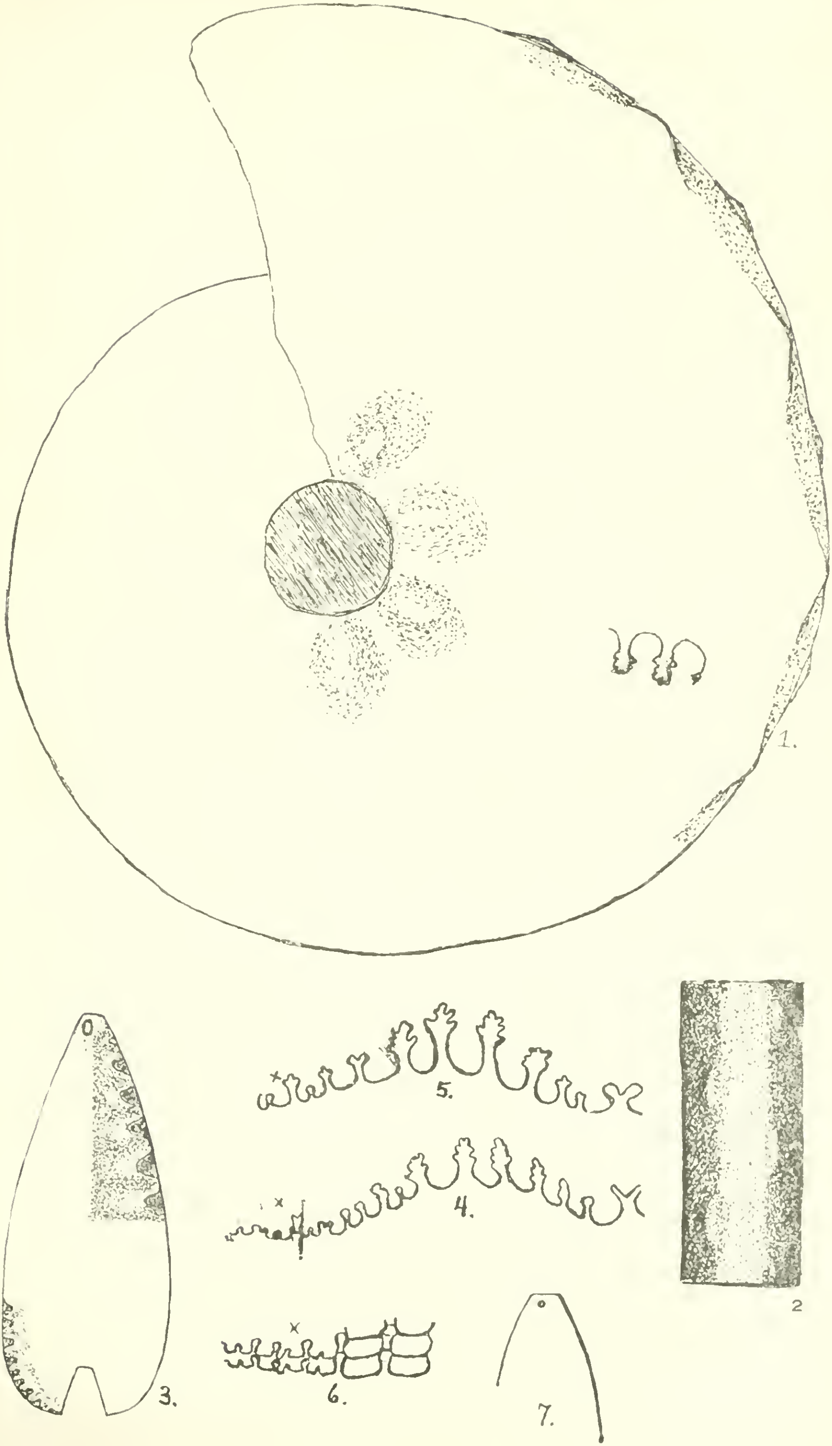
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THE CAPRICORNS, MAMMALS OF AN ASIATIC TYPE, FORMER INHABITANTS OF THE PIKE'S PEAK REGION.

[A popular lecture delivered before the Colorado College Scientific Society on the 27th of October, 1899, and here reprinted with some revision from the *Colorado Springs Gazette* of November 12th, 1899.]

By F. W. Cragin.

As most of my audience are aware, the granite of the Rocky Mountains, though commonly building our highest peaks, has been raised to such elevated positions from an origin deep down in the earth's crust beneath the stratified rocks.

These latter members of the earth-crust have been deposited almost wholly by water, largely oceanic, in small part fresh, as mechanical, chemical and organic sediments, long since consolidated into rock, that covered the granite like a series of thick, more or less nearly horizontal blankets.

The upward movement is attested by a thousand basset edges of strata, of which the Gateway of the Garden of the Gods and the hog-backs north and south of Colorado City are conspicuous examples, upturned by the ascending portion of the granite, and which still are seen, though profoundly cut down and reduced to the merest remnants, the mountain-blanket having been almost wholly cut away in the case of Pike's Peak itself, by erosion.

The granite and its burden of stratified rocks were thrown into mountain folds in the region of the Rockies chiefly at the close of the Cretaceous age, the last of the three ages of the great Mesozoic era; which means that, for the most part, the Rocky Mountains were born at the end of the ocean's sway over interior North America, though that sway, prior to that time, had not been uninterrupted, for the Pike's Peak region stood above the sea during parts of at least three earlier geological ages — the Devonian, the Carboniferous and the Jurassic — and probably during parts, also, of others, as is witnessed by the absence from it of Devonian rocks (sediments), the presence near Maniton of thick Carboniferous

beach-deposits, and the fact that the Jurassic is represented here solely by a great fresh-water lake deposit in whose border-muds, near Canon City, Colorado City and Morrison and elsewhere, bones of huge walking reptiles, the Dinosaurs, including some, like Atlantosaurus and its allies, eighty or perhaps even one hundred feet long, the most ponderous land animals known ever to have trod the earth, have lain stranded to the present day.

Of all the stratified rock-systems represented at the eastern base of our front-range, the Manitou limestone, of Silurian age, is nearly the oldest, only one, the underlying formation of red sandstone, gravel and lime, which, from its deep-green glauconitic variegations and its typical occurrence about Rainbow Falls, may well be called the Rainbow formation, belonging to an earlier age, the Cambrian. The extensive system of caves found in the Manitou limestone, and of which the Pickett cave, or Cave of the Winds, and the Grand Caverns, besides several smaller caves, are a part, have been made primarily by the solvent and secondarily by the transporting action of carbonic-acid-laden waters that formerly circulated through the joints and other fissures of the limestone. Subsequently, as happens to most caves sooner or later, those of the Manitou limestone have been to a greater or less extent filled with a reddish sediment, known as cave-earth, which it is necessary to remove in order to display the subterranean galleries and chambers in anything like their original dimensions. The cave-earth in many instances is barren, or contains only sparingly the remains of recent animals; but in others it yields the bones of extinct animals, throwing light on the history of both the cave and the surrounding region. In having the cave-earth removed from a cave in this limestone in Glen Eyrie a few years since, General Palmer saved, with his usual foresight, the organic remains, consisting of a number of bones, thrown out with the earth by the workmen, and these he very kindly submitted to me for determination. Two of the specimens were at once recognized as proximal phalanges of an extinct species of Horse, whose remains occur abundantly elsewhere with those of Elephants, bulky, armored Ground-sloths, Llamas as large as Camels. Saber-toothed Tigers

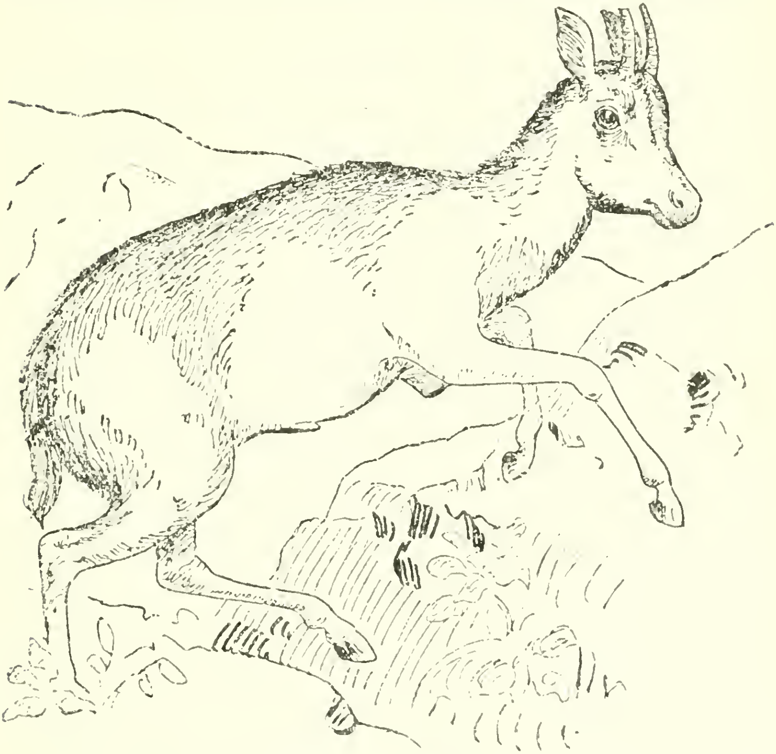
and other extinct animals of late Pliocene to Pleistocene age. The other remains found in the cave at Glen Eyrie were identified last summer, when I was first able to compare them with extensive series of skeletons in the United States National Museum. The smaller bones—a jaw and two femurs—were soon found to belong to a species of Woodchuck, different from the common one of eastern North America, and not improbably so from the Yellow-bellied Woodchuck, which is the present species of the central Rocky Mountains. The large bones pertained to the right fore limb of a young ruminant, or two-toed ungulate, which some ancient beast of prey had doubtless dragged into the cave as a choice morsel to feed upon at leisure. They were humerus and cannon-bone (metacarpus), in which part of the epiphyses were missing, not yet having united with the shaft. It was at first thought that they might have pertained to a Rocky Mountain Sheep, or Big-horn. From the skeleton of this, however, they widely differed, as they also did from the Mountain Goat of Northwestern America, and from skeleton after skeleton with which they were compared, until the whole range of modern two-toed ungulates of North America had been gone over. Then Asiatic forms were tried, and it was soon found that the bones closely agreed with those of the Capricorns, or Goat-antelopes, a genus of animals technically known as *Nemorhædus*, and represented to-day by several species living in the Himalayas and other mountains of Asia, Japan and Formosa. They belong to the family *Bovidae*, which includes also the Cattle, Sheep, Goats and Antelopes. They are intermediate in their characters between the Goats and Antelopes, whence the alternative name, Goat-antelopes; they are also sometimes called Mountain Antelopes. It is interesting to note that they range at altitudes of 3,000 to 8,000 feet, the cave at Glen Eyrie being within these limits.

There are two sections of the genus *Nemorhædus*. One includes clumsier built animals, which, however, resemble the deer in having a tear-pit in the face, and which are solitary in habit; these are the Serows, of the sub-genus *Capricornis*. The other, of the section or sub-genus *Kemas*, includes the graceful forms, the Gorals, which lack tear-pits, and go in

small parties. It is impossible, in the absence of a skull from the Glen Eyrie material, to be quite sure whether the particular Capricorn that inhabited the Pike's Peak region was a Serow or a Goral. Of the Serows, the osteological collections of the National Museum included a skull, but no skeleton; but the agreement of the two limb-bones with the corresponding ones of the species of Goral in the museum (*Nemorhædus crispus*, or *Kemas crispus*, of Japan) is such that the slight differences can hardly be considered of more than specific value, and it seems probable that our Rocky Mountain Goat-antelope was a Goral. A glance, therefore, at the species of Goral that inhabits an interior mountain region of Asia corresponding with ours of North America may be of interest. The following account of the Goral of the Himalaya is derived in part from Jerdon's "Mammals of India" and in part from Lydekker's "Chapters on Hoofed Animals."

The animal is very caprine in appearance, the back somewhat arched, the limbs stout and moderately long. It is well adapted for both climbing and jumping. It stands some twenty-seven to thirty inches high at the shoulder, the head and body measuring fifty, the tail four, and the horns eight inches in length. The horns, which are present in both sexes, and only a little larger in the male than in the female, incline backward and slightly inward, and are a little recurved; they are shorter than the skull, black in color, round in cross-section, and ornamented with twenty to twenty-five encircling raised folds. The fur is somewhat rough, of two kinds of hair, and there is a short, semi-erect mane in the male. The color is brown, with a more or less decided gray or ruddy tinge, a little lighter beneath. The throat is white. A dark line runs down the back from crown to tail, and the front surfaces of the legs are also marked with dark streaks. Though found considerably higher and lower, the Himalayan Gorals are commonest at elevations of 5,000 to 6,000 feet above sea-level. They inhabit rugged, grassy hills and rocky ground in the midst of forests, and are usually found in small family parties of three to eight. If one Goral is seen, you may be pretty sure that others are not far off. They rarely or never

forsake their own feeding grounds. In cloudy weather they feed at all hours of the day; in fair weather, only morning and evening. When one is alarmed it gives a short hissing sound, which is answered by all within hearing. Goral stalking is with many a favorite sport in the middle ranges of the Himalaya. The illustration of the Himalayan Goral herewith given is enlarged from Lydekker's work above cited; for the use of it here I am indebted to the *Colorado Springs Gazette*.



HIMALAYAN GORAL.

If the range of the Pike's Peak Capricorn corresponded nearly with that of the Himalayan, and the cave of the Capricorn-eating carnivore was conveniently located within the zone of greatest abundance of the quarry—5,000 to 6,000 feet above sea-level—the Rocky Mountain plateau must have stood something like one or two thousand feet lower in its Capri-

corn epoch than to-day, as the present elevation of the cave approaches 7,000 feet. And as the two conditions above predicated are those most likely to have prevailed, it seems quite probable that *Nemorhœdus* as an element of the North American fauna, belonged to the Champlain phase of the Glacial epoch.

Though the differences may be due to variant conditions of preservation in the cave-earth in which they were imbedded, and hence not significant of difference in age, the Horse phalanges from the cave at Glen Eyrie are darker, heavier, and much more thoroughly mineralized than the bones of the Capricorn, and are, seemingly, much older. They belong, however, to rather a slender-limbed type of Equus that occurs in Kansas in deposits which are probably late Pliocene or early Postpliocene, and thus tend to show that the caves in the Manitou limestone date back at least as far as the beginning of the Quaternary, and not improbably had their origin in the later part of the Tertiary age.

The finding of Goat-Antelopes as members of the Pleistocene or earlier fauna of the Rocky Mountains was quite unexpected, but is no more remarkable than that Elephants, which, not only by present habitat but also by the very place of their origin from the Mastodon stock, are Asiatic types, abounded in Pleistocene times throughout North America, to which the Asiatic fauna doubtless had measurable access by way of lands now interrupted in the vicinity of Behring's Straits. The Asiatic Mammoth occurs in transsierran America, and Elephant remains of an extinct species related to the Mammoth and to the Modern Asiatic, rather than the African Elephant, are common on the Great Plains, and not rare in the old river gravels which underlie Denver, Colorado Springs and Colorado City, and have been found as high in the Rocky Mountains at least as the bogs of Grassy Gulch, in the Cripple Creek district. It serves to show how imperfect still is our knowledge of even the most recent chapters of the earth's history, to emphasize the interest of the great stone book of nature which too often we pass by without an attempt to read, and to show how the darkest cave, carefully studied, may prove a torch to illumine a page of that book as well as its own mystic interior.

BUCHICERAS (SPHENODISCUS) BELVIDERENSIS AND ITS VARIETIES.

By F. W. Cragin.

Sphenodiscus belviderensis. Crag.

Shell of medium size, flattish-lenticular, the venter (periphery) truncate, narrowly so on the younger whorls, broadly and less sharply and less evenly so on the oldest one, particularly on the body-chamber; body-chamber occupying three-eighths to one-half of a volution; umbilicus narrow, the greater part of the height of the second whorl being embraced within one-third to one-half of that of the body-chamber; suture "ceratitic," much like that of *Sphenodiscus pedernalis*, Von Buch, the part corresponding to the outer saddle of (strictly so-called) *Buchiceras* being divided into five saddles by means of four leaves, of which the very unequal outer two are much smaller than the subequal inner two; all of the saddles of the suture rounded to flattish or, in case of some of the inner ones, emarginate at fundus, some symmetrically, some obliquely so, and all much broader than the leaves, excepting the next to the outermost of those formed by the lobing of the outer buchiceran saddle; the leaves little cleft at the summit; ornamentation of the shell consisting usually of at least two revolving series of low tubercles on either side; one just outside of the umbilical shoulder, and consisting of few tubercles, one or two of the newer of which sometimes become more prominent than any other tubercles on the shell; the other, presented ventrally on the ventro-lateral margin, and consisting of numerous tubercles so compressed as to trend with that margin, and so arranged that those of the right alternate with those of the left margin, half (consisting of every other one) of these tubercles constituting the termini of broadly and feebly accentuated lateral ribs, which are confined to the outer part of the flank, and at whose inner ends (one at the end of each) the very low and diffuse tubercles of a third revolving series may be developed.

Measurements.—The shell of this ammonite attains a height of at least 125 mm.; that of the body-chamber about half as much.

Occurrence and History.—In Texas this is the commonest Sphenodiscus of the Comanche Peak limestone; and in Kansas it is, so far as the writer has observed, the only representative of its genus in the Champion shell-bed and the Kiowa shales, being associated there with the common Fredericksburg and Kiowa ammonite, *Schloenbachia peruviana*, Von Buch. All of the truncate Sphenodisci hitherto found either in the Comanche Peak limestone and Champion shell-bed, or in later rocks of the Comanche series, are here provisionally included under this species, though some of the supposed varieties may prove to be distinct, and some are doubtless of stratigraphic importance.

Messrs. Stanton and Hill have recently* called attention to the fact that a Sphenodiscus differing from *S. pedernalis*, Von Buch, by its truncate venter, occurs in the Belvidere beds of Southern Kansas.† I had observed the same fact in 1889, as witnessed by figures 1, 2 of Plate I, here reproduced from rough lead-pencil sketches which I made at that time, of a Champion shell-bed specimen that I was once inclined to separate from *S. pedernalis* and describe as new. I afterward gained the impression that the truncate character of the periphery was inconstant, and did not signify a species distinct from *S. pedernalis*, but only a variety. Hence the broad use that I have generally made of the name *Sphenodiscus pedernalis*.

I now agree fully with the above-cited statement of Messrs. Stanton and Hill, and confirm also my own former supposition of the distinctness of these truncate Sphenodisci from *S. pedernalis*.

The specific name, *belviderei* (afterward changed to *belviderensis*), was first applied to this species in 1890 in my article "On the Cheyenne Sandstone and the Neocomian Shales of Kansas" ‡ where I listed "*Ammonites Belviderei*" as

*Except the two figures noted as sketched in 1889, the manuscript and drawings of the present article were prepared in the spring of 1897.

† Am. Jour. Sci., Third Series, Vol. L, June, 1895.

‡ Bul. Washburn Coll. Lab. Nat. Hist., No. 11, March, 1890.

occurring in No. 5 of my Blue Cut Mound section (the Fullington horizon of the Kiowa shales). The specimens so recorded were small ones, of which the drawings were made in 1889, and published, without comment, in 1894.* They were assigned a specific name because smaller, plainer and simpler-sutured than those recorded in 1889 and 1890 as "*Ammonites pedernalis*" from the Champion shell-bed;† but the Fullington and Champion forms are now regarded as belonging to one variable species, which must, therefore, take the name *belviderei*, or its modified form, *belviderensis*.

Of this species there appear to be five varieties that call for notice. These may be described as follows:

Var. MONS-COMANCHEANUS.

Plate I, fig. 5.

Suture relatively complex for this species, having even the smaller leaves more or less cut at the summit, usually with two or three simple, obtuse lobules, and the larger leaves cleft into a larger number (4—6) of processes which are either simple and short (tooth-like) to somewhat longer (subdigitiform), or show a tendency to secondary toothing, one or two of the processes being expanded at the extremity and abruptly truncated or notched. Of the saddles centripetally succeeding the five secondary saddles,‡ the first three are simple and subrotund (the first one a little compressed), with rounded to truncate extremity, the next two (respectively just outside of and opposite the circum-umbilical tubercles) are broader than deep and strongly emarginate or bilobate, being parted into two lobes by a small and short clavate leaflet. The type is from the Comanche Peak limestone of Tarrant County, Texas: and to this variety the common Sphenodiscus of the Champion shell-bed (illustrated in figures 1 and 2 of Plate I) seems to belong.

* New and Little Known Invertebrata from the Neocomian of Kansas. American Geologist, XIV, Plate I, figs. 3-5. July, 1894.

† Geological Notes on the Region South of the Great Bend of the Arkansas. Bul. Washb. Coll. Lab. Nat. Hist., No. 9, February, 1889; and, On the Cheyenne Sandstone and the Neocomian Shales of Kansas. L. C. No. 11.

‡ The term "secondary saddles" is used as a convenient designation for the five saddles into which, by foliolation, the external saddle of typical *Buchiceras* is subdivided in *Sphenodiscus*.

Var. CLAVATUS.

Plate II, figs. 1-3.

Suture considerably less complex than in similarly sized specimens of *mons-comancheanus*, the leaves being capitate-clavate, the summits of the larger leaves departing less (or at most, not more) from a simple outline, in a specimen exceeding 100 mm. in height, than do those of the little (27 mm.) specimen of *Sphenodiscus belviderensis* figured in Vol. XIV of the American Geologist, Plate I, figs. 4, 5. In a smaller specimen (which, if the missing portion of the body-chamber were restored, would have a height of about forty-seven or forty-eight mm.), the first four (all that the specimen shows) of those saddles that succeed the secondary, are not emarginate, the first and third of them being narrower (the first narrower than the third), and of rounded outline, the second and fourth being broader and truncate. The two type specimens were obtained, with the type of *mons-comancheanus*, from the Comanche Peak limestone of Tarrant County, Texas.

Var. UDDENI.

Plate I, figs. 3 and 4.

Suture relatively complex, the primary lateral and the larger secondary lateral and auxilliary leaves little different from those of var. *mons-comancheanus*, the leaves and saddles interior to the secondary ones being as follows: First and second lateral leaves irregularly and obtusely dentate, inclosing a large, simple, subrotund saddle; these followed in succession by a deeply emarginate saddle: an intermediate-sized, asymmetrical, feebly denticulate leaf: three simple, subrotund, subequal saddles, parted by two small, narrow, simple, clavate leaves; a small emarginate leaf; a small deeply emarginate saddle; a narrow emarginate leaf; a broad emarginate saddle (this in the line of the series of circum-umbilical tubercles); and finally, a small leaf and saddle, both emarginate. The type-specimen of this variety is in the museum of Augustana College, and was kindly loaned me for study by Prof. J. A. Udden, who collected it in the Kiowa shale, a few miles west of Lindsborg, Kansas, and for whom it is named.

Var. MENTORIANUS.

Plate I, figs. 6 and 7.

This variety is distinguished by having the two simple, truncate saddles second and third exterior to that which is in the course of the circum-umbilical tubercles, remarkably broad and shallow. The type is from the Mentor beds, near Brookville. Kansas.

Var. SERPENTINUS.

Plate II, figs. 4-6.

Venter of entire body-chamber strongly and rather narrowly rounded and sinuous, the convexities of the sinuosity being correlated with pronounced, rounded or somewhat suddenly flattish-crested ribs on the outer part of the body-chamber, the tubercles seen on the ventro-lateral margin of typical *belviderensis* being nearly lost in the ventro-lateral shoulders of the ribs; the ribs separated by broadly round-bottomed valleys of about their own breadth, and disappearing about midway between the venter and the umbilicus; a series of obliquely compressed tubercles encircling the umbilicus. Of the septate portion of the shell, only the anterior part has well-developed ribs and tubercles, and a distinctly sinuous and rounded venter, the posterior part of the venter being abruptly truncate, as in typical *belviderensis*.

This sub-species is common in and characteristic of the Denison division, though its occasional occurrence in the Washita division also is probable. The types are from the Pawpaw clays near Denison.

THE NUMBER CONCEPT.

BY FLORIAN CAJORI.

For the benefit of teachers who do not have access to a modern mathematical library it is proposed in this compilation to present the views of mathematicians of the present time relating to number, its origin and nature. We shall begin by giving what is considered to be the primary number concept; then we shall briefly indicate how the original and primitive idea of number is extended so as to apply to measurement.

PRIMARY NUMBER.

"Separateness or distinctness is a primary cognition, being necessary even to the cognition of things as individuals, as distinct from other things. The notion of number is based immediately on this primary cognition.

"Number is that property of a group of distinct things which remains unchanged during any change to which the group may be subjected which does not destroy the distinctness of the individual things. Such changes are changes of the characteristics of the individual things or of their arrangement; for these do not cause one thing to split up into more than one, nor more than one to merge in one. . . . . The number of things in any two groups of distinct things is the same, when for each thing in the first group there is one in the second, and reciprocally, for each thing in the second group, one in the first.

"Thus, the number of letters in the two groups, A, B, C; D, E, F, is the same. In the second group there is a letter which may be assigned to each of the letters in the first: as D to A, E to B, F to C: and reciprocally, a letter in the first which may be assigned to each in the second: as A to D, B to E, C to F.

"Two groups thus related are said to be in *one-to-one* (1—1) correspondence. . . . . The fundamental operation of arithmetic is counting. To count a group is to set up a one-to-one correspondence between the individuals of this group and the individuals of some representative group. Counting leads to an expression for the number of things in any group in terms of the representative group: if the representative group be the fingers, to a group of fingers; if marks, to a group of marks: if the numeral words, or symbols in common use, to one of these words or symbols."—PROF. H. B. FINE, of Princeton, in *The Number-System of Algebra*. 1891, pp. 3, 4, 5.

"To count things means to consider them as alike, to take them together, and to associate with them, singly, other things which are also considered alike. Each of the things with which other things are associated in counting is called a *unit;* each of the things which in counting are associated with other things is called a *one*. The result of counting is called *number*. On account of the likeness (Gleichartigkeit) of the *units* among themselves and of the *ones* among themselves, the number is independent of the order in which the *ones* are associated with the *units*.

"If in a number we specify to what extent the units were considered as being alike, by assigning a collective namė to the units, then we have a *concrete number*. By completely ignoring the nature of the things counted we pass from the notion of a *concrete number* to the notion of an *abstract number*.

"By the word *number*, unqualified, we shall always understand an abstract number."—BURKHARDT AND MEYER'S *Encyklopädie der Mathematischen Wissenschaften*. Part I, Vol. I, 1898, pp. 1-3.

"To count distinct things means to make of them an artificial individual or group, and then to identify its elements with those of a familiar group." PROF. G. B. HALSTED, *in Science*. N. S., Vol. VIII, 1898, p. 909.

"Counting presupposes the comparison of multitudes (Vielheiten). By multitudes we mean a group of objects alike among themselves; that is, we convey the notion of disjunctive things, the differences of which remain unnoticed, and the mode of arrangement of which is disregarded. Two multitudes are called *equal* when with each object of the first there is associated, singly, one of the second and none remain disassociated. That multitude is called *greater* than another which has some objects left disassociated after every object of the second (smaller) multitude is associated singly with some object of the first. The common mark (Merkmal) of all multitudes which are equal to a definite one is expressed by a numeral word. . . . . The natural number is a multitude of units, that is of ones."—OTTO STOLZ, of the University of Innsbruck, in *Vorlesungen über Allgemeine Arithmetik*. 1885, pp. 9, 10.

"In fine, we live. We are surrounded by objects; the idea of comparing these objects, to consider the groups which they form with each other, is natural to man, and has, we are sure, presented itself to him since early times. Attentive observation reveals to us the fact that objects do not exist exactly equal to one another; but, through an operation of the mind which demands no effort, although it involves the entire secret of mathematical abstraction, we consider as alike the bodies which seem to us to resemble each other, and we wave momentarily the examination of the differences which distinguish the one from the other. Hence arises the origin of calculation; the process of *counting* is in general quite simple, even to those who are the least mathematical. Thus, if we count the trees in a park, we know full well that the trees may be of different kind, that they are not the same in shape, that they have not the same age, nor the same number of branches and leaves. Some grains of wheat are placed on a table; we say, here are twenty-five grains of wheat, and if we set ourselves to examine them with a magnifying glass, we perceive that each has its special properties which enable us, if necessary, to distinguish it from all the rest. But by a conventionality that is natural and even instinctive, we have created in the term "tree" or "grain of wheat" an abstraction which is indispensable in the act of counting."—C. A. LAISANT, *La Mathématique, Philosophie-Enseignement.* Paris, 1898, pp. 15-16.

"Primary number is an abstraction from a group of objects which represents their *individual* existence."—A. LEFEVRE, *Number and Its Algebra.* 1896, p. 20.

"The word which stands at the head of this chaper ["number"] contains six letters. In order to find out that there are six, we count them; *n* one, *u* two, *m* three, *b* four, *e* five, *r* six. In this process we have taken the letters one by one, and have put beside them six words which are the first six out of a series of words that we always carry about with us, the names of numbers. After putting these six words one to each of the letters of the word *number*, we found that the last of the words was *six;* and accordingly we called that set of letters by the name six.

"If we counted the letters in the word 'chapter' in the same way, we should find that the last of the numeral words thus used would be *seven;* and, accordingly, we say that there are seven letters. But now

a question arises. Let us suppose that the letters of the word *number* are printed upon separate small pieces of wood belonging to a box of letters; that we put these into a bag and shake them up and bring them out, putting them down in any other order, and then count them again: we shall still find that there are six of them. For example, if they come out in the alphabetical order, *b*, *e*, *m*, *n*, *r*, *u*, and we put to each of these one of the names of numbers that we have before used, we shall still find that the last name will be six. In the assertion that any group of things consists of six things, it is implied that the word six will be the last of the ordinal words used, in whatever order we take up this group of things to count them. That is to say, *the number of any set of things is the same in whatever order we count them.*

"Upon this fact, which we have observed with regard to the particular number six, and which is true of all numbers whatever, the whole of the science of number is based."—W. K. CLIFFORD, *Common Sense of the Exact Sciences.* 1891, pp. 1, 2.

"The oldest calculations were probably achieved by a certain arrangement, either of the objects themselves, which were the subject of calculation, or of other things more easily handled. Pebbles, small shells, may have served as representatives, as they still do at the present time among certain tribes, and these marks . . . , when brought into smaller or larger heaps, arranged in rows, will have facilitated materially the adding together or the division of a collection of objects. As long as only small numbers had to be dealt with, man carried the simplest mode of visualization with him; namely, the fingers of his hands and the toes of his feet. To be sure, he could not thereby advance very far without some new device. Certain tribes of South Africa still show us to-day how friendly coöperation may be used to overcome the difficulty of visualizing larger numbers by using the fingers only. 'In counting beyond one hundred the difficult task must, as a rule, be performed by three men. One of them counts the units on his fingers, by raising one finger after the other and pointing out the object counted or, if possible, touching it. The second man raises a finger (always beginning with the little finger of the left hand and proceeding continuously toward the little finger of the right hand) for every ten, as soon as it is completed. The third man counts the hundreds.' *

"Whatever explanation may be offered for the fixed order of using the fingers, the fact of its existence remains, and in the course of our researches we shall repeatedly encounter this fixed order as the foundation of the so called *finger counting.*" M. CANTOR, of Heidelberg, in *Vorlesungen über Geschichte der Mathematik.* Vol. I, p. 6. 1894.

* SCHRUMPF, in *Zeitschrift d. deutsch. morgenländ. Gesellschaft* XVI, 463.

"But men did not arrive at this use of the fingers till they had already made some little progress in calculation without them. That this is the true history of the art of counting is evident, if we consider the following facts in order:

"*First*, there is hardly any language in the world in which the first three or four numerals bear, on the face of them, any reference to the fingers. *Secondly*, there are many savage languages in which these numerals are obviously taken (not from the fingers, but) from small symmetrical groups of common objects. Thus, 'two' is, among the Chinese, *ny* and *ceul*, which also mean 'ears'; in Thibet, *paksha*, 'wing'; in Hottentot, *t'Koam*, 'hand': and so, also, among the Javanese, Samoyeds, Sioux and other peoples. So, again, with the Abipones, 'four' is *geyenkñaté*, 'ostrich toes'; 'five' is *neenhalèk*, 'a hide spotted with five colours'; with the Marquesans 'four' is *pona*, a 'bunch of four fruits,' etc. *Thirdly*, there are also many savages who, having only a very few low numerals, count to much higher numbers dumbly by means of the fingers.

"But just as, in the examples quoted above, the name of the pattern group (e. g. *ears* or *hands*) becomes the name of the number which that group contains, so with finger-counting, the savage, advancing in intelligence, begins to name the gesture with or without performing it, and this name becomes the symbol of the number which the gesture is meant to indicate. Hence, all the world over, in nearly every language under the sun where names for the higher units exist and show a clear etymology, the word for 'five' means 'hand,' and the other numbers, up to ten or twenty, as the case may be, are merely descriptive of finger-and-toe-counting."—J. Gow, *History of Greek Mathematics*. 1884, pp. 6 and 7.

REMARKS.

(1) From the above citations, copied from representative books of our time, it appears that the mathematicians of the present day are unanimous in describing the earliest notions of number as being free from ratio and measurement. It is worthy of notice, moreover, that the great historians of mathematics are led from archæological and ethnological study to results in agreement with those of the mathematicians.

(2) The only data which must be supplied to the mind through the senses for the cognition of number are the *separateness or distinctness of objects*. For the earliest cogni-

tion of number, it is necessary and sufficient to see, hear or feel things as differentiated one from the other. The child may, at the same time, be conscious of form, of the fact that one body is larger or smaller than another, but *form* and *size* are not *necessary* for the primitive number-concept.

(3.) Counting is the simplest mathematical act. For *measuring*, the senses must supply to the mind more data than are needed in *counting*, and the data must be more accurate. For *counting*, the child only needs to see one object as separate from another; for *measuring*, he must also see one as greater than another. Take two rods, one twice as long as the other. For measurement, the child must not only see them as distinct rods of unequal length, but he must apply the smaller rod to the larger, either in imagination or by actual manipulation. The fact that the smaller rod can be marked off twice on the larger conveys no idea of ratio, unless the child has, beforehand, the primitive number-idea. If this idea is present, then he may recognize the parts of the longer rod as "two," and may obtain the idea of ratio. But if the child has not the number-concept for "two," then it seems impossible for him to acquire an idea of the ratio between the lengths of the rods. The child must know that the length of one rod is *some* number of times the length of the other, before he can find out how many. If the primary concept of number is a prerequisite to any attempt at measurement, then one cannot find the origin of number in measurement.

EXTENSION OF THE NUMBER-CONCEPT. (RATIO, FRACTIONS.)

"The first extension of the concept of number is the identification of the ratio of any two magnitudes of the same kind, and without qualitative distinction for the purposes of the comparison, as a *number*."

"The *measurement* of any magnitude (concrete or abstract) is the process of finding its ratio to another magnitude of the same kind, arbitrarily chosen as a unit. The measure of a magnitude is this ratio—a number."—A. LEFEVRE, *Number and Its Algebra.* 1896, pp. 61, 125.

"Measuring in the ordinary sense — the process which leads to the representation of *continuous* magnitudes as lines or surfaces, in terms of some unit of measure — deserves all the prominence which our authors [McLellan and Dewey] would give it in arithmetic. We do not mean measuring in the exact mathematical sense, of course, but the rough measuring of common life, in which the magnitude measured and the unit are always assumed to be commensurable. Compared with counting, or even addition and multiplication, an operation which involves the use of an arbitrary unit, and the comparison of magnitudes by its aid, is artificial. But this metrical use of number is of immense practical importance, and of great interest to any child mature enough to understand it. No doubt a child may use a twelve-inch rule to advantage when practicing multiplication and division of integers. Certainly, such an aid is almost indispensable in learning fractions. Without it, the fraction is more likely to be a mere symbol to him, without exact meaning of any kind. 'Two-thirds' has a reality for the child who can interpret it as the measure of a line two inches long in terms of a unit three inches long, which it quite lacks for him who can only repeat that it is 'two times the third part of unity.' Mathematicians now define the fraction as the symbolic result of a division which cannot be actually effected, but that definition will not serve the purposes of elementary instruction. It is as certain that the fraction had a metrical origin as it is that the integer had not, and in learning fractions, as in learning integers, the child cannot do better than follow the experience of the race."—PROF. H. B. FINE, in *Science*. N. S. Vol. III, 1896, p. 136.

The most ancient mathematical hand-book known to our time—the Ahmes papyrus, about 2000 B. C., which claims to be founded on much older Egyptian documents—begins with fractions. It was probably written for the advanced mathematicians of its day. The study of this document shows how difficult fractions were to the ancients. Ahmes confines himself to *unit-fractions* having unity for their numerators. If another fractional value was to be considered, it was always expressed as the sum of two or more unit-fractions. Thus: $\frac{2}{21} = \frac{1}{14} + \frac{1}{42}$. And how did the necessity of the introduction of fractions arise? The document contains problems like this, "Divide 2 by 3," and Ahmes solves this by means of his fractions. Thus dividing 5 by 21 gives $\frac{1}{7}\ \frac{1}{14}\ \frac{1}{42}$. That the idea of measurement was predominant in the use of these fractions follows from such problems in Ahmes as

the computation of the average daily produce of fat, when the yearly produce is ten "besha," and from the geometrical examples dealing with areas of land. For instance, a triangular piece of land is computed as having half the area of the corresponding parallelogram.

Apparently, number first suggests itself to the child in connection with discrete things. He sees three distinct objects, and learns to count *one*, *two*, *three*. In measurement (of lengths, for instance) the mind marks off some unit of length (the foot) along the length to be measured (a yard-stick, let us suppose), and thereby comes to imagine the yard as made up of parts; the measured length is thought of as composed of three equal parts; that is, by a mental act the continuous length is pictured as made up of discrete parts of a group of like objects. The original or primitive yard-stick is differentiated by the mind into an artificial group of three equal lengths. A pencil is found to be six inches long; the mind at once pictures a group of six equal lengths, which have become discrete objects of thought. In the counting of separate objects, we, by a process of abstraction, consider the objects as alike; in measuring, by a further mental process, we consider a continuous magnitude as made up of separate like parts.*

In measurement we extend the number concept so that it is applicable not only to things that are discrete or discontinuous, but also to things which are continuous. We can now say that *ratio is a number*, but we are not allowed to say that *number is always ratio*. Number in general is a broader term than ratio.

While in the crude measurements of every day life all magnitudes appear to us as commensurable with one another, mathematical reasoning shows that incommensurability may exist. The keen intellect of the Greeks first detected the fact that the side of a square and its diagonal are incommensurable with each other. Hence in the refined reasoning of the mathematician it is not sufficient for measurement to deal with ratios which are integers or ordinary fractions, but with numbers which are incommensurable to the measuring

*Consult an article by PROF. G. B. HALSTED, in *Science*, N. S., Vol. III, pp. 470, 471, to which I am indebted. See also CLIFFORD, *loc. cit.*, p. 95.

unit. Thus, the connotation of ratio (embracing both the rational and irrational) as number enables us to make number *continuous*, so that. starting with a ratio as small as we please, we can conceive number to increase continuously, passing through all the stages of primary number and all the interspersed stages of irrational number, to a ratio as large as we please.

But the notion of continuity is not as simple as it, at first, appears to be. Few notions in mathematics are more subtle. It is in considerations of this sort that the logical superiority of the reasoning based on numbers is asserting itself. In advanced mathematical research, the greatest rigor of treatment is secured. not by relying on intuition, not by depending on geometrical figures, but by an entire separation from the world of our senses, and making mathematical demonstrations wholly arithmetical. Through reliance on their intuitions, mathematicians have been led to some erroneous results; for instance, that every continuous function must have a derivative at all points in a given interval.

The tendency at the present time is to *arithmetise mathematics*. The earlier explanation of irrational numbers, like that of fractions, involved the idea of measurement. Formerly an irrational number was defined as the expression of the incommensurable ratio of two geometrical quantities — that is, as the ratio between two quantities having *no common measure*. For the purpose of removing certain logical difficulties, G. Cantor, K. Weierstrass, R. Dedekind and others have treated irrational number in a manner free of ratio and measurement and of all geometrical considerations. This arithmetical theory of the irrational is now about one quarter of a century old; but our college text-books contain nothing of these new ideas; the opinion strongly prevails among teachers everywhere that the arithmetical theories of the irrational are not suited for elementary instruction in the differential and integral calculus, or in analysis in general.

To the teacher of elementary arithmetic the chief point of interest of these remarks on higher mathematics lies in the fact that the use of the number concept, which is free of ratio and measurement, is assuming a more and more central position in the rigorously logical exposition of the advanced branches.

REMARKS UPON CLIFFORD'S PROOF OF MIQUEL'S THEOREM.

By F. H. Loud.

The name of Auguste Miquel, on the tongues or pens of geometers of the present day, occurs most frequently in connection with the remarkable theorem* which forms the concluding proposition of the following statement.

Given five lines in a plane, they form ten triangles, whose circumcircles meet by fours in five points, and *these points lie on a circle.*

For convenience of statement, I have combined with Miquel's theorem proper an antecedent truth upon which it is based, relating to four lines. On examination it will be perceived that if we would build up the theorem from its elements we must begin with *two* lines, in the following fashion:

Given *two* lines, they have one point of intersection, P_2.

Given *three* lines, we may leave out one at a time, and thus form three pairs. Each pair has its point of intersection, P_2, and the three points lie on a circle C_3.

Given *four* lines, leaving out one at a time forms four sets of three, each with its circumcircle C_3, and these four circles meet in a point P_4.

Given *five* lines, they form in the same way five sets of four, each determining one point P_4, and these five points lie on a circle C_5.

The interest of Miquel's theorem was much increased when it was shown by W. K. Clifford,† and later, (though, it seems, independently) by S. Kantor,‡ that the series of propositions thus begun continues true in an indefinite prolongation, defining, for every *even* number, $2n$, of arbitrarily

* Liouville's Journal, vol. x, p. 349.

† "Synthetic Proof of Miquel's Theorem," *Mathematical Papers*, p. 38.

‡ "Ueber den Zusammenhang von n beliebigen Geraden in der Ebene," Sitzungsberichte, Wien, 1878, p. 789.

given lines in a plane, a point P_{2n}, and for every odd number a circle C_{2n+1}, with the property that always the point P_{2n} is common to all the $2n$ circles C_{2n-1}, and always the circle C_{2n+1} passes through all the $2n+1$ points P_{2n}.

Clifford's method is indicated by Salmon,* but as the latter author does not enter upon the above-mentioned extension to more lines than five, I take the liberty of summarizing the proof of Clifford, though his original paper, which no one of geometrical tastes should omit to read, is fairly accessible.

An *n-fold parabola* is defined as a curve of class $n+1$, touching the line infinity n times. Such a curve is rational and of order $2n$, and is determined by $2n+2$ finite tangents. It has only one focus; for from the circular point I there can be drawn only one tangent beside the line infinity, and this tangent meets its conjugate from J in the single real point F. If the number of given finite tangents is only $2n+1$, a single infinity of curves can be drawn, and the focus of each is the intersection of a ray from I by the projectively corresponding ray from J; hence the locus of F is a conic through I and J; that is, it is a circle. Among the curves of the above pencil† there are $2n+1$ cases of disintegration, viz , an n-fold parabola may consist of the point in which one of the given lines meets the line infinity, together with the $(n-1)$-fold parabola touching the other $2n$ lines; and its focus is the focus of the latter parabola. Hence the $2n+1$ foci of these special curves lie on the one circle determined by the $2n+1$ given lines; while, when $2n+2$ lines are given, we may from any $2n+1$ of them determine one circle as above, and all these circles will pass through one point, viz., the focus of the n-fold parabola determined by all the lines.

Clifford does not consider, (as do Kantor and P. Serret‡) the condition under which Miquel's circle breaks up into a right line *plus* the line infinity, but a problem essentially similar is treated by Salmon.§ The statement for the general

* *Higher Plane Curves*, p. 128.

† Curves forming a *pencil* are usually understood to have in common a number of *points*, one less than suffices to determine the curve; here, however, and throughout this paper, substitute for common *points* in this definition, common *tangents*.

‡ Comptes Rendus, 189.

§ *Higher Plane Curves*, § 145, p. 127.

case may be shortly made as follows: The circle which is the locus of the foci of n-fold parabolas will break up in this way if the pencil of such parabolas contains one that has the circular points as two of its contacts with the line infinity. For that parabola can have no finite focus. Thus the Miquel circle of five given lines is replaced by a right line if the five lines are all tangents to a curve of class three, order four, touching the line infinity at the circular points. This curve is the hypocycloid of three cusps, by some called the deltoid.

A number of theorems may readily be reached from the suggestion afforded by Clifford's demonstration.

Thus we might discuss pencils of curves having two real foci, one of which is fixed, and seek the locus of the remaining one. A series of theorems would thus be derived where the locus found would in general be a circle; though in the first member of the series, namely, the case of the conic touching two lines and having a fixed focus, the locus is rectilinear.* Passing by such partial, though perfectly valid, applications of Clifford's method, I wish to notice a case which I regard as interesting, in which a true result is suggested rather than proved by an argument framed in imitation of Clifford's.

Let us consider a curve of third class and fourth order, having a point of contact with the line infinity, and also meeting the same in the two circular points. It is rational, and has no inflection, one double tangent, three cusps and no other double point. No tangent which shall touch the curve elsewhere can be drawn through I or J, save the line IJ itself; there is therefore no simple focus analogous to that of the conic parabola, such as any n-fold parabola possesses, but there is one and only one focus, the meeting point of tangents at I and J, and thus of the type exemplified by the center of a circle. The number of conditions imposed by the definition of the curve is four,—two given points, one given line as tangent, and an unspecified double tangent. Five more are required to determine a curve, or

* Salmon, *Conics*, p. 320. Ex. 3.

rather, to reduce the problem of determining it to a finite number of solutions. Let four of these be supplied by assigning, in the finite region, lines which the curve must touch, and a fifth by requiring that the double tangent to the curve meet the line infinity in a specified point, K. Then a set of curves is determined, to each one of which one tangent is drawn at I and one at J, meeting at a definite point. Now let K move along the line infinity. We may regard each curve of the set as undergoing a continuous displacement and distortion; and if the attention be fixed upon one such varying curve, we shall see its focus describing a locus, defined by the intersection of a tangent at I with a conjugate tangent at J. This separation of the tangents at I and J belonging to one curve from those belonging to any other affords an indication that the whole pencil of tangents through I is resolvable into partial pencils. in each of which a ray through I is met by a single ray through J projectively corresponding to it. If this be the case, the locus of each focus is a circle. But when the point K, in its motion along the line infinity, reaches the point at which that line is cut by one of the four given tangents, the curve has four contacts with lines through K, and being of third class only, it must break up into the point K and a curve of second class cutting the line at infinity at I and J, and touching the three remaing given lines. Such a curve, of course, is a circle, and its center is the focus of the disintegrating third-class curve. We have thus the theorem that, when five lines are given, the centers of the circles which touch three of these lines lie by fours upon circles which have a common point; that is, if four given lines be a, b, c, d, the center of a circle touching a, b, c and those of one touching a, b, d, one touching a, c, d and one touching b, c, d lie on a single circle; moreover, if a fifth line be added, and the proper single circle selected for each of the five sets of four lines formed by dropping one line from the given five, these five circles pass through a point.

This theorem is true, as I have elsewhere shown analytically;* and is, moreover, the first term of a series of theorems which would be obtained by replacing the curve of third order in the foregoing argument by curves of higher order precisely according to the analogy of Clifford's demonstration. The argument here given is, however, incomplete as a proof, until it is shown that the pencils of I-tangents and J-tangents break up, as indicated, into separate simple pencils whose rays have a one-to-one projective relation.

To do this, by purely geometric reasoning would probably not be easy; at least it may be expected that the demonstration would acquire a length and cumbersomeness which would render it entirely unlike the elegantly simple argument in which the analogous theorem was established by Clifford.

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* "Sundry Metrical Theorems concerning n Lines in a Plane;" a paper read before the Am. Math. Soc. April 28, 1900, and subsequently published in the *Transactions* of that Society, Vol. I, No. 3.

MAY 1, 1900.

LA FEMME DANS LES CHANSONS DE GESTE.

BY H. A. SMITH.

I.—INTRODUCTION.

Le temps qui est compris dans cette investigation peut se placer, nous croyons, entre 1100 et 1250. Excepté La Chanson de Roland il n'y a guère de chansons de geste qui remontent plus haut que 1100. et si nous donnons aux mots "chansons de geste" le vrai sens, nous n'en avons pas beaucoup après la première moitié du XIII^e siècle. A partir de cette date de 1250 la plupart de ces poèmes écrits en imitation de la vieille chanson de geste sont mieux compris sous le nom de romans d'aventures. Ce n'est pas donc notre intention d'appuyer aucune assertion importante sur un texte evidemment composé depuis cette dernière date. s'il n'est pas clair qu'un tel passage vient d'une époque plus ancienne. Sur ce point il faut peut-être un peu d'explication.

Le temps supposé de la plupart des chansons de geste est beaucoup plus ancien que la date de la composition. Presque toutes qui sont citiés ici prétendent avoir pour temps le règne de Charlemagne ou de son fils Louis. Mais il ne faut pas qu'on s'y trompe. Les écrivains à cette époque ne saivaient pas ce que c'est la couleur locale et sauf la légende il n'y avait pas d'histoire. Les conditions et les moeurs alors sont presque toujours celles du temps de la composition. Mais il y a des exceptions notables qui viennent du fait que les chansons de geste sont le plus souvent basées sur des légendes, ou elles sont tout bonnement des remaniements des versions plus anciennes, et quand une coûtume fait une partie nécessaire de l'histoire. on pourrait s'attendre qu'une telle coûtume se préserverait. Une preuve concluante de ceci se trouve dans le poème Girart de Rossillon. Il y en a deux versions, l'une beaucoup plus ancienne que l'autre. Maintenant tous les faits les plus utiles pour notre sujet. qui

sont cités de la version la plus récente, se trouvent aussi dans la version la plus ancienne.

Peut-être un mot d'explication est aussi nécessaire sur la valeur de nos textes. Il ne faut pas qu'on s'attende à trop grandes choses de ce titre. Les chansons de geste sont des poèmes guerriers par excellence, et naturellement on n'y trouve pas beaucoup de renseignements sur la femme et la vie domestique. Mais par cela même, ce peu doit être d'une valeur plus grande.

Nous avons préparé une petite bibliographie des poèmes, qui aura lieu à la fin.

II.—La Jeune Fille.

Si l'on commença avec la femme comme M. Gautier commence avec le chevalier dans cet ouvrage superbe "La Chevalerie," c'est à dire avec l'enfant dans son berceau, on serait très embarassé à tirer quelque chose sur la petite fille de nos vieilles chansons de geste. Plusieurs de nos chevaliers les plus renommés *parviennent* à avoir des *enfances* dans nos poèmes—cela est l'expression exact parceque *l'enfance* est toujours la dernière chose dans la vie d'un héros—mais il n'y a pas, que je sache, une seule enfance d'une femme. Même, nous croyons que la petite fille n'était pas toujours le hôte le plus bienvenu dans la famille d'un vieux baron. Ce qu'on voulait avant toute autre chose, c'était un fils, un héritier pour bien maintenir le fief et l'honneur de la famille; et chose triste pour notre sujet, c'est ce qui arrive toujours. Il n'y a pas d'exceptions dans les chansons de geste.

Cependant on peut deviner que la vie de la petite fille ne différait pas beaucoup de la vie de ses frères. Elle était baptisée dans la même manière et nous savons aussi, plus tard, qu'ils étaient instruits également dans beaucoup de choses. Mais la petite fille devient intéressante dans les chansons de geste seulement quand elle arrive à l'âge du mariage.

Si nos vieilles chansons de geste nous laissent dans une obscurité regrettable à l'égard de la petite fille, en revanche elles nous disent beaucoup quand elle devient jeune femme.

Mais avant que nous commencions à décrire l'instruction d'une jeune fille, peut-être serait-il bien à faire son portrait. C'est ce qu'on fait toujours dans les poèmes. Voici un passage de Doon de Maience qui décrit la belle Nicolette, une jeune fille qui se fiait, comme toujours, trop volontiers à la bonne foi et à l'amour de l'homme, et qui le suivit trop justement aux dangers et à la mort, en renonçant à ses parents, à ses amis et à sa patrie:

"Une pucelle vit aus sa couche seant,
La plus très bele rien de chest siècle vivant
Vestue d'un samin à terre traïnant.
Les iex ot amoreus et la bouche riant
Le vis lonc et traitis, bien fet et avenant:
Blanche et vermeille fu et de si bel jouvent
Qu'ele n'ot que .XI. ans et .I. mois seulement
Longue fu et gresleite et de bel estement,
Par ses espaulez sunt ses biaus chevaus gesant
Qui plus sunt esmeré que fin or qui resplent."[1]

C'est là un portrait d'une jeune fille carâcteristique. Et il faut le dire que les poètes emploient presque toujours les mêmes mots pour la peindre. Elle est toujours grêle et blanche "comme la neige" ou "comme une fleur," et a des tresses comme "or esmeré," les yeux sont toujours *vairs*, comme dans Raoul de Cambrai: "vairs ot les ex, ce samble toz jors rie."[2] La bouche est petite et les lèvres rouges. Nous ne savons trop pourquoi, mais c'est vrai qu'on ne trouve pas peut-être une seule femme brune dans toutes les chansons de geste.

Le mot *vair*, qui est employé toujours pour décrire les yeux d'une femme. mérite un paragraphe à lui-même. Sa signification la plus ordinaire est de couleurs différentes ou changeantes. Il est employé pour une espèce de fourrure de couleur gris blanc melé.[3] Mais en décrivant les yeux, il semblerait certainement avoir un sens plus précis que changeant ou brillant, parce qu'on n'y emploie jamais un autre mot pour indiquer la couleur. Maintenant comme les femmes sont toujours des blondes on attenderait le plus naturelle-

[1] Doon de Maience .V. 3623.
[2] Raoul de Cambrai .V. 5568.
[3] Voy. Gautier p. 401. La Chevalerie.

ment un oeil bleu, et nous croyons que ceci est son vrai sens. Peut-être une autre cause pour employer un mot qui signifie de couleurs différentes pour les yeux bleus, est que dans un tel oeil nous avons le bleu mêlé de petits points blancs, et si cette explication est la vraie ce semblerait indiquer plutôt un oeil bleu blanc.[1]

Il est digne de remarque que dans les chansons de geste les plus anciennes, on ne décrit pas longuement les beautés physiques des femmes. On emploie là des epithètes homeriques comme "Aëlis au clair visage"[2] et quand on veut donner une idée plus frappante de sa beauté on dit que "Tut le palais de sa bealté resplent."[3] Nous n'avons trouvé qu'une seule femme laide dans les chansons de geste. C'est dans Aymeri de Narbonne, et elle était la femme de cet original Hernaut qui parlait toujours des choses qu'il ne ferait jamais et qu'il faisait tout de suite. Entre autres Hernaut dit, "Que fame rouse n'avroit en son aé,—puis en ot une, ainz lonc terme passé,—Qu'il n'ot plus laide en une grant cité:—D'un pié clochait: s'ot .I. oil avuglé.—Si estoit rouse et il rous par verté."[4]

On entend beaucoup aujourd'hui des femmes qui deviennent plus instruites que les hommes, mais il semble que c'était vrai aussi au XII[e] siècle. Au moins c'est l'idée que les chansons de geste nous donnent et on ne sait aucune raison à douter de leur témoinage. On peut comprendre comment cela se pourrait. Le métier d'un homme au moyen âge était la guerre, et il y avait un si grand nombre de choses à apprendre pour être guerrier et chevalier accompli, qu'on n'avait pas de temps pour apprendre à lire et à écrire, des choses dont après tout le chevalier ne faisait pas grand cas. Mais ces mêmes conditions ne tenaient pas pour la femme, et on trouve le plus souvent dans les descriptions des jeunes filles qu'elles savaient lire et écrire *roman* et quelque fois

[1] Nous trouvons dans le provençal que le mot *vair* se dit du raisin qui se colore, et nous y avons la phrase: "A l'uei vaire comme uno pruno vairo." (Mistral dic. prov.)

[2] Raoul de Cambrai .V. 115: "Aëlis au vis clair," et mille fois dans celui-ci et d'autres poèmes.

[3] Otinel, V. 343, et dans plusieurs autres poèmes.

[4] Aymeri de Narbonne, V. 1551.

latin. Très souvent les poètes vont plus loin même et ajoutent des choses qui sont évidemment des exaggérations. Il sera utile de citer quelques-uns de ces textes. Le poète dit de Flordespine en Gaufrey:

> "Bien sot parler latin et entendre romant
> Bien sot jouer as tables as eschés ensement,
> Et du cours des estoilez, de la lune luisant
> Savait moult plus que fame de chest siècle vivant."[1]

Dans Aiol:

> "Et des cours des estoiles, del remuer
> Del refait de la lune, del rafermer
> De chou par savoit il quant qu'il en est:
> Avisse la ducoise l'en ot moustrée;[2]

et dans Berte aus Grans Piés:

> "Avoit une coustume ens el tiois pays,
> Que tout li grant seignor, li conte et li marchis
> Avoient entour aus gent françoise tous dis,
> Pour aprendre françois lor filles et lor fils."[3]

Mais la plus remarquable est Mirabel, amie d'Aiol:

> Ele sut bien parler de XIIII latins:
> Ele savoit parler et grigois et hermin,
> Flamenc et borgengon et tout le sarrasin,
> Poitevin et gascon, se li vient a plaisir."[4]

Mais ce sont beaucoup de langues même pour une femme.

Il faut se tênir compte des exaggérations, mais il y a ici beaucoup qui est évidemment vrai. Sans aucun doute savait-elle lire et écrire, et peut-être un peu de latin. Au moins elle pouvait comprendre des mots dans son psautier. Elle pouvait calculer et donner les noms des principales constellations. Sur ce dernier point il n'y a guère de doute. Il y a trop grand nombre de passages qui indiquent l'intérêt dans l'astronomie, si l'on peut la nommer ainsi à cette période. Aussi etait-elle un peu médecin, parcequ'elle connaissait des herbes qui donnaient des guérisons merveilleuses.[5]

[1] Gaufrey, V. 1793.

[2] Aiol, V. 268

[3] Berte aus Grans Piés, V. 149.

[4] Aiol, V. 5420.

[5] Gaufrey, V. 3923. La Mort Aymeri de Narbonne, V. 1988, où on fait mention d'un onguent, d'un vin blanc et de phrases réligieuses et du signe de la croix, etc.

Quant aux choses qui sont particulières à la femme, elle savait coudre, filer et broder.[1] Elle taillait les vêtements de la famille. Pour ses amusements elle jouait aux eschecs[2] comme ses frères et aussi comme eux elle savait monter à cheval et chasser avec le faucon. Alors il y avait la danse et le chant. La dernière chose, et ce qu'elle savait le mieux, c'était sa religion. A croire les chansons de geste, il n'y a personne plus dévote que la femme du XII[e] siècle.

Nous croyons que les jeunes filles étaient le plus souvent instruites dans la maison par la mère ou par une institutrice. Mais même à cette époque quelques-unes étaient élevées dans les couvents.[3]

Les devoirs de la jeune fille étaient sans doute essez nombreux. En outre de ceux que nous venons de citer, elle faisait de coûtume beaucoup d'ouvrage de la maison. Avec sa mère elle avait la tenue des lits,[4] et des habits de la famille, et s'il y en avait besoin elle n'hésitait pas d'étendre ses soins jusqu'à l'écurie. Elle aidait à armer et à désarmer son seigneur et les hôtes,[5] et même à les déshabiller et leur donner des bains.[6] A l'égard de ces derniers devoirs il ne faut pas juger des actions semblables par les mêmes règles que celles de notre société. La société au XII[e] siècle était à certains égards plus libre que la nôtre. Mais surtout il

[1] En parlant de Berte devenue contourière, on dit:

"De ce faire en s'enfance avait été aprise
Bien sout tailler et coudre et braïes et chemise.
S'elle sout tel mestier ce ne fut pas mervoille
Quar Augustus Césars fist bien le cas paroille
Il fut vaillans et saiges et regna moult grant pièce:
Mas il n'out oncques file ne cosine ne nièce;
Qu'il ne féist aprendre à quelque mestier faire."
(Girart de Rossillon, V. 2371).

[2] "J'ai une fille qui moult a de biauté;
Des eskiés set a moult grande plenté;
Ainc ne le vi de nul homme mater."
(Huon de Bordeaux, V. 7427).

[3] Dans Raoul de Cambrai une jeune fille d'un baron est brûlée dans un couvent. V. 1493. M. Gautier cite d'autres autorités.

[4] Aiol, V. 7306.

[5] "File" dit il, "ce paien te comant—Donez li armes trestout à son talent—Ces III puceles armerent Otinel" (Otinel, V 344). De tels passages sont assez nombreux dans les poèmes.

[6] ".LX. furent filz de moult haute gent,
Aprester font les bains tot maintenant
Ni a si poure tant ait poi tenement
N'ait sa pucelle devent lui enpresant
Fille de conte ou de prince ansument
Qui bien les servent et font à lor talent"
(Les Enfance Vivien, V 5147).

n'y avait pas de pruderie, et nous croyons que ces choses-là sont, pour la plupart, de bonne foi, quoique il soit possible qu'on en ait fait des abus. Au moins on les trouve dans les textes les plus irrécusables.

Sur plusieurs de ces points il sera utile de donner quelques passages d'Aiol, tirés de la première partie, oeuvre du XII^e siècle, dont nous croyons l'autorité assez établie. C'est où Aiol est reçu dans la maison de sa tante par sa cousine, Luisane, qui ne sait pas qu'il est son parent:

El le prist par l'estrier par grant amor:
"Amis ostés vostre elme, donés le nous,"
La bele Luisane al cors legier
Un escuier commande le sien destrier
Le maistre senescal a apelé:
Se li fist le mengier bien conreer,
Et vint a Marchegai par esgarder,
S'aplanoie ses crins et ses costés:
La Pucele s'en torne al cors gentil
La sist le lit Aiol par grant delit;
Aiol en apela, se li a dit:
"Damoiseus, venés ent huimais dormir"
Par le poin le mena dessi al lit.
Puis le fist descauchier, nu devestir
Et quant il se coucha bien le couvri:
Douchement le *tastone* par endormir.
Mais bele Luisane bien le servi:
Douchement le *tastone* la demoisele:[1]

Ces derniers vers font mention d'une coûtume si singulière qu'on pourrait en douter si les textes le permettaient. Pour faire endormir les hôtes les femmes les *tâtonnent* ou les massent.[2] Il ne semble pas qu'il y avait aucune mauvaies intention dans cet usage, parceque nous y voyons des dames comme Guiborc qui sont audessus de tout soupçon. mais c'était aussi une coûtume très dangereuse, comme on voit dans le cas d'Aiol où Luisane est éprise d'amour et lui fait des avances qu'il repousse.

Un des devoirs de la jeune fille que M. Gautier donne

[1] Aiol, V. 2044.

[2] "Quens Aimeris est ses lis aprestés—en une cambre où molt avait biautés;—toute nuit fu de Guiborc tastones." (Aliscans, V. 4361). C'est quelque fois fait par un homme comme on voit de ce passage dans Li Charrois de Nymes: "Looys, sire, dit Guillaume li bers —moult t'ai servi par nuit *tastoner*." M. Gautier cite aussi uu passage en Girart de Rossillon.

dans "La Chevalerie" était le service de la table. Pour des preuves il donne les memoirs de Sainte-Palaye. C'est l'occupation qu'on attendrait le plus naturellement de la femme, mais nous n'avons pu en trouver des preuves convaincantes dans les chansons de geste. Dans Fierabras nous avons un passage qui dit que "Les pucieles les servent à joie et à bonté—à manger et à boire eurent à grant plenté."[1] Mais dans ce cas comme dans tous les autres qu'on trouve où il est certain qu'une femme a servi à la table les circonstances sont telles qu'il était nécessaire. Ils sont ici dans la prison et il n'y a pas d'autres personnes pour les servir. Ce n'est pas un répas ordinaire.

De l'autre côté il y a mille passages où un jeune homme, un écuyer ou même un chevalier découpe et sert à manger. On sait de plus que c'est toujours une des choses citées pour l'instruction d'un page ou d'un aspirant à la chevalerie, celle d'apprendre à découper et à servir à la table.

Aussi tous les cuisiniers que nous avons pus trouver sont des hommes.[2] Que la châtelaine se chargeait du gouvernement de la cuisine et du menu, on ne peut douter, mais qu'elle et ses filles faisaient l'ouvrage ou qu'elle avait même des servantes pour le faire, on ne peut trouver aucune indication. Le contraire serait indiqué. On sait que c'était du plus vieux temps un honneur à servir à la table d'un roi ou chef, et on voit de cette circonstance comment cette coûtume se préserverait; et c'était au XII^e siècle toujours un service le plus honorable. Mais du fait que les femmes ne cusaient pas, si c'était un fait, nous ne voyons aucune explication suffisante. Si c'est vrai, il y avait après tout de petites choses qui feraient desirer aux femmes d'aujourd'hui de vivre au XII^e siècle.

Avant que nous arrivons à la question du mariage de la jeune fille, il reste encore à parler d'elle comme amante: et c'est ici qu'on voit une grande différence entre les femmes des plus vieilles chansons de geste et celles de la première moitié du XIII^e siècle, ou même de la dernière partie du XII^e. C'est ce qui paraîtra plus tard.

[1] Fierabras, V. 2215. [2] Voy. Aiol, cité audessus.

D'abord nous n'entendons pas dire que les mariages étaient toujours des mariages d'amour, ou même que c'était la condition ordinaire. On peut voir que la nécessité décide très souvent le mariage et que les volontés des partis contractants comptent pour peu de chose. Cependant il est facile à voir dans les poèmes que le mariage d'amour est l'idéal, s'il n'est pas toujours la pratique.

La première question qui se présente est sur quelles qualités est basé l'amour au XII^{e} siècle. Nous croyons qu'il est tout facile à répondre. Un homme aimait une femme pour sa beauté, mais certainement une femme aimait un homme à cause de son courage. On trouve cette idée partout. Qu'un chevalier donne de bons coups, c'est l'essentiel. Par exemple, prenez le cas de la belle Beatrix, qui fut éprise d'amour pour Bernier en Raoul de Cambrai. Si Bernier avait des qualités exceptionnelles plus qu'un autre au commencement, nous n'en savons rien. Mais il aurait eu certainement des petits défauts selon les idées de notre temps. D'abord il était bâtard et avait été l'ennemi acharné de père de Beatrix. Il avait tué le cousin Raoul et les frères de la jeune fille. Aussi ses combats n'avaient pas été tels qui augmentent un bel extérieur. Sans compter ses nombreuses blessures au corps et aux membres, pour commencer Raoul lui avait brisé la tête d'un gros bâton, Gueri avait percé son visage de l'os de la jambe d'un cerf, Gautelet avait ôté une partie généreuse de son cuir chevelu, et pour finir avait découpé son oreille droite avec la chair d'un "demi-pied" de son visage. Certainement des choses comme cela feraient penser deux fois à une belle d'aujourd'hui.

Mais de l'autre coté il était un des plus grands combattants du pays et quand lui et Gueri se reconcilièrent, dès que Beatrix a entendu son nom, elle dit "heureuse la dame qui sera sa mie car il a grand prix de chevalerie," et elle va l'embrasser dans la première entrevue. L'auteur ajoute une phrase qui dit tout en deux mots: "Ci s'entracolent nus n'en doit mervillier,—car ele est *bele* et il *bons chevaliers*."[1]

Voulez-vous un example plus fort! Il n'en manque pas.

[1] Raoul, V. 5666.

Dans le poème, Aiol, le héros prend la jeune fille d'un roi païen et l'emmène captive. Mais, chose pas ordinaire, elle ne veut pas devenir la femme du jeune chevalier, et cherche toujours à échapper. Il y avait un roi païen dont elle était amoureuse. Un jour tandis qu'Aiol fatigué dort sous un arbre elle voit approcher quatre de ses parents qui les poursuivent. Elle est joyeuse, elle va échapper, mais en vrai femme elle ne veut pas laisser égorger un chevalier en sommeil. Elle le secoue et lui dit de s'enfuir. Vous comprendrez sans doute la consequence. Aiol se lève terrible, monte sur son cheval et tue l'oncle, le frère et les deux cousins devant les yeux de la belle captive. Vous croyez qu'elle pleurait de belles larmes. Sans doute, mais voici ses pensées: "Dieu! quel bon chevalier! C'est mon avis s'ils avaient été vingt, il les aurait tués tous:" et elle ajoute: "Sire venés vous ent car je sui vostre drue." C'est une conversion soudaine et par explication le poète nous dit: "Que feme aime tost home qui bien fiert en bataille."[1]

C'est donc l'homme le plus brave qui a toujours la plus belle dame, et cela ne doit nous surprendre comme nous savons, plus tard, au moins au temps des chevaliers errants, que l'homme le plus brave s'était accoutumé d'enforcer ce petit point avec la lance et l'epée. Aussi dans le douzième siècle la plus belle femme était très souvent à celui qui avait la force pour la prendre.

Est-ce que la femme avait un vrai amour pour ce chevalier duquel elle pense principalement au renom? Dans les plus vieux poèmes cela est sans doute. Voici une incident de la chanson de Roland, la plus vieille et la plus belle de tous. Après sept longues années de batailles, l'armée française revena d'Espagne. Tristement, parcequ'ils ont laissé en Roncevaux le plus grand chevalier du monde, Roland, le héros de la France, et avec lui Olivier, tous les douze pairs et vingt mille hommes, la fleur de l'armée. L'empereur vient à sa capitale, "Muntet el palais, est venuz en la sale,—as li venue Alde, une bele dame,—ço dist a l'rei: "U est Rollanz li catanies,—Ki me jurat cume sa per à prendre?"—Carles

[1] Aiol, V, 5597.

en ad e dulur et pesance,—Pluret des oilz, tiret sa barbe blanche:—"Soer. chere amie, d'hume mort me demandes,—jo l'en durrai mult esforciėt escange:—c'est Loewis, mielz ne sai jo qu'en parle:—il est mis filz et si tiendrat mes marches."—Alde respunt: C'est moz mei est estranges,—ne placet Deu ne ses seinz ne ses angles—apres Rollant que jo vive remaigne!"—Pert la culur, chet as piez Carlemagne.—Sempres est morte, Deus ait mercit de l'anme!"[1]

La douleur d'Heluis, la fiancée de Raoul de Cambrai, quand elle le regarde pour la dernière fois sur la bière est presque aussi grande. "Biax dox amis" dist la bele en plorant—"n'avrai signor en trestout mon vivant," voeu qu'elle tiendra toujours.[2]

Malheureusement il faut se tourner de ces beaux exemples au type de la jeune fille que les poètes du XIII[e] ou de la dernière partie du XII[e] siècle nous ont donné. Il faut comparer Florete en Floovant avec Aude pour voir combien sont plus bas les sentiments. Florete a été l'amie de Floovant mais il avait donné sa parole à Maugalie, et Florete se plaint à son père en bons termes. "Elle n'aura jamais mari en trestot son vivant" sauf Floovant. Mais ce dernier s'avance au roi, allègue son engagement avec Maugalie et demande la main de sa fille pour son écuyer. Le roi y consent et demande à Florete son avis. "Sire," dit la pucelle, "je le voul ausimant;—Quant autre ne pout estre, à Richier me commant."[3] C'est vrai que Richard n'était pas moins courageux que son maitre!

Dans Elie de Saint Gilles on vois même scène repété dans tous les détails où Rosamonde la liberatrice d'Elie se console sans beaucoup de peine et immédiatement demande Galopin son écuyer;[4] et on pourrait répéter à plaisir des passages pareils. Quelle différence entre ces réponses et celle de la belle Aude! On voit qu'il n'était plus le mari voulu mais un mari quelquonque.

[1] La Chanson de Roland, V. 3705.

[2] Raoul de Cambrai, V. 3713.

[3] Floovent, V. 2348.

[4] Elie de Saint-Gilles, V. 2689.

Cependant si la légèreté était la seule faute qu'on pourrait reprocher à la classe des héroïnes créée à ce temps on serait bien heureux. Mais les poètes n'hésitent point à les peindre d'une effronterie et d'une impudicité horribles. Il y a une ressemblance bien forte entre toutes ces jeunes filles. et l'on peut bien croire qu'elles étaient copiées après quelque mauvais type qui jouissait d'une grande célébrité. Elles sont on ne peut plus déshonnêtes et font toujours des avances à leurs amants. Elles n'hésitent point à employer tout moyen pour arriver à leur but. Floripas en Fierabras s'efforce à faire tuer son père parcequ'il ne se convertit pas, et sa méchanceté est tout-à-fait incroyable.[1] Il y a une foule d'autres presque aussi méchantes. Sans doute les poètes ont dit beaucoup de mensonges dans ces poèmes; mais s'il y a des exaggérations là-dessus, nous croyons qu'il y en a aussi des raisons. Nous y reviendrons plus tard.

III.—Le Mariage.

Dans un des vieux poèmes, l'auteur se fache beaucoup contre les mariages prématurés. Ses mots sont dignes d'être cités:

> "Baron a icel tant dont vous m'oés conter
> Nus hom ne prendrait feme, s'avoit .XXX. ans passé
> Et la pucele encontre aussi de bel aé;
> Mais puis est avarisse et luxure montés,
> On fait mais .II. enfens de .XII. ans asambler."[2]

On est souvent en peine à dire dans les passages semblables si l'auteur parle rèelement d'un fait ou si ce n'est pas tout bonnement une expression du régret, qu'on trouve partout, pour "le bon vieux temps," même quand "le bon vieux temps" est le plus mauvais. Mais nous croyons qu'il dit ici, avec un peu d'exaggération, une vérité historique. La date du passage est à peu près 1200, et à ce temps sous l'influence du système féodal beaucoup de mariages étaient sans doute de très bonne heure. L'amante de Doon de

[1] Fierabras, V. 5955.

[2] Aiol V. 1696.

Maience, Nicolette "n'ot que XI. ans et .I. mois seulement."[1] Flordespine en Gaufrey avait ".XIII. ans et demi"[2] L'héroine de Parise la Duchesse n'avait que XV ans quand elle fut chassée de son pays et elle était mariée peut-être une année.[3] Il y a une foule d'autres cas où la femme est aussi jeune.

On sait qu'il était à l'intérêt du roi ou du seigneur féodal au moyen âge, quand un de ses barons mourut et laissa une jeune fille heritière, de la donner en mariage aussitôt que possible, pour obtenir un homme pour défendre le fief et pour ses guerres. Et c'est cette circonstance qui faisait un si grand nombre de mariages prématurés.

Mais avant le XII[e] siècle le fief n'était pas héréditaire. Nous croyons même voir qu'il ne l'était pas toujours au XII[e] siècle. Alors il se pourrait très bien qu'avant ce siècle les mariages n'étaient pas de si bonne heure. De plus, un examen soigneux des textes les plus vieux, comme Roland et Aliscans, appuie cette idée. Le rôle des femmes dans ces poèmes est plutôt le rôle d'une femme d'un âge mûr, que d'une jeune fille de douze ans. C'est vrai que les femmes à cette époque étaient de bonne vigoureuses et cet âge de douze ans n'était pas du tout aussi jeune qu'il est à présent.

Nous avons déjà vu que dès que le fief devint héréditaire on arrivait à se marier très jeune. Cela évoque la question de qui faisait le mariage; et en théorie au moins, si non toujours dans la pratique, c'est facile à répondre. C'était le roi ou le seigneur féodal. La femme et le fief étaient inséparables et c'était tout naturel que le seigneur eût ce pouvoir. C'était son moyen de payer ses chevaliers comme on le voit dans tous les poèmes. Le passage où Charlemagne dit qu'il donnerait un autre mari à Aude a été déjà cité, et dans le même poème l'émir incite les paiens à la bataille par la promesse de belles femmes et des fiefs.[4] Enfin toute cette

[1] Doon de Maience V. 3630.
[2] Gaufrey V. 1797.
[3] Parise la Duchesse.
[4] Roland V. 3397.

histoire de batailles terribles et de brutalité en Raoul de Cambrai se base sur ce droit du roi. Ce n'était pas seulement un droit mais c'était aussi le devoir du roi de donner des femmes à ses chevaliers et des maris aux jeunes filles héritières, et on voit ces dernières qui viennent à la cour "pour demander mari." C'est ainsi que la belle Ayglentine de Gui de Nanteuil est venue "pour querre mariage" et quand l'empereur allait la donner à un homme qu'elle ne voulait pas—Il eut reçu un petit cadeau de mille "mars d'or. Comment les historiens se sont sottement trompés en Charlemagne!—il s'éleva une guerre pour sa possession. Cependent il n'y a guère de doute que de bonne heure dans le XIII^e siècle ce droit n'était pas aussi rigoureusement enforcé, et peut-être ce vers trouvé dans le passage que nous venons de citer était à peu près exact: "A moillier la prendra si le roi le *consent*."[1] C'est évident que le consentement du roi était toujours supposé nécessaire mais comme ici il ne pouvait très souvent l'enforcer. Sur cette question du mariage la femme avait beaucoup de fois de petites préférences et quelquefois elle savait les obtenir.[2] Mais il y a plusieurs exemples où la femme était forcée à se marier à un homme qu'elle haïssait.[3]

Très souvent sans doute pouvait-elle se soustraire de la dure nécessité d'un tel mariage, mais il semble que ce ne serait ordinairement qu'avec la perte de son héritage. Il fut ainsi qu'Ailis était déshéritée en Raoul de Cambrai. Le droit du roi est évident. Il parle d'Ailis qu'il voulait donner à un chevalier: "Et s'ele i faut. trèstot par son outraige—S'irai saisir la terre et l'eritaige."[4] Comme on pourrait croire c'était la cause la plus fréquente des guerres féodales comme c'était aussi un moyen de confirmer la paix.

[1] Gui de Nanteuil V. 586.

[2] La demoiselle a par la main saisi,—Gui dist le rois, ceste aurez en baillie;—Lors elle respondi com fame courroucie:—"Par ma foi, Gui, voz pensez grant folie,—Qu ja n'aurez de moi seignorie,"—Dist l'empereres: "voz ditez grant folie,—Qu'il voz convient faire ma commandie"—Mais uns proverbes noz aprent et chastie:—Engiens de fame maint saige home cunchie." C'était cela qui arriva ici et ce n'était pas le seul exemple. (Gaydon V. 8569).

[3] Raoul de Cambrai V. 6836.

[4] Daurel et Beton V. 644. Raoul de Cambrai V. 151.

En dehors des conditions citées il faut de plus que les partis ne soient pas des parents, ni des parents du sang ni des parents spirituels,[1] parcequ'il semble que les parrains du baptême contractèrent une parenté qui interdisait au mariage de même qu'une parenté du sang.

Dans cette connection il faut mentionner qu'un grand nombre de mariages était entre des barons chretiens et des princesses païennes, et quelque fois entre une chretienne et un païen. Il va sans dire que dans ces cas les païens s'étaient convertis et baptisés. L'idée qu'il pouvait être une union entre deux partis de réligions différentes n'entra jamais dans la tête d'un homme du moyen âge. Quand une femme fut baptisee il était la coûtume de changer son nom. Dans la Prise d'Orenge le nom d'Orable est changé en Guiborc.[2] Au baptême, la femme était dévêtue et plongée dans les fonts. Elle avait des parrains et des marraines comme un enfant.[3]

Selon les idées de l'église il fallait le libre consentement des partis contractants pour faire mariage légal, mais sous un régne de la force les officiaux de l'église étaient très souvent forcés à se plier,[4] et il y avait des mariages très contre la volonté de la femme. Mais au moins ce n'était pas l'idéal.[5] Enfin comme concession à la femme, on peut ajouter que les partis seront à peu près de même âge. Il y a beaucoup de passages ou la femme exprime son aversion à un vieux homme.[6]

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[1] "Que je suis ses cousins moi ne peut avoir mie" (Aiol V. 8120). Dans Elie de Saint-Gilles après Rosamonde eut sauvé Elie, et il était sur point de finir leurs amours en l'épousant il apprit que le mariage était impossible parceque lui et sa famille eurent été de ses parrains au baptême. "Signor" che dist Elies à m'amie m'espousés"—"vasal" dist archevesque, "de folie parlés"—che ne poroit soffrir sainte crestientés,—voyant vos ieus trestous l'as aidiet a lever—et es saintismes fous beneir et sacrer" (Elie de Saint-Gilles V 2672).

[2] "Il la baptisent en l'enor Dame dé:—Li nom li otent de la paieneté—A nostre loi la font Guibor nomer (Prise d'Orenge V. 1868).

[3] "Adonc s'est desvestue la bele o le chief blon:—En la cuve l'ont mise li nobile baron.—L'archevesque Turpin commenche une lechon:—la bele batisa u nom Saint Symeon,—Garins fu son parrain et Do et Salemon-Onques au baptizier son nom n'y canja on—Que ne le vout souffrir Berart le gentis hom" (Gaufrey V. 9146). Otinel V. 626. Fierabras V. 5998.

[4] Raoul de Cambrai V. 6455.

[5] Mes une chose sachieg qu'est veritez—molt est li hom fox et musarz provez.—Qui feme prant outre-ses volentez." (Aymeri de Narbonne V. 2392).

[6] "Car il est vieux, s'a la barbe florie-ne le prendroie por a perdre la vie." (Aymeri de Narbonne V. 2474). " N'ai cure de viellart qui le pel ait froncie" Elie de Saint-Gilles V. 1735. Prise d'Orenge V. 628.

Il reste encore à décrire la façon d'un mariage au XII^e siècle. Un bon example est le mariage d'Hermenjart dans Aymeri de Narbonne dont il n'y a aucun point important qui ne soit pas appuyé par nos meilleurs poèmes.

Aymeri envoya une ambassade des plus hauts barons de son pays au roi, Boniface, frère d'Hermenjart. Les ambassadeurs firent la demande de la main de la soeur en lui promettant un riche douaire et en judicieusement menaçant le frère de la guerre s'il refusa. Le roi acceda volontiers et demanda l'avis de sa soeur. Bien qu'elle ne désirait mieux elle dit que son amant devrait venir la prendre. Ce fut là une modestie qu'on ne voit pas chez beaucoup d'héroïnes.

Aymeri vint et ils furent fiancés. Les fiançailles étaient tout bonnement une promesse faite entre les deux partis de se prendre comme mari et épouse. Le plus souvent c'était en forme d'un serment fait sur les reliques des saints ou sacré par un prêtre.[1] Quelquefois le père et l'amant s'engageaient mutuellement, l'un à donner et l'autre à prendre la femme comme épouse.[2]

M. Gautier fait une différence ici et nomme ce dernier cas une *fois* tandis que les fiançailles sont le libre engagement des amants où entre la volonté de la femme. Mais le plus souvent les deux étaient le même, et où le serment était seulement entre les amants c'était presque toujours où la femme n'avait pas de parents présents. Ordinairement le roi ou parent prenait la jeune fille par la main et la donnait au chevalier;[3] et quelque fois on ajoute que le prêtre le sacra, ou qu'ils "s'entrejurèrent." C'est possible que les fiançailles étaient souvent faites dans l'église,[4] mais il faut garder de les

[1] Turpins li archevesque, à la chiere membrie
A demandé Berart si la dame li grée.
Oïl, chen dist Berart, de cuer et de pensée.
Et vous? dist archevesque douche dame senée.
Oïl, dist Flordespine, bien me plest et agrée.
Adonques la li a l'archevesque affiée.
(Gaufrey V. 7170).

[2] "Sor une table font les sains aporter; Ileuques font les sairement jurer. Bernier del prendre et Gueri de donner" (Raoul de Cambrai V. 5838).

[3] (Aymeri de Narbonne V. 3404).

[4] "Sire R. dist la franche pucele,—vos me jurastes dedens une chapele." (Raoul de Cambrai V. 3683).

confondre avec le mariage. C'était tout bonnement un engagement publique, et il n'y avait pas de temps fixe de là jusqu'au mariage.

On parle quelque part de la coûtume de donner un anneau aux fiançailles, mais nous n'avons pu en trouver des preuves suffisantes que c'était un usage commun. En Gaydon cité pour appuyer ce point, ce fut envoyé par la dame au amant pour le reconnaitre,[1] et en Girart de Rossillon où la reine eut donné un anneau à Girart, ce n'était qu'un témoinage d'amitié. Dans les deux ou trois instances où il était réelement donné comme gage d'amour on ne voit aucune raison à le croire une coûtume générale plus qu'un autre cadeau dont nous avons beaucoup d'exemples; et nous croyons que l'usage générale était plus tard. Car s'il avait été ordinaire au XII^e siècle rien n'aurait fait une plus grande impression sur les esprits des poètes, comme on voit à l'égard de l'anneau du mariage.

Les mariages étaient presque toujours dans l'église et l'heure ordinaire était le matin. On allait à l'église à cheval avec un cortège nombreux.[2] Les femmes montaient des mules qui étaient presque universellement la monture de la dame.[3] Elles montaient une selle de femme comme aujourd'hui et on parle souvent de sa magnificence.[4]

Dans l'église le mariage était sacré par un prêtre et le mari donnait un anneau à la femme. Ce semble avoir été une partie frappante de la cérémonie puisque la phrase "dame épousée de l'anneau" est un lieu commun.

Après le mariage il y avait une messe et le parti retournait au château. C'était pendant cette rentrée de l'église que les jongleurs, toujours si nombreux aux mariages—des gens rusés qui savaient cueillir les "deniers"—jouaient

[1] Gaydon V. 8668.

[2] Pour tous ces faits: Aymeri de Narbonne V. 4420. Raoul de Cambrai V. 6070. Aye d'Avignon V. 4100.

[3] "Une moult riche mule li ont apparellie—La sele fu d'ivoire, s'est à or entaillie" (Gaufrey V. 2021).

[4] "Ele met le jamble outre par grant nobilité; le piet met es estriers, esperous ot dorés" (Aiol V. 7483). Sans doute ordinairement la femme portait une robe différente pour monter à cheval: "Jehennete et Martine out lor dame levée—come pour chevaucher l'ont bien atournée" (Gui de Nanteuil V. 1563).

leur plus belle musique. C'était alors que le jeune mari devenait si liberal et donnait quelquefois son cheval ou son manteau au chanteur.[1]

Pour les grandes fêtes après le mariage qui duraient de huit jusqu' à quinze jours on avait la coûtume de dresser des tables au milieu de la plaine. Il va sans dire qu'il y avait des joûtes, des danses, des chants et beaucoup de réjouissances. La dernière scène était très belle. C'était où le prétre venait bénir le lit des nouveaux mariés.[2] Alors les femmes couchaient la jeune dame[3] et la journée était finie.

Il n'y a guère des mesalliances dans les chansons de geste. Le cas de Bernier qui était bâtard et qui se maria à la fille d'un comte a été déjà cité.[4] Aussi quand Aiol allait chercher fortune à la cour de Louis un baron voula lui donner sa fille qu'il refusa parcequ'il était pauvre.[5]

Un exemple plus parfait est dans Hugues Capet. Le héros était surnommé le boucher, parceque c'était le métier de sa famille. Le poème est une histoire des exploits merveilleux par lesquels un homme de si basse famille pouvait arriver à se marier avec l'héritière du trône de la France.[6] Mais ce poème fut écrit vers le milieu du XIV[e] siècle quand

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[1] Raoul de Cambrai V. 6089.

[2] L'evesque va l'estole à son col afubler—Lor lit vint beneir le soir après souper" (Aye d'Avignon V. 4115).

[3] Passe Rose couchièrent les dames du roion." (Gaufrey V. 7115).

[4] "Dist B. vos savez bien que je sui de bas lin,
Trop est haus hom li riches sors Gueri.
D'avoir sa fille n'iert ja par moi requis."
(Raoul de Cambrai V 5704).

[5] "Sire che dist aiols, onques mais n'oi tel:
Ja me douge forment que vos ne me gabes.
Je n'ai en nule ters ne chastel ne chité,
Trop povre mariage avés or esgardé."
(Aiol V. 1781).

[6] Dans Hugues Capet il y a un point utéressant à l'egard des jeunes filles héritières. Hugue Capet épousa l'héritière du trône et devint roi. L'auteur dit qu'après son avènement un parlement de tous les barons décida que dorènavant seulement les héritiers mâles pouvaient prendre la couronne. Il y a un air historique dans le passage.

Les éditeurs ont établi que ce poème fut écrit après 1312, et on trouve dans l'histoire qu'il y avait un états généraux en 1316 qui donna un tel arrêt et que pendant les quatorze années suivantes les femmes héritières étaient excluées trois fois de la couronne de France, les premières examples où la loi s'appliquait.

Il semblerait très probable alors que le passage cité fut fondé sur l'histoire et que le poème fut écrit après 1316 et puet-être plu sieurs ans plus tard.

la condition de la bourgeoisie était beaucoup plus élevée. Un pareil sentiment aurait été impossible au XII^e siècle où on voit un tel mépris du petit peuple que les poètes avaient toujours soin de faire une naissance noble pour les héros vulgaires comme Renouart en Aliscans. Nous n'avons pas trouvé un seul texte honorable où un homme noble se maria à une femme de basse naissance.

(To be concluded in Vol. X of Colorado College Studies.)

A STUDY OF SOME TELEOSTS FROM THE RUSSELL SUBSTAGE OF THE PLATTE CRETACEOUS SERIES.

By F. W. Cragin.

Science owes to the Reverend Mr. H. C. Bradbury the careful preservation of a specimen of *Syllæmus latifrons* from the Cretaceous of Kansas: and the writer is especially indebted to him for the use of it in paleontological research. It is but justice to Mr. Bradbury to say that it was submitted for study long ago, at a time when all that was known of this interesting fish-form was comprised in Cope's description and figures of the type-specimen, and that preliminary studies and drawings of Mr. Bradbury's specimen were made by the writer some years ago, but were not then published. Important supplementary knowledge has quite recently been added by Mr. Alban Stewart in his paper on Kansas Cretaceous Teleosts, in Volume VI. of the University Geological Survey of Kansas. As the Bradbury specimen throws still further light on both genus and species, as well as confirms in most respects Cope's and Stewart's studies, affording together with these a fairly complete knowledge of this fish, the writer here presents revised generic and specific diagnoses in which the results of his own and renewed studies are incorporated with the data of the authors named.

Family SYLLÆMIDÆ.

Type-genus, *Syllæmus* Cope.

As regards the systematic position of the genus Syllæmus, Prof. Cope rightly considered it allied to the Mugilidæ. Dr. Zittel* placed it in the "Familie. Mugiliformes. Harder"; but Mugiliformes is a group of higher than family rank and practically equivalent to the suborder Percesoces, since it

* Handbuch der Palæontologie, Vol. III, p. 312.

was made to include the type-genera of the families Sphyrænidæ, Mugilidæ and Atherinidæ.

In studying the Cretaceous teleosts in the collection of the University of Kansas, Mr. Alban Stewart* has recently treated Syllæmus under the Mugilidæ, but adds that its position in that family is very doubtful. The present writer believes that Syllæmus, though presenting relationships to the Mugilidæ, can not be referred to that family, and should be made the type of a distinct family, Syllæmidæ, differing from the Mugilidæ and resembling the Sphyrænidæ in having the body elongate and subcylindrical and the head long, pointed and pike-like, and in the presence of a lateral line, the latter extending along the middle of the sides; differing from the Mugilidæ and resembling the Atherinidæ in having the vertebræ considerably more than 24; differing from the usual condition in the suborder Percesoces by the subinferior position of the pectoral fins, and from that in the order Acanthopteri by the relatively posterior abdominal position of the ventrals, and apparently by having the anterior part only of the mouth border formed by the premaxillary.

Genus SYLLÆMUS.

Syllæmus Cope. Report U. S. Geological Survey of the Territories, Vol. II, p. 180: 1875. Stewart, University Geological Survey of Kansas, Vol. VI, p. 383.

Type, *Syllæmus latifrons* Cope.

Body subcylindrical or fusiform, not compressed; skull depressed, flattish-convex above from right to left, broad across the occipital region, in advance of which it is somewhat contracted and produced to form a bill-like muzzle, tapering to a narrow, truncate extremity; cranial bones, for the most part, of subtriangular outlines as seen from above, those on the occipital region short, those of the muzzle elongate; inferior side of head contracted, the coracoid bones forming a keel and the lower borders of the dentaries and also those of the large opercular bones meeting at the inferomedian line; mouth-cleft long, extending to two-thirds of the

* University Geological Survey of Kansas, Vol. VI, p. 383.

distance from tip of muzzle to occiput; maxillaries apparently toothless; premaxillaries armed with very small conical teeth; orbits large; vertebræ about 36 in number; dorsal fin having a long basis, consisting of an anterior triangular, more elevated, closely rayed lobe, which is at least for the most part soft-rayed, having not more than three or four of the anterior rays simple and regardable as slender spines, and of a posterior less elevated lobe in which the rays are few, widely interspaced, and much shorter and slenderer than those of the anterior lobe, the anterior and posterior lobes being connected by a portion in which the rays are obsolescent; pectoral fins subinferior, consisting of numerous small rays; pelvic fins abdominal, placed posterior to the front lobe of the dorsal; anal fin with short basis, its rays articulated and distally dissected, except the first, which is simple; caudal fin deeply forked; scales cycloid; lateral line extending along the middle of the sides.

SYLLÆMUS LATIFRONS Cope.

Plate I, figs. 1–6.

Syllæmus latifrons Cope. Report (Hayden) U. S. Geological Survey of the Territories, Vol. II, pp. 181 and 273; 1875. Report (Wheeler) U. S. Geographical Surveys West of the One Hundredth Meridian, Vol. IV, Pt. II, p. 27, Plate XXIII, figs. 1, 1a; 1877. Stewart, University Geological Survey of Kansas, Vol. VI, p. 384, Plate LXXII, fig. 2; 1900.

Body relatively stout in the anterior and middle region, broadest a little in advance of the middle. The rostrum, viewed from above, has the form of an elevated, apically truncated isosceles triangle. As to length of basis, the dorsal fin equals at least nearly a fourth of the total length of the fish, and has the posterior rayed lobe about equal to the anterior, the subrayless interval being somewhat less extended than either lobe. There are about 13 or 14 rays in the anterior and at least 7 in the posterior lobe, those of the latter lobe being placed (in the Lincoln county specimen) at intervals of about 5 mm. All but the anterior three or four of the rays of the anterior lobe are distally dissected, and all except the rudimentary first are cleft at base, each embracing

the posterior part of the ray in front of it by the basal cleft. Pectoral fin composed of 15 to 20 small rays which are strongly bent at the proximal ends. Caudal fin strongly and deeply forked, though much less widely so than in either of the species of Pelecorapis herein described, the lobes very long and narrow, the lower a little longer than the upper. The scales are large and moderately thick, subrhomboidal, their vertical extent about twice the longitudinal, smooth or exhibiting only delicate concentric line-sculpture, and are arranged in 13 longitudinal rows on either side, the lateral line occupying the eighth row below the dorsal fin. The intermediate verterbræ are nearly as deep as long.

Measurements.—Those of the specimen studied by the writer are: total length about 370; length of skull, measured along median line to posterior limit of roof, 64; length of head to posterior limits of opercular apparatus 82; tip of snout to caudal peduncle 285; tip of snout to anterior limit of dorsal basis 123; same to posterior preserved limit of dorsal 220; same to anterior insertion of anal fin about 250; length of longest ray of anterior lobe of dorsal about 34; greatest breadth of base of dorsal between exteriors of the right and left integumental grooves (being that of the anterior lobe) 4.5; average length of verterbræ in mid-region about 7; height of scales on middle-posterior region about 9 mm.

Occurrence.—The example studied by the writer is from the Benton stage of the Platte series, in the "Fencepost limestone" of the Russell substage, near Lincoln Center, Kansas. Professor Cope's example was given him as hailing from the "Summit of Pike's Peak"; but as that is granite, the Professor deemed the specimen more likely to have been derived "from the Cretaceous or possibly Jurassic beds" at the foot of the peak. Later, he ascribed it to the Niobrara or Fort Pierre ("Cretaceous No. 3 or 4, of Colorado"); and still later, he cites an early, verbally expressed opinion of his, that it was "probably of the Niobrara", adding that "more full information leads to the belief that it was obtained from some point in New Mexico."*

* See above cited Report of Hayden Survey, pp. 181, 182 and 273; and that of the Wheeler Survey, p. 28.

In his table of stratigraphic range of genera, page 386 of University Survey VI, Mr. Stewart, apparently intending to use one of the above cited statements of Prof. Cope, and inadvertently substituting *and* for *or* in the latter's expression "Cretaceous No. 3 or 4", gives the range of Syllæmus as Niobrara and Fort Pierre; but on page 384 he states that the two specimens in the Kansas University collection are "from the Fort Benton Cretaceous, the exact locality of which is unknown". There seems to be no satisfactory evidence that *Syllæmus latifrons* has hitherto been found except in the Benton stage.

Remarks.—All of the above stated characters that have not been previously recorded by Cope or Stewart (also not a few of those that have) are indicated by the Bradbury specimen. The latter, though the torsus is somewhat flattened by pressure, so as to appear considerably wider than deep, is in a condition of preservation exceptionally favorable to showing a large number of characters. The head and the caudal fin are preserved practically entire. The greater part of the inferior surface, all of the left side, and half (at the front, all) of the superior surface are exposed, though the scales and lateral line are indicated only by their imprints, and these well defined on only a part of the posterior region, several scale-prints also appearing at one point on the nape. The nature of the dorsal fin is shown by the double insertions and anteriorly, some of the cleft basal remnants of the rays and, outside of these, by the basal groove for the insertion of the integument, this groove making the complete circuit of the fin and demonstrating the continuous character of the latter. The matrix over this fin is broken vertically and in such a manner as to display an imprint-elevation of the fore-lobe, showing that all but three or four of the anterior rays were flattened and dissected soft rays. Along the interval separating the two lobes, the rays appear at first glance to be lacking, but were probably present, at least as basal rudiments, as is indicated by a number of faintly impressed insertions, intervalled about as in the posterior lobe. The pectoral and pelvic fins have been broken away. An

imprint of the middle and distal parts of five anterior rays of the anal fin, indicates that the spine as well as the rays following it were articulated, and preserves distally some of the fibres of four post-spinal dissected rays. The vertebræ have fallen to the ventral surface, the corrosion of which has displayed their approximate form and size in molds. The posterior borders of the scale-prints usually present one or more shallow, rounded emarginations.

On Plate I is given a diagram of the osteology of the skull, based on the Bradbury specimen. Both the arrangement of the bones and their morphological interpretation as indicated in this diagram, are to be regarded as provisional. The more or less weathered-off and broken or scaled-off condition of the cranial bones in the Bradbury specimen, is such that it is impossible to be sure that the arrangement indicated in the diagram is entirely correct; but it is believed to be at least nearly so. The position of the posterior part of the parieto-occipital suture is not satisfactorily shown on the specimen, but the surfaces of stone which were apparently covered with the parietals are feebly elevated, and the faintly defined inner-posterior boundaries of these slightly raised areas are indicated in the diagram by dotted lines, as probable approximate limits of the parietals. The bone called parethmoid should possibly be called the prefrontal. On the other hand, a prefrontal may perhaps be included as the anterior part of the bone called sphenotic. The latter bone presents uncertain indications of being crossed by a jagged suture just anterior to the position represented on the diagram by the letters *sph.* If this suture is really present, the posterior and anterior bones into which the one here provisionally called sphenotic would so be divided, may be respectively either sphenotic and postfrontal or postfrontal and prefrontal.

Family PELECORAPIDÆ.

Type-genus, *Pelecorapis* Cope.

Teleosts of a generalized type, combining characters of the families Exocœtidæ, Clupeidæ, Albulidæ, etc.; gill-arches

5; pelvic bones distinct, presenting considerable resemblance to those of Exocœtus, plate-like, in part concave below, bearing an anterior spine-like process; fins large, without rigid spines (unless in case of the dorsal, which is doubtful), subtended by large bony scales or scutes; the dorsal fin with long basis, placed midway of the length of the body, wholly anterior to the pelvic fin; pectoral fin subinferior; pelvic fin abdominal and very posterior; caudal forked; scales ctenoid; lateral line placed high on the sides in the anterior region when present.

Genus PELECORAPIS.

Pelecorapis Cope. Report (Hayden) U. S. Geological Survey of the Territories, Vol. II, p. 182. 1875.

Form compressed and elongate; head naked, orbit large, jaws toothed throughout, the lower jaw projecting beyond the upper; maxillary distinct from premaxillary, forming lateral margin of upper jaw; premaxillary short; pelvic bones consisting of a pair of irregular plates in contact at the median plane chiefly by the long edges of thin wing-like expansions; anterior process of either bone long, tapering, acute, the two of the opposite bones convergent but not meeting; bases of median fins subtended on either side (and in front?) by a single row, those of the paired fins subtended at the lower side by a patch or two- to three-fold row of imbricated bony scutes; anterior ray of fins (including anal?) simple and, in case of the pectoral and pelvic at least, articulated; dorsal fin with moderately long basis, attaining a moderate height in the anterior part and considerably diminished in the posterior, placed wholly anterior to the pelvic fin; the latter situated far back on the abdomen, remote from the large and long, inferiorly placed pectoral; pectoral fin intermediate in length between that of Exocœtus and that of Albula; scales small, in numerous longitudinal rows; lateral line high on anterior region, apparently partial, (sometimes wanting?); vertebræ of the posterior to middle trunk region large, those of the anterior region becoming smaller and much shortened.

PELECORAPIS VARIUS Cope.

Plate II, figs. 1 and 2.

Pelecorapis varius Cope. Report (Hayden) U. S. Geological Survey of the Territories, Vol. II, p. 182. 1875.

Body rather elongate, tapered-elliptic, compressed, deepest in the front-middle region near and in front of the dorsal fin and tapering thence slowly backward to caudal peduncle; head about $4\frac{1}{3}$ in total length; muzzle not elongate; mouth-cleft reaching to about opposite posterior border of orbit; premaxillary extending backward a little more than half of the distance from tip of snout to below anterior border of orbit, apparently articulated so as to be movable with reference to maxillary; premaxillaries and anterior portion of dentaries armed with moderately large, recurved, intervalled teeth, maxillaries with small and narrowly interspaced teeth; clavicle stout; pelvis of two broad, irregular, lamelliform bones, either of which presents toward the median plane, along which their edges are in contact, a large thin, longitudinally oblong, inferiorly concave wing, or scoop-shaped portion, thickened posteriorly, the portion exterior to which is subquadrangular, thicker, and in part transversely convex on its under surface. The scoop-like portion extends considerably further backward than the quadrangular, and from the angle formed by the outer border of the former and the posterior border of the latter a strong, anteroposteriorly trending articular process springs rather abruptly downward. A portion of the bone, chiefly of the thicker quadrangular portion, is produced far forward as a sharp splinter-like process, or spine, the inferior surface of which is traversed near its inner margin by a shallow groove. The spine is directed as a radius from the articular process, and a little obliquely, so that the two spines of the pelvic bones slightly converge. Dorsal fin consisting of about 20 rays, the first one of which is apparently simple or spinous, the others dissected, the 8 or 10 anterior dissected rays being moderately tall and strong and placed in rather close succession, while the posterior are small and well intervalled, the change from larger to smaller rays being apparently quite gradual. The

base of the dorsal fin is subtended on either side by an imbricated series of rather large smooth bony scutes. The exposed parts of these scutes are rhomboids whose two longer sides are directed obliquely forward and downward, as determined by the direction of the anterior scute-border, there being one scute on either side for each dorsal ray. The scutes slope outward and downward and their upper borders are in contact with the dorsal rays, while the lower are nearly on a level with the articulations between those rays and the blade-like interneurals. It is probable that these scutes constituted a sheath into which, in life, the dorsal fin was depressible. Pectoral fin elongate-triangular, reaching nearly half way to beginning of pelvic and to a point nearly under beginning of dorsal, composed of at least 16 rays, of which all are articulated and all, save the first, dissected; pelvic fin abdominal, placed well back of the middle of the torsus and beginning about under posterior limit of dorsal, consisting of one large simple distally articulated ray and 12 or 13 dissected soft rays; caudal fin large, strongly and widely forked; pectoral and pelvic fins subtended (apparently on lower side only, of insertion) by scutes similar to the dorsal ones, those of the pectoral arranged in a partly triseriate group. A double series of scutes, representing the fore-base of the anal fin, begins at a distance of about one and one-fifth times the pelvic fin-length back of the insertion of the first pelvic ray; vertebræ upward of 50, their length and depth subequal, gradually becoming relatively small and short in the anterior region; intermuscular bones numerous, especially above vertebral column; scales small, arranged in 50 or more rows, larger anteriorly than posteriorly, (a few especially large ones on nape?), minutely and closely concentric-striate on the anterior (concealed) part, radiately punctate-rugose on the exposed part, which is thickened with cementum, the posterior border toothed; lateral line short (?), very high up on the flank in the region anterior to the dorsal fin (where alone it has been seen), its distance from the summit there contained about four and a half times in the half girth of the body.

Measurements.—Length (roughly) about two feet; height, exclusive of fins, 121 mm.; tip of snout to insertion of pelvic fin 351; distance between insertion of first ray of pectoral and that of pelvic fin 190 to 195; length of basis of dorsal about 80; length of pectoral, from insertion of first ray to tip of fin, about 95; same of pelvic fin about 52; length of vertebræ in posterior and middle region of body 8.5 to 9, diminishing in cervical region to 4.5 mm. or less. There are usually one and a half to two scales in 6 mm. in the oblique rows of the anterior, and two and a half to four in those of the posterior region. One completely exposed scale on the nape has a diameter of 6 mm.

Occurrence.—Benton stage of the Platte series, in the Fencepost limestone of the Russell substage. All of the known specimens of this fish are from Kansas: those examined in this study being, one from near Bazine. in the eastern part of Ness county, one from near Lincoln Center, and one from an unknown locality probably in or near Russell county. The "Sibley," two miles west of which Prof. Mudge obtained Prof. Cope's type-specimen, is the old Cloud county post-village of Lake Sibley, near the lagoon of that name which is a former northern meander of the Republican river, northwest of Concordia, the present postoffice of Sibley, Kansas, being in Douglas county.

Remarks.—The writer secured the Bazine specimen and that from Lincoln Center,—the former by purchase, the latter as a gift from Mr. W. S. and Mrs. A. C. Wait.—about 1888–1890, for the Museum of Washburn College, to which he is indebted for the use of them. The Russell (?) county example was of a collection personally purchased by the writer about 1895 from the late Mr. Martin Allen, a former State Forester of Kansas and for many years a resident of Hays City, interested in science and much esteemed by those who knew him. The Allen collection is part of the large collection purchased a few years ago by General William J. Palmer and the Colorado Springs Company and by them presented to the Museum of Colorado College.

The following are the parts shown in the three specimens. In the Bazine specimen: the body; a pectoral and a pelvic fin, both practically complete; part of the lower lobe of the caudal fin; an imprint of the dorsal fin; a series of scutes at origin of the anal fin. Faint traces of part of the dorsal scutes can be seen in this and the specimen next mentioned. In the Lincoln Center or Wait specimen: nearly all of the head and body; the pelvic bones, several pelvic scutes and most of a pelvic fin; numerons intermusculars. In the Martin Allen specimen: the entire body; in greater or less part, a dorsal fin, a pectoral fin, and both pelvic fins, with the basal scutes of the two former, and a pelvic bone supporting one of the latter; the clavicle; a part of a lateral line.

Ribs, vertebræ and scales are shown more or less satisfactorily in all three of the specimens. In the dorsal fin, the number of rays preserved is 19; there are doubtful indications of one or two smaller spines in front of the one mentioned, and of an additional soft ray at the posterior end. The condition of preservation of most of the scales is such that their ctenoid character might be overlooked on a casual inspection. The lateral line is displayed, showing the tubes, in but one of the specimens, and in that for only the short distance covered by twelve consecutive scales. On this specimen, its former presence is doubtfully indicated further forward, after an interruption, on two or three scales of the same row.

PELECORAPIS MICROLEPIS sp. nov.

Plate II, fig. 3, and Plate III.

Represented by a specimen which includes most of the trunk and the tail.

Much smaller than *Pelecorapis varius*: the body more elongate and less deep than in that species, compressed, the ventral line and posterior half of dorsal line indicating a fusiform lateral profile: caudal isthmus rather contracted. Of the dorsal fin, only about 10 of the rays are preserved: these slender and apparently soft, representing the anterior part of the fin, which is anteriorly placed forward of the midway point between beginnings of pectoral and anal fins.

and is well elevated, attaining a height equal to about two-thirds of that of the pectoral fin. The pectoral, which is preserved entire, is subinferiorly placed, large, and if in appressed position, would reach more than one-third of the distance from the insertion of its anterior ray to the beginning of the anal fin; it is composed of 16 strong and slender and of about 4 posterior short and feeble rays, of which both the first, which is simple, and the following or dissected rays are articulated. The pelvic fin is not preserved, nor its position indicated. The anal fin is represented by rather a long insertion at the front of which and preceding (or partly embracing?) it are preserved six (more, perhaps originally) overlapping pairs of bony scutes similar to those described as embracing the base of the dorsal fin in *Pelecorapis varius*. The caudal fin, which is large, is strongly forked, much more widely so than that of *Syllæmus latifrons*, its lobes long and narrow, lanceolate, though considerably less slender than those of the *Syllæmus*. Vertebræ estimated to exceed 50; ribs well developed. Scales ctenoid, very small, much smaller than those of *Pelecorapis varius* relatively to size of body, estimated to form about 60 longitudinal rows on either side of the body, there being about 6 scales in 6 mm. in any of the oblique rows on the middle of the flank, glistening, seen under the hand-lens to be ornamented each with three or four coarse, strongly accentuated, concentric undulations on the anterior part and with several less pronounced, longitudinal or subradiate grooves on the posterior part, the posterior margins being toothed. I am unable to detect any trace of a lateral line on the parts on which scales are preserved, but these represent but a small portion of the surface and do not include the region in which the lateral line is known to be developed in *Pelecorapis varius*.

Measurements.—Length from insertion of anterior ray of pectoral to extremity of tail 338; from same to beginning of anal scutes 173; from same to point under insertion of first (or first preserved) ray of dorsal fin about 85; maximum height about 60; length of upper lobe of tail, measured from caudal isthmus, about 90; its breadth, midway of same length,

19; average length of vertebræ in mid-body 6.1 mm. (15 vertebræ included within 82 mm.)

Occurrence. — Benton stage of the Platte Cretaceous series, in the Downs limestone ("Fencepost limestone", or "Postrock") of the Russell substage, near Bazine, Kansas.

Remarks.—The type and only known specimen of this species was submitted to the writer late in the eighties by Mr. Samuel G. Sheaffer, for study and to be placed in the Museum of Washburn College, subject to return, if called for. It has since been recalled by Mr. Sheaffer. On the block containing it, is carved the locality record, "T. 19, R. 22, Sec. 3," together with the name of the probable former owner or collector, "J. C. Long." For the photograph, from which the illustrations have been made, the writer is indebted to the kindness of Mr. Albert A. Blackman.

The species is readily distinguished from *Pelecorapis varius*, as well as from all other known teleosts of the Platte series, by the fineness of its scales, which is such as to give the surface of the body a seedy appearance, the exposed parts of the scales being only about as large as flax seeds, several of them together covering only the area of a single scale of the *varius*.

EXPLANATION OF PLATES.

PLATE I.

Syllæmus latifrons Cope.

Figure 1. Provisional diagram of head, natural size, showing approximate arrangement of the bones visible from above. (See comments on same, under *Remarks* on the species, page 30.) Explanation of reference letters in the figure: *so*, supraoccipital; *i-so*, interparietal portion of supraoccipital; *p*, parietal; *epo*, epiotic; *pto*, pterotic; *sph*, sphenotic; *f*, frontal; *me*,* mesethmoid; *pe*, parethmoid; *mx*, maxillary; *pmx*, premaxillary; *op*, upper part of opercular apparatus.

Figure 2. Anterior border of premaxillaries, enlarged, showing three of the teeth.

Figure 3. Plan of the dorsal fin, natural size.

Figure 4. Elevation of the dorsal fin, natural size; the posterior part restored in supposed approximate outline, the anterior lobe drawn from an imprint which is practically complete in the specimen, lacking only a narrow zone at base.

Figure 5. A group of scale-prints, across the lateral line, natural size.

Figure 6. Caudal fin, natural size, the outline in part restored; made from a considerable part of the fin and an imprint of most of the remainder.

*This lettering, which refers to the median shaded bone in advance of the frontals, has been accidentally omitted from the plate.

PLATE II.

Figure 1. Greater portion of lateral view of head of *Pelecorapis varius* Cope, natural size. This and the following figure have been drawn by Mrs. Dr. J. C. Shedd, after sketches by F. W. Cragin.

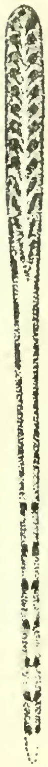
Figure 2. Inferior side of left pelvic bone of *Pelecorapis varius*, restored, natural size; the restoration based on a right and left in the Wait specimen and a right in the Allen; with base of fin.

Figure 3. Scales from mid-flank of *Pelecorapis microlepis*, sp. nov.; enlarged to natural size from the Blackman photograph. See Plate III.

PLATE III.

Type-specimen of *Pelecorapis microlepis*, sp. nov., one-half natural size. From a photograph by Mr. Albert A. Blackman.

[Note.—The writer takes this opportunity to make an acknowledgment, omitted by oversight from Volume VIII of the Studies. The three figures of *Sphenodiscus serpentinus* in that volume (Plate II, figs. 4 to 6) are from drawings very kindly made in 1897 by Mr. Robert T. Walker, Ph. B., of the Colorado College class of 1900.]



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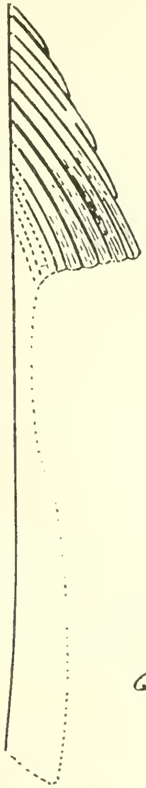

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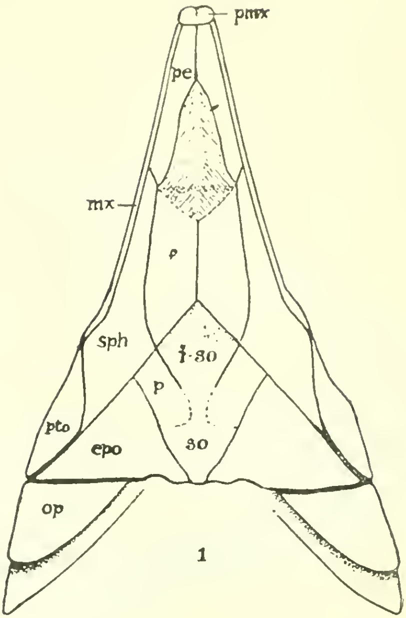

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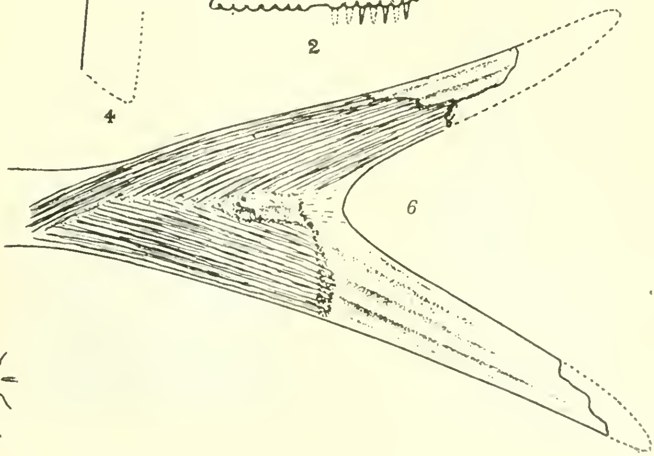

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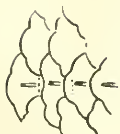

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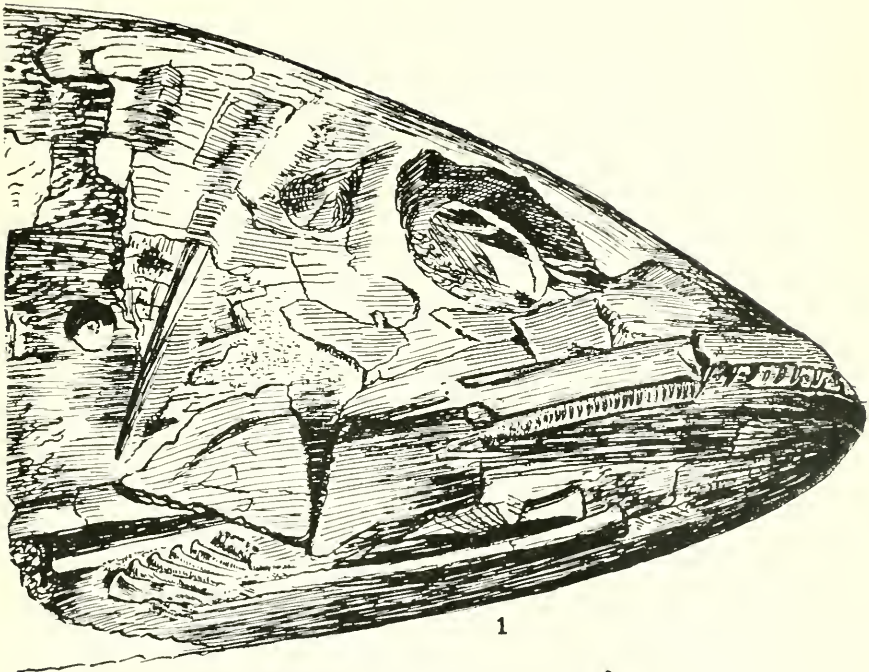

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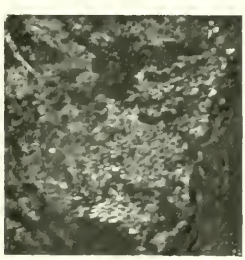

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